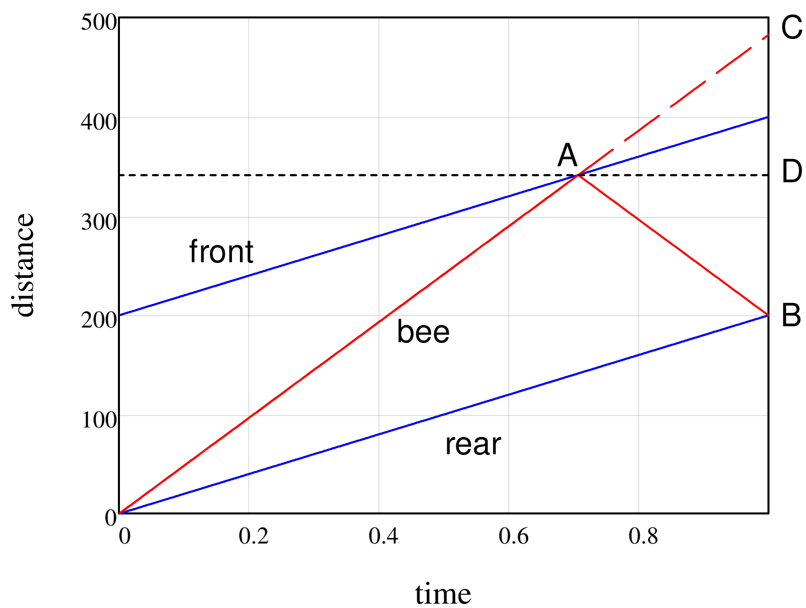
A freight train 200m long travels along a straight track at constant speed. In the time it takes the train to move a distance equal to its own length, a bee, initially sitting at the rear of the train, flies forward to the front of the train, turns instantly and returns to the rear, flying at constant speed all the while. What is the total distance travelled by the bee?

On the distance-time graph below the blue lines represent the front and back of the train, the solid red line represents the path of the bee, and the dotted red line represents the path the bee would have taken if it hadn't turned back. (Note: all speeds are relative to the ground.)



Suppose the train is travelling at 200 metres/minute. After 1 minute the front of the train is 400 metres from where the bee started, while the rear is 200 metres from the start.

The bee must be travelling faster than the train (it's a high-speed bee!) and it reaches the front of the train at point A in the above diagram. If it didn't turn back at that point it would carry on to point C at the end of one minute.

It does turn back, but continues ***at the same speed relative to the ground*** to point B (which we are told is the rear of the train i,e. 200 metres from the start if the train is travelling at 200 metres per minute).

Because the speed (not velocity) of the bee is constant it travels back from A to B the same distance it would have travelled forward from A to C if it hadn't turned back. That is, the distance DB equals the distance DC.

This means the total distance travelled by the bee in 1 minute is the distance from the start to A plus the distance from A to B which is the same as the distance from the start to A plus the distance from A to C.

Without doing any calculations we can see that the point C is further away from the start than is the front of the train. Since the front of the train is 400 metres from the start, C is more than 400 metres from the start so the bee must have travelled further than 400 metres.

Numerically, it is easy to determine that point A occurs at time $\frac{\sqrt{2}}{2}$ minutes from the start and is a distance $200\cdot\left(1+\frac{\sqrt{2}}{2}\right)$ metres from the start.

(*Equate distances when bee reaches front of train at time τ:*

$$\text{speed}_{\text{bee}}\cdot\tau = 200 + 200\cdot\tau \qquad (1)$$

Equate gradient magnitudes:

$$\frac{\text{speed}_{\text{bee}}\cdot 1}{1} = \frac{\left(\text{speed}_{\text{bee}}\cdot\tau - 200\right)}{1-\tau} \qquad (2)$$

Two independent equations for two unknowns)

Since B is 200 metres from the start the total distance travelled by the bee is given by

$$200\cdot\left(1+\frac{\sqrt{2}}{2}\right) + \left[200\cdot\left(1+\frac{\sqrt{2}}{2}\right) - 200\right] \rightarrow 200\cdot\sqrt{2} + 200 \text{ metres}$$

or approximately 482.8 metres.

The result is, of course, independent of the speed of the train; the assumption above was made purely to allow the use of simple numbers. Any other speed for the train would produce the same end result.