Epicyclic Motion Analysis for API M8 Bullet Firing Sidewise from a High Subsonic Air Vehicle

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Abstract

The present study investigates the epicyclic pitching and yawing motion phenomenon of spinning symmetric projectiles applying the 6-DOF free flight dynamic simulation modelling for small-yaw flat-fire trajectories and compares it with the corresponding classical second order linear differential solution. The ammunition will be used is the high spin-stabilized .50 API M8 bullet type firing from M2 machine gun launched horizontally from different firing angles on high-speed subsonic aircraft. This analysis includes constant coefficients of the most significant aerodynamic forces, moments and Magnus effects, which have been taken from official tabulated databases, in addition to gravity acceleration.

Keywords: Bullet Projectile Type, Epicyclic Pitching and Yawing Motion, Trajectory Dynamics Simulation, Symmetric Projectiles, Subsonic Aircraft.

1. Introduction

External ballistics (see for example McShane et. al. 1953; McCoy 1999; Gkritzapis & Panagiotopoulos 2008) deals with the behavior of a non-powered projectile in flight. Many forces act upon the projectile during this phase including gravity, air resistance and air density.

Pioneering English ballisticians Fowler, Gallop, Lock & Richmond (1920) constructed the first rigid six-degree-of-freedom projectile exterior ballistic model. The trajectory deflection from the aircraft on a .30 caliber machine gun bullet was first addressed in 1943 by T. E. Sterne (1943). This reference was written of mathematicians who were tasked with generating firing tables for the machine guns, used sidewise fire from high speed airplanes.

The present work addresses the application of the full six degree of freedom (6-DOF) projectile flight dynamics modelling to the accurate trajectory prediction of high spin-stabilized bullets firing sideways at different angles (30°, 60° and 90° test cases) from a high subsonic air vehicle. It takes into consideration the influence of the most significant forces and moments, based on appropriate constant mean values of the aerodynamic coefficients. The angular oscillatory motion of the 6-DOF simulation with initial small yaw firing angles and flat-fire trajectory motions is compared with the corresponding classical second order linear differential solution for high-spin spinning projectiles of the 12.95 mm M8 API bullet projectile type.

2. Projectile Model

The M8 API (armor piercing incendiary) was put into service in 1943 to replace the M1 Incendiary, and is still in service today. The M8 is built nearly identical to the M2 Armor Piercing except the M8 has 12 grains of incendiary mix in the nose instead of lead filler, and a lead caulking disc in the base acting as a seal. Having the same hardened steel core as the M2, the M8 matches the armor piercing capability of the M2 with the added advantage of incendiary effect. While it has considerably less incendiary mix than the M1, the performance of the M8 was greatly superior to the M1 because of its ability to penetrate the target and ignite the material inside rather than just flash on the surface like the M1 often did, making for a greater first-shot effect. Pyrotechnic performance of these projectiles is only slightly less than the M1 Incendiary. The present analysis considers this type of representative flight bullet vehicle.

Physical and geometric characteristics data of the above mentioned 12.95 mm M8 API bullet and Browning M2 .50 Caliber machine gun are illustrated in Table 1 and Figure 1, respectively.

Characteristics	API M8 Bullet
Reference diameter D, mm	12.95
Total length L, mm	57.76
Mass m, kg	0.0419
Center of gravity $x_{C.G.}$, mm	23.18 (from the base)
Axial moment of inertia, kg·m^2	$0.7843 \cdot 10^{-6}$
Transverse moment of inertia, kg·m^2	$0.7389 \cdot 10^{-5}$

TABLE 1. Physical and geometrical data of 12.95 mm API M8 bullet

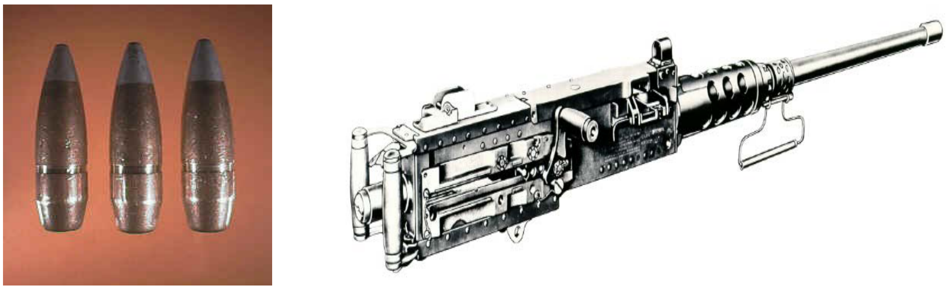


FIGURE 1. .50 Caliber M8 API and Browning M2 .50 Caliber machine gun

3. Trajectory Flight Simulation Model

The projectile can be modelled as a rigid body possessing six degrees-of-freedom (6-DOF) including three inertial position components x, y and z of the system mass centre as well as the three Euler orientation angles φ, θ and ψ (see for example Amoruso 1996; Hainz & Costello 2005).

Two mean coordinate systems are used for the computational approach of the atmospheric flight motion. The one is a plane fixed (inertial frame, IF) at the ground surface, whose center O1 lies on the projection of the firing point onto ground surface, as depicted in Figure 2. The other is a no-roll rotating coordinate system that is attached to, and moving with, the projectile's center of mass O2 (no-roll-frame, NRF, φ = 0) with X_{NRF} axis along the projectile's axis of revolution positive from tail to nose (Figure 2). Y_{NRF} axis is perpendicular to X_{NRF} lying in the horizontal plane. Z_{NRF} axis is oriented so as to complete a right-hand orthogonal system.

Newton's laws of motion state that rate of change in linear momentum must equal to the sum of all the externally applied forces and the rate of change in angular momentum must equal to the sum of all the externally applied moments, respectively. The force acting on the projectile comprises the weight, the aerodynamic forces and the Magnus force. On the other hand, the moment acting on the projectile comprises the moment due to the standard aerodynamic force, the Magnus aerodynamic moment and the unsteady aerodynamic moment.

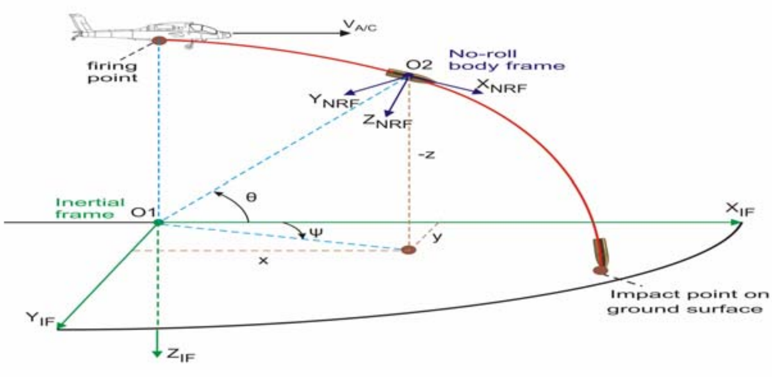


FIGURE 2. No-roll (moving) and fixed (inertial) coordinate systems for the atmospheric flight trajectory of flat-firing projectiles from a high-speed aircraft vehicle.

The twelve state variables x, y, z, φ, θ, ψ, $\hat{u}$, $\hat{v}$, $\hat{w}$, $\hat{p}$, $\hat{q}$ and $\hat{r}$ are necessary to describe position, flight direction, linear and angular velocities at every point of the projectile's atmospheric trajectory. Introducing the components of the acting forces and moments expressed in the no-roll-frame ($\hat{*}$) with the dimensionless arc length l expresses the projectile downrange travel in calibers, as an independent variable

$$l = \frac{1}{D} s = \frac{1}{D} \int_0^t V dt \tag{3.1}$$

the following modified linear equations of motion are derived (see Hainz & Costello 2005):

$$\frac{dx}{dl} = \frac{D}{V}\left(\cos\theta\,\hat{u} - \psi\,\hat{v} + \sin\theta\,\hat{w}\right) \tag{3.2}$$

$$\frac{dy}{dl} = \frac{D}{V}\left(\psi\cos\theta\,\hat{u} + \hat{v} + \psi\sin\theta\,\hat{w}\right) \tag{3.3}$$

$$\frac{dz}{dl} = \frac{D}{V}\left(-\sin\theta\,\hat{u} + \cos\theta\,\hat{w}\right) \tag{3.4}$$

$$\frac{d\varphi}{dl} = \frac{D}{V}\left(\hat{p} + \tan\theta\,\hat{r}\right) \tag{3.5}$$

$$\frac{d\theta}{dl} = \frac{D}{V}\hat{q} \tag{3.6}$$

$$\frac{d\psi}{dl} = \frac{D}{V\cos\theta}\hat{r} \tag{3.7}$$

$$\frac{d\hat{u}}{dl} = -\frac{D}{V} g \sin\theta - \frac{\rho S_{ref} D}{2m} C_D V \tag{3.8}$$

$$\frac{d\hat{v}}{dl} = -\frac{\rho S_{ref} D}{2m}\left(C_{LA} + C_D\right)\left(\hat{v} - \hat{v}_w\right) - D\hat{r} \tag{3.9}$$

$$\frac{d\hat{w}}{dl} = -\frac{\rho S_{ref} D}{2m}\left(C_{LA} + C_D\right)\left(\hat{w} - \hat{w}_w\right) + D\hat{q} + \frac{D}{V} g\cos\theta \tag{3.10}$$

$$\frac{d\hat{p}}{dl} = \frac{\pi\rho D^5}{16 I_{YY}} C_{RD}\hat{p} \tag{3.11}$$

$$\begin{aligned}\frac{d\hat{q}}{dl} = {} & \frac{\pi\rho D^4}{16 I_{YY} V} L_{CGCM} C_{MaM}\hat{p}\left(\hat{v} - \hat{v}_w\right) + \frac{\pi\rho D^3}{8 I_{YY}} L_{CGCP}\left(C_{LA} + C_D\right)\left(\hat{w} - \hat{w}_w\right) + \\ & + \frac{\pi\rho D^5}{16 I_{YY}} C_{PD}\hat{q} + \frac{\pi\rho D^4}{8 I_{YY}} C_{OM} - \frac{I_{XX}}{I_{YY}}\frac{D}{V}\hat{p}\hat{r}\end{aligned} \tag{3.12}$$

$$\begin{aligned}\frac{d\hat{r}}{dl} = {} & \frac{\pi\rho D^4}{16 I_{YY} V} L_{CGCM} C_{MaM}\hat{p}\left(\hat{w} - \hat{w}_w\right) - \frac{\pi\rho D^3}{8 I_{YY}} L_{CGCP}\left(C_{LA} + C_D\right)\left(\hat{v} - \hat{v}_w\right) + \\ & + \frac{\pi\rho D^5}{16 I_{YY}} C_{PD}\hat{r} - \frac{\pi\rho D^4}{8 I_{YY}} C_{OM} - \frac{I_{XX}}{I_{YY}}\frac{D}{V}\hat{p}\hat{q}\end{aligned} \tag{3.13}$$

where L_{CGCP} and L_{CGCM} are the distances of the center of gravity (CG) from the center of pressure (CP) and center of Magnus (CM), respectively. Equations (3.2–3.13) form the modified linear theory equations of motion for a projectile and must be numerically integrated to estimate a trajectory. The following set of simplifications was employed for the previous flight dynamic model:

1) The station line velocity $\hat{u}$ and roll rate $\hat{p}$ are large in relation to the side velocities $\hat{v}$ and $\hat{w}$, yaw angle ψ, pitch and yaw rates $\hat{q}$ and $\hat{r}$, and wind velocity components $\hat{v}_w$ and $\hat{w}_w$. Products of small values and derivatives of small values are treated as negligible.

2) The yaw angle ψ is small, allowing the simplifications

$$sin\psi \approx \psi \qquad cos\psi \approx 1 \tag{3.14}$$

3) The aerodynamic angles of attack α and sideslip β remain small (< 15°) for the main part of the atmospheric trajectory (see for example Hainz & Costello 2005):

$$a \approx \frac{\hat{w} - \hat{w}_w}{V} \qquad \beta \approx \frac{\hat{v} - \hat{v}_w}{V} \tag{3.15}$$

4) The Magnus force components are small in comparison with the weight and aerodynamic force components, and so they are treated as negligible in the modified linear theory computations. The Magnus force does create a non-negligible moment, and so it is maintained in the moment computations.

5) The projectile is geometrically symmetrical about the station line. This allows the inertia matrix to be simplified as

$$I_{XY} = I_{YZ} = I_{XZ} = 0 \qquad I_{YY} = I_{ZZ} \tag{3.16}$$

6) The projectile is aerodynamically symmetric.

7) The wind velocity component $\hat{u}_w$ parallel to the projectile station line is negligible in comparison to the projectile total forward velocity.

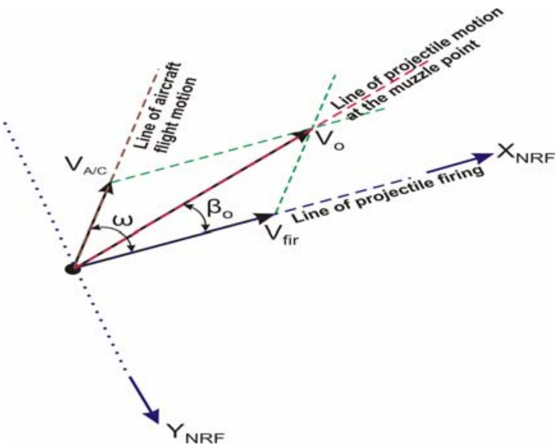


FIGURE 3. Top view of the initial muzzle velocity at the firing point from M2 automatic gun placed on aircraft's structure

8) The station line velocity $\hat{u}$ is replaced by the projectile total velocity V as

$$V = \sqrt{\hat{u}^2 + \hat{v}^2 + \hat{w}^2} \approx \hat{u} \tag{3.17}$$

9) The total initial muzzle velocity of the projectile V_o firing sidewise with V_{fir} at an angle ω relative to the helicopter's flight path motion $V_{A/C}$ (Figure 3), is

$$V_o = \sqrt{V_{fir}^{\ 2} + V_{A/C}^{\ 2} + 2\, V_{fir}\, V_{A/C}\, cos\,\omega} \tag{3.18}$$

3.1 Initial Spin Rate Estimation

In order to have a statically stable flight projectile trajectory motion, the initial spin rate $\hat{p}_o$ prediction at the gun muzzle in the firing site is very important. According to McCoy (1999) definitions, the following form is used:

$$\hat{p}_o = \frac{2\pi V_o}{\eta D} \quad \left(rad/s\right) \tag{3.19}$$

where V_o is the initial firing velocity (m/s), η the rifling twist rate at the $M2$ gun muzzle (calibers per turn), and D the reference diameter of the bullet type (m), see Table 1. Typical value of rifling η for the .50 caliber API M8 bullet type is 29.41 cal/turn.

3.2 Aerodynamic Model

For the projectile trajectory analysis, a constant flight dynamic model is proposed for the examined test cases. The above calculations are based on appropriate constant mean values of the experimental average aerodynamic coefficients variations C_D, C_{LA}, C_{MaM}, C_{PD}, C_{OM} and C_{RD} taking from official tabulated database McCoy (1990), as shown in Table 2.

Aerodynamic Coefficients	**API M8 Bullet**
Drag force C_D	0.63
Lift force C_{LA}	3.52
Magnus moment C_{MaM}	0.27
Pitch damping moment C_{PD}	-6.6
Overturning moment C_{OM}	2.6
Roll damping moment C_{RD}	-0.009

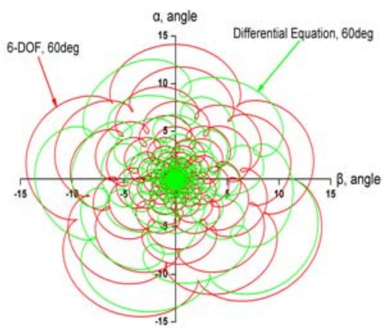
TABLE 2. Aerodynamic parameters of the applied atmospheric flight dynamic model

4. Computational Results

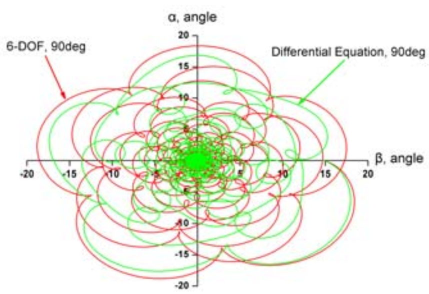
The flight dynamic model of 12.95 mm M8 API (see for example Gkritzapis et. al. 2007; Gkritzapis et. al. 2008) type involves the solution of the set of the twelve nonlinear first order ordinary differential, Eqs (3.2-3.13), which are solved simultaneously by resorting to numerical integration using a 4th order Runge-Kutta method and the solution of the second order differential equation of motion.

Initial flight conditions for the dynamic trajectory bullet model with constant aerodynamic coefficients are illustrated in Table 2, assuming different firing angles ω = 30°, 60° and 90° with the supersonic firing velocity of $V_{fir} = 900\ m/s$ relative to the aircraft's high subsonic flight motion of almost $V_{hel} = 230\ m/s$ (Mach number flight M = 0.67).

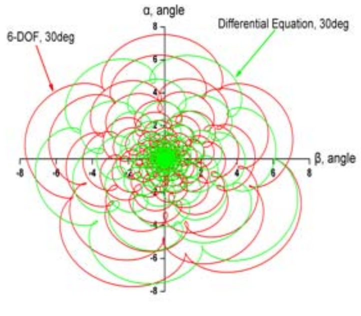
The angle of attack α versus angle of sideslip β flight path trajectory motions with the numerical 6-DOF analysis (red color lines) and classical analytical differential equation (green color lines) are indicated in Figure 4a for the 12.95 mm bullet with initial firing velocity of 900 m/s, firing sidewise at 30 deg with aircraft's velocity 230 m/s. The produced initial yaw (or sideslip) angle is approximately 6 deg. The corresponding initial yaw angle for firing sidewise at 60 deg relative to the aircraft's velocity flight path is approximately 11.2 deg (Figure 4b). Firing sidewise at 90 degrees (perpendicular) of the aircraft's flight path motion gives initial yaw angle at about 14.7 deg, as shown in Figure 4c.



(a)

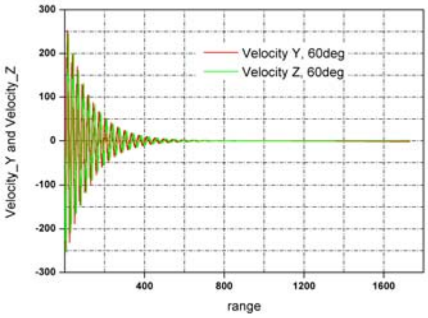


(b)

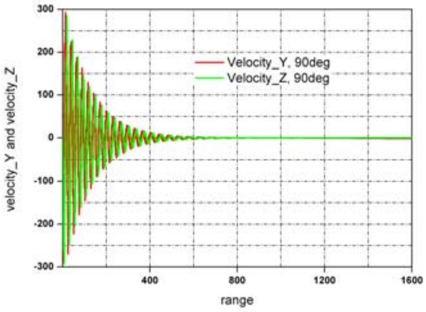


(c)

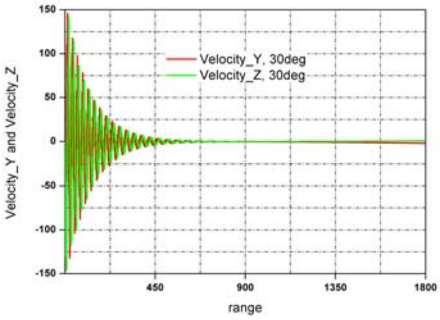
FIGURE 4. Coupled pitching and yawing motion for the .50 API M8 bullet fired sidewise at different angles ω from a high subsonic air vehicle flying at 230 m/s.



(a) 30 deg



(b) 60 deg



(c) 90 deg

FIGURE 5. Cross velocities in Y and Z directions (m/s) versus range (m) for the .50 API M8 bullet fired sidewise at different angles ω from a high subsonic air vehicle flying at 230 m/s

The side velocities in Y and Z directions versus range are also investigated for the examined 12.95 mm bullet diameter, using the numerical 6-DOF flight motion analysis firing sidewise from the aircraft's machine gun at angles 30, 60 and 90 deg, as indicated in Figures 5a, 5b and 5c, respectively.

5. Conclusions

The epicyclic pitching and yawing motion of the 6-DOF flight dynamic modelling for flat-fire free flight trajectories is compared with the corresponding classical analysis by using the linear second order differential equation of external motion. The above are applied for the prediction of spin-stabilized projectile trajectories, as the 12.95 mm M8 API bullet projectile type, launched horizontally (from different firing angles) from high-speed subsonic airplanes with constant aerodynamic force and moment coefficients based on official tabulated database for the examined bullet.

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