

Historical Data in Modelling Warships Battle Damage Survival Probability

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ABSTRACT

When modelling or otherwise analysing naval combat one needs a means of assessing the damage done or the probability of survival when a ship is hit by ordnance that hits and detonates. This can be done by detailed modelling of the target structure and systems, the distribution of hits and the effects of hits on the structure. This is a time consuming process and its validity is limited by our lack of detailed knowledge of the arrangements of many targets of interest. An alternative approach, particularly suited for historical analysis, is to use the results of damage inflicted by a particular type of ordnance on a class of targets of broadly similar characteristics. It is this latter approach that is pursued in this study.

Here I investigate the probability of some identifiable level of damage (say mission kill or sinking damage) as a function of target size and a measure of the quantity of ordnance (in particular bombs or torpedoes in the context of WW2) that has hit it. The general procedure is to find the parameters for the probability model using a numerical maximum likelihood estimate. The particular examples given here are of aircraft carrier mission kill damage from bombs in the Pacific campaign of WW2, and of Royal Navy cruisers and sinking damage from torpedoes also in WW2.

1. INTRODUCTION

In this note I wish to discuss simple models of the probability of weapon hits rendering a ship a mission kill (or inflicting some other level of damage). We desire a relatively simple model suitable for top level or rough and ready battle modelling. There is some literature on modelling at this level [1][2][3]. These authors use historical data from naval actions to derive their models, this is their strong suit. However the methods used to derive their models seem unsatisfactory in some respects and some of the dependencies seem implausible. These problems are dealt with in more detail in section 2 below.

In what follows I present what I regard as a suitable generic model of the relationship between the yield of ordnance that a vessel is hit by, its displacement and the probability of it suffering mission kill. The parameters of the model are then fitted to the historical data numerically by maximising the data's likelihood.

2. REVIEW OF SOME PREVIOUS MODELS

Beall [2] develops a model of the quantity of ordnance (measured in Thousand Pound Bomb Equivalents TPBE) required to render a ship mission killed, this is a power law of the ship's displacement. The exponent in the power law is derived by looking at plots of mission killed platform's displacement against yield of weapons causing the damage. This is not an ideal method as there is at least as much if not more information to be gleaned from those platforms that were hit but survived. For example Beall's data set includes *Warspite* at Jutland which is judged to have been mission killed. However how the comparable damage to *Tiger* also at Jutland is less relevant (judged not mission killed) is difficult to see [7]. Beall also mixes data from armoured and un-armoured ships and aircraft carriers while using the total energy of the hits to measure the level of attack. I feel this is unjustified since the difference between hits that penetrate and those which do not penetrate the protective schemes of armoured ships is important and this is correlated with size of projectile. Also the damage-causing mechanism of torpedo hits may well be different from that of shells and bombs and not adequately represented as a multiplier on warhead yield. So damage from such different types of ordnance should if possible be isolated in any analysis. Finally I find the conclusion that staying power (in this context this is the amount of ordnance that the ship can take without becoming a kill) is proportional to the cube root of displacement (with the same coefficient of proportionality) for all types of man-of-war and all types of attack implausible.

A-priori I would expect the staying power to be directly proportional to displacement if the mechanism of mission kill requires the destruction of volume. The corresponding volume is directly proportional to the warhead yield (Washburn and Kress tell us that if the warhead causes damage primarily by overpressure then the maximum distance from the detonation point at which some critical level of damage occurs is proportional to the cube root of the warhead yield, and so the affected volume is proportional to the mass of explosive). If the mechanism is area destruction I would expect staying power to be proportional to the two thirds power of displacement (possibly the mechanism of carriers with hangars which are super structure rather than part of the hull or even armoured). Finally if the mechanism is linear destruction I would expect staying power to be proportional to the one thirds power of displacement (this may be relevant in some circumstances to torpedo

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damage as reported in Kimball and Morse for merchant shipping in the Battle of the Atlantic). In reality a combination of mechanisms may be at work so intermediate values of exponents would not be surprising, Less likely but still not completely implausible would be exponents outside the range above if the damage mechanism changes with ship size, say making larger vessels proportionately more or less vulnerable.

Humphrey[3] uses the same data set as Beall but models things differently, trying to determine the probability of mission kill from the attack yield. His model gives a similar cube root of displacement dependence when the staying power is characterised by the ordnance yields required for a 50% probability of mission kill. Other than the implausibility to the present author of this as a global conclusion I cannot say much more about Humphrey's model as I have been unable to locate the reference and am relying on accounts from Hughes[1] who mentions the above but concentrates more on Beall's results.

3. PROPOSED MODEL STRUCTURE

There are a number of approaches to modelling the probability of mission kill. The first involves detailed modelling of the platform structure and systems and their reaction to a warhead detonating at some point within the platform or close by. This might be suitable for assessing the survivability of a platform in design but is not suitable for the purposes that I am interested in. A higher level model might suppose that a platform consists of a volume and a number of vulnerable point features, and that a warhead will destroy all such features within some radius related to its size. I will call this a *Type A model*. Another type of model might proceed along the lines that we do not know anything about the mechanism by which a weapon renders a platform a mission kill but we do know that it is some function of the size of the platform and the size of the warhead which is zero for a zero sized warhead and goes to one as the warhead size increases without bound. Similarly the probability goes to zero as the size of the platform increases without bound and is unity for a platform of zero size. I will call the particular form of such a dependence that I use here a *Type B Model*. In what follows I will pursue *Type B Models* since it is more generic and we may observe that *Type A Models* produce very similar results.

Type B Models

Here we seek a functional form to represent the probability of a kill (mission or sinking damage depending on context) p_{mk} that goes to 0 when the weapon yield v goes to zero, and to 1 when the yield becomes large. Similarly for fixed non-zero weapon yield we want p_{mk} to go to 1 as the platform displacement D goes to zero, and to 0 as the displacement becomes large. The form that we use here is:

$$p_{mk} = 1 - \exp\left(-k \frac{v^\alpha}{D^\beta}\right)$$

which gives the yield for a 50% probability of mission kill of:

$$v_{0.5} = \left[-\frac{\log_e(0.5)}{k}\right]^{1/\alpha} D^{\beta/\alpha}$$

which may be interpreted as staying power is proportional to displacement to the power β/α .

4. MODEL FITTING PROCESS

In an ideal Bayesian world we would have a model of the probability of mission kill with unknown parameter vector Θ but known to represent reality and data (v_i, r_i, D_i) where v_i is the yield of the ordnance which has hit platform i of displacement D_i with result $r_i = 0$ if the platform is not mission killed and $r_i = 1$ if it is. Then we would want to calculate the posterior distribution of Θ given the data and our prior for Θ . If we want a point estimate of Θ we could choose the Θ which maximises the posterior distribution. The posterior distribution for Θ is given by Bayes' rule:

$$P(\Theta|data) = \frac{P(data|\Theta)P(\Theta)}{P(data)}$$

where:

$$P(data|\Theta) = \prod_i (r_i p_{mk}(v_i, D_i, \Theta) + (1 - r_i)(1 - p_{mk}(v_i, D_i, \Theta)))$$

and:

$$P(data) = \int P(data|\Theta)P(\Theta) d\theta$$

is a constant dependent only on *data* and the prior $P(\Theta)$.

To simplify things if we use a uniform prior for Θ over a sufficiently large volume of parameter space we are left with the posterior probability being proportional to the likelihood $P(data|\Theta)$ inside that volume. Now I take as my point estimate the maximum likelihood estimator for Θ . This is what we will use from now on.

For our convenience we use non-linear optimisation software to find the maximum of the posterior probability or likelihood. Initially I used the non-linear optimiser in the solver tool of the spreadsheet Gnumeric. This appears to handle this problem somewhat better than the similar tool in Excel, later I converted to using the Nelder-Mead algorithm from a matrix language system which produces identical results to the Gnumeric non-linear solver.

5. CARRIER DAMAGE FROM BOMBS IN THE PACIFIC DURING WORLD WAR TWO

To illustrate the method of estimating “staying power” (the number of standard weapon hits to render a platform a mission kill or alternatively sunk) we need a data set of number of and type of hits on more or less similar platforms differing principally in displacement. A data set that sort of meets these requirements is that of carrier damage from bombs in the Pacific in World War 2. Here we will assume that the carriers are subject to the same vulnerabilities and that the state of damage control was not relevant for medium term mission kill. Also I will assume that the TPBE of multiple hits is the sum of the TPBE of the individual weapons. Table 1 below shows the data.

ship	battle	disp (tons)	TPBE	result, 1=disabled
Soho	Coral Sea	14200	2	1
Shokaku	Coral Sea	32105	2	1
Lexington	Coral Sea	47700	0.8	0
Yorktown	Coral Sea	25484	0.4	0
Kaga	Midway	43650	4	1
Soryu	Midway	19800	3	1
Hiryu	Midway	21900	4	1
Akagi	Midway	42750	2	1
Yorktown	Midway	25484	1.2	0
Ryujo	Eastern Solomons	13650	4	1
Enterprise	Eastern Solomons	25484	1.6	0
Zuiho	Santa Cruz	14200	1	1
Shokaku	Santa Cruz	32105	4	1
Hornet	Santa Cruz	25484	1.6	1
Enterprise	Santa Cruz	25484	0.8	0

Table 1: Carrier Damage from bomb hits in the Pacific in WW2 (TPBE denotes the quantity of ordnance ship hit by in units of Thousand Pound Bomb Equivalent)

Some of the data in Table 1 may look unfamiliar, some of the results are based on an interpretation of narratives of the battle and a judgment call as to the result of the bombing alone at a particular stage of the battle. Note: All displacements are from [5] other than for Lexington for which there is inconsistency between sources. Also the thousand pound bomb equivalent of the hits is the sum of the explosive yields (TNT equivalent) of the ordnance divided by that of the TNT equivalent of a 1000 pound US heavy case bomb (660 pounds)[2], there is some ambiguity here in the identification of model of Japanese 250 kg anti-ship bomb used and so its explosive content.

Figure 2 shows a plot of the data from Table 1 and plots of the lines corresponding to 0.75, 0.5 and 0.25 probability of mission kill for the maximum likelihood *Type-B Model*. These show that the model does a reasonably good job in identifying regions of the plot where the platforms in Table 1 suffered mission kill and regions where they survived. However the trained eye can see that a variety of other lines on this plot would do an equally good job separating the data points.

The results of running the log-likelihood maximising process over a Type-B Model with the ratio of β/α taking a prescribed value are shown in Figure 1, these show that the log likelihood is maximised for a ratio of β/α of about 0.7. This could be interpreted as indicating that it is destruction over area rather than volume or length that is critical for mission kill. However the variation in the maximum likelihood in Figure 1 is relatively low and so caution should be used in coming to any very firm conclusion about the exponent ratio. To produce a firmer estimate of the exponent ratio would require a larger data set, if possible of higher quality.

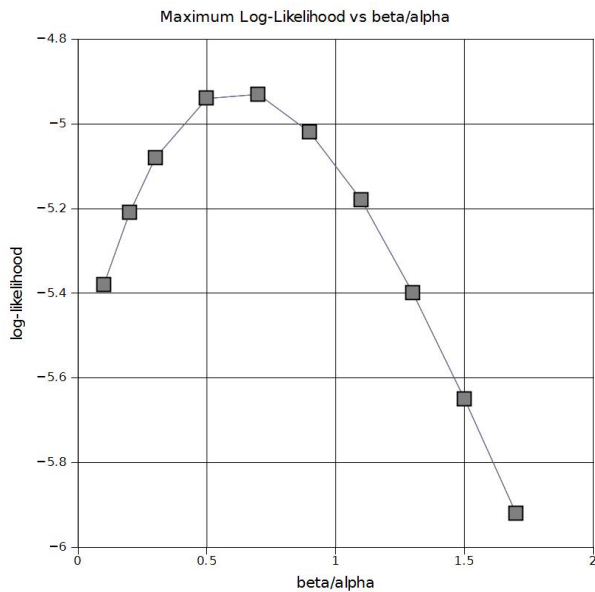


Figure 1: Plot of the maximum of the log-likelihood as a function of β/α (the exponent of displacement in the expression for the TPBE for 50% probability of kill) for the Carrier Data Set

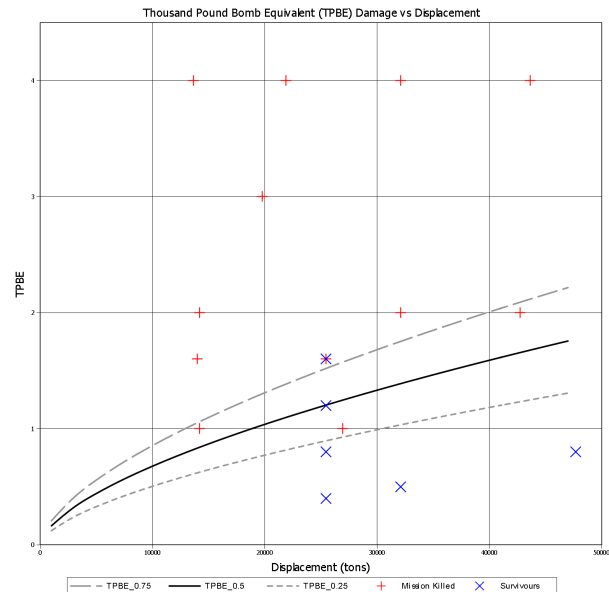


Figure 2: Plot of the Data from Table 1 and Lines from the Maximum Likelihood Solution for a Type-B Model Indicating the TPBE for Probability of Mission Kill of 0.75, 0.5 and 0.25 for the Carrier Data Set

6. ROYAL NAVY CRUISER DAMAGE FROM TORPEDOES IN WORLD WAR TWO

As a second illustration of the use of fitting historical data to a *Type-B Model* I will examine torpedo damage to RN cruisers in World War 2, mainly because this data is reasonably easy to find on the Internet in individual ship histories [8]. In this case we will divide the results of a hit (by a torpedo that detonates) into ship damaged (usually severely) and ship sunk since for torpedo hits on cruiser sized ships damage less than mission kill from a torpedo is rare. The data for RN cruisers is shown in table 2, where the TPBE is the total explosive yield of all the hits in Thousand Pound Bomb Equivalents. The main part of table 2, above the first broken line, is the basic cruiser data, below that is the extra data that allows an extension of the range of displacements first by including the Abdiel cruiser mine-layers, and secondly by including RN carriers.

Figure 3 shows the plot of the maximum of the log-likelihood for the Cruisers+Abdiels data set for constrained β/α which shows the maximum around $\beta/\alpha = 0.66$, and Figure 4 shows the plot of the data set and the lines corresponding to 0.75, 0.5 and 0.25 probability of the ship sinking which seem to separate the regions where there is a high probability of the target sinking from that where the probability of sinking is low.

The result of minimising the likelihood for a type B model for the strict cruiser data set has a value of $\beta/\alpha = 1.33$. This is possibly misleading because of the limited range of displacements in the data set. Adding in the Abdiel cruise-minelayers *Manxman* and *Welshman* changes this ratio for the maximum likelihood model to $\beta/\alpha = 0.66$ and if pressed this is the value that I would use. Adding in the Carriers gives $\beta/\alpha = 0.32$, but even if the carriers were sufficiently similar to the cruisers for this to be a valid process this data set is dominated by the loss of *Ark Royal* to a single torpedo.

7. COMPARISON WITH PREVIOUS WORK

The previous work most comparable with this current paper is Humphrey's [3], at least that would seem to be the case from what Hughes [1] writes since so far I have been unable to locate copies of Humphrey's original papers.

If we look at table 6.1 of [1] we may compare the 1.6 TPBE of bombs for a 50% probability of mission kill on a 15000 ton ship attributed to Humphrey's with the ~0.75 TPBE of bombs found here for a 50% probability of mission kill on a similar sized carrier. A argument could be made that the data base used by Humphrey contains many ships more robust than carriers so it is not surprising that his model requires more ordnance to achieve the same effect as the present model.

ship	battle	disp (tons)	TPBE	result, 1=Sunk
Leander	7/43	9100	1.60	0
Perth	3/42	9000	6.40	1
Hobart	6/43	9000	1.06	0
Sydney	11/41	9000	1.21	1
Arethusa	11/42	6700	0.70	0
Galatea	12/41	6700	1.21	1
Penelope	2/44	6700	2.42	1
Birmingham	11/43	11350	1.21	0
Glasgow	11/40	11350	1.39	0
Newcastle	6/42	11350	1.21	0
Liverpool	10/41	11650	0.70	0
Liverpool	6/42	11650	0.70	0
Manchester	7/41	11650	0.70	0
Manchester	8/42	11650	0.70	1
Edinburgh	5/42	13175	3.64	1
Bonaventure	3/41	6850	1.08	1
Hermione	6/42	6850	1.21	1
Naiad	3/42	6850	1.21	1
Phoebe	8/41	6850	0.70	0
Phoebe	10/42	6850	1.21	0
Charybdis	10/43	6850	2.42	1
Cleopatra	6/43	6850	1.08	0
Argonaut	12/42	6850	1.08	0
Fiji	9/40	10450	1.21	0
Nigeria	8/42	10450	1.08	0
Trinidad	3/42	10450	1.59	0
Newfoundland	7/43	10450	1.21	0
Kent	9/40	13500	1.08	0
Dragon	7/45	4850	1.21	1
Dunedin	11/41	4850	2.42	1
Manxman	12/42	3415	1.21	0
Welshman	1/43	3415	1.21	1
Courageous	9/39	22560	3.64	1
Ark Royal	11/41	27720	1.21	1
Indomitable	7/43	29730	0.70	0
Eagle	8/42	26000	4.85	1

Table 2: RN (including RAN & RNZN) Cruiser Damage from torpedo hits in WW2 (TPBE denotes the quantity of ordnance ship hit by in units of Thousand Pound Bomb Equivalent)

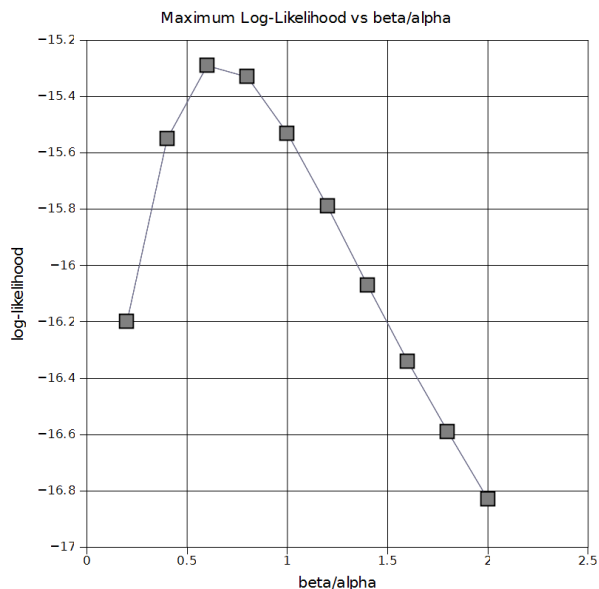


Figure 3: Plot of the Maximum of the Log-Likelihood as a Function of β/α (the exponent of displacement in the expression for the TPBE for 50% probability of kill) for the Cruiser (+Abdiels) Data Set.

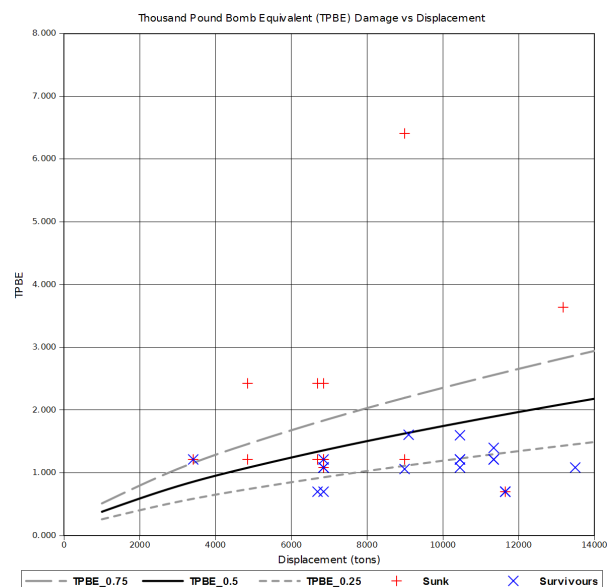


Figure 4: Plot of the Data from Table 2 and Lines from the Maximum Likelihood Solution for a Type-B Model Indicating the TPBE for Probability of Mission Kill of 0.75, 0.5 and 0.25 for the Cruiser (+Abdiels) Data Set.

For a comparison of torpedo lethality we need to look at Humphrey's results quoted in table 6.2 of [1]. This quotes 1.6 US 21" torpedo equivalents to sink a 3000 ton ship with a probability of 80%, and 3.5 US 21" torpedo

equivalents to sink a 15000 ton ship with a probability of 80%. The corresponding numbers for sinking RN cruisers with a probability of 80% are 0.88 and 2.48 US 21" torpedo equivalents (the figures used earlier in this note are in TPBE which is just the scaled equivalent explosive yield of the warhead, these are converted into torpedo equivalents by the appropriate scaling corresponding to the relative yield of the warheads). These figures are relatively closer to the corresponding figures of Humphrey, but still a long way out, but now we do not have the excuse of the fragility of carriers to explain the difference away.

8. SUMMARY

In this note a generic model of the relationship between the probability of mission kill/sinking and ship displacement and the Thousand Pound Bomb Equivalent of the ordnance that hit has been proposed. This model has been fitted to data sets of Carriers subject to bomb damage in the Pacific in World War 2 and Royal Navy cruisers hit by torpedoes also in World War 2 using a maximum likelihood method. The fitted models give plausible probabilities of kill given the data, as is shown by the curves in figures 2 and 4.

When the results are used to investigate the dependence of the quantity of ordnance required to give a specified probability of kill as a function of target displacement then the combination of models, data and fitting method used here proves inadequate. There is insufficient data and the range of displacements in the data sets are not sufficient to give confidence in such results. A qualitative analysis of such investigations (which this paper is too short to expand on) is that anything from a cube root to linear dependence on displacement for the 50% probability of kill is consistent with both data sets.

To widen the data sets we could be less restrictive about which types of ships are included but we should not attempt to derive simple models like the present one with data on platforms whose damage mechanisms are significantly different and weapons which have a significantly different damage causing mechanism. This would preclude including damage to Royal navy carriers in the carrier data set as a significant sub-population of those have armoured flight decks and even more have the hangars internal to the hull rather than as superstructure as the US and Japanese carriers. It may be worth while however extending the cruiser data set to include more nation's cruisers and this should be pursued in any further work on such models.

A third data set has been compiled and investigated, this is for warships hit by anti-shipping missiles in the post WW2 period. The results for this data set are not presented here as in general the data represents hits with more than enough energy to disable/sink the targets, and so the data does not give reliable results for the model fitting process.

From the carrier data set we may observe that there is some indication that the Japanese would have been better served by bigger bombs, and that ~18 inch torpedoes are a bit too small to ensure a decisive outcome against cruiser sized warships with a single hit.

REFERENCES

- [1] Hughes W. P., Fleet Tactics and Coastal Combat, Naval Institute Press, 2000
- [2] Beall T. R., The Development of a Naval Battle Model and its Validation Using Historical Data, Masters Thesis, Naval Postgraduate School, Monterey CA, 1990
- [3] Humphrey R. L., "Warship Damage Rules for Naval Wargaming" ORSA/TIMS Joint Meeting, Las Vegas, NV, May 1990
- [4] Washburn A., Kress M., Combat Modelling, Springer, 2009
- [5] Chesneau R. (ed), Conway's All the World's Fighting Ships 1922-1946, Conway maritime Press, 1980.
- [6] Kimball G. E., Morse P. M., Methods of Operations Research, Peninsular Publishing, 10th printing 1970.
- [7] Wikipedia Contributors, Damage to major ships at the battle of Jutland, Wikipedia the Free Encyclopedia (accessed March 7th 2011)
- [8] Mason G., Service Histories of Royal Navy Warships in World War 2 and other Papers, <http://www.naval-history.net/xGM-aContents.htm>, 2007.