CALCULENG FOR VIRTUAL MATHSAID : THE DEVELOPMENT AND EVALUATION OF A SET OF ON-LINE TUTORIAL RESOURCES FOR ELEMENTARY CALCULUS

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Abstract: Development of proficiency in Mathematics is an essential aspect of most scientific and technical degrees, and also in subjects such as economics, business and finance, having an impact on many other areas of such students' curricula. However, Mathematics is an area which many students find difficult, particularly those from "non-traditional" academic backgrounds, including disabled and mature students, and they often do not realise its relevance and importance to their other courses. Thus, students often neglect to devote as much effort as they should to practicing mathematical exercises, and the time allocated for face-to-face tutorial sessions is frequently very limited. This poses challenges for both students and academic staff. Although there have been previous attempts to develop on-line tutorial exercises, with automated marking and feedback, to assist students with their mathematical studies, most of these have either been restricted to multiple choice or numerical answer questions, or have only addressed the most elementary of topics – notably simple algebra, functions & graphs and trigonometry. In this paper, we describe our efforts to remedy this situation. We have produced a set of resources, including a variety of exercises, on single variable differential and integral calculus, with applications, which can be delivered on-line or using a Virtual Learning Environment such as Blackboard or Moodle. These exercises, developed using the QTI framework, allow algebraic input from the student, which is checked for consistency with the "correct" solution, and with the outcomes of anticipated "common errors", using the MAXIMA computer algebra engine. We discuss the design and implementation of resources we have produced, and their preliminary evaluation when used on a first year undergraduate Engineering Mathematics module. These resources should be of particular benefit to students who, due to disabilities or family commitments, may have difficultly attending classes in person.

Keywords: Accessibility, Calculus, e-learning, Mathematical skills, QTI, self-study

1. Introduction

Mathematical skills are core aspects of most scientific, technical, business and finance degrees – especially those in Engineering disciplines (Jaworski et al., 2012). In Engineering, the theory of topics such as dynamics, control, fluid mechanics, thermodynamics, electrical circuit theory, electromagnetism and communications are all described in mathematical terms. Similarly, in Chemistry and the Life Sciences, in addition to thermodynamics of chemical reactions, rates of change and integration occur in the theory of reaction kinetics and evolution, and manipulation of power laws into a form representable in a straight line form occurs in many areas such as diffusion, radioactivity, attenuation of radiation (including all kinds of EM radiation – visible light, IR, UV, etc.). Proficiency in mathematics – at least at an intermediate level – is essential to any professional engineer or scientist. However, these mathematical topics are often found difficult by students, particularly those from "non-traditional" academic backgrounds, including disabled and mature students, who may have problems attending classes due to mobility, working or family commitments, and in accessing traditional support resources (James & Gardner, 1995). Furthermore, the class time available for face-to-face tutorial support for mathematics in other disciplines is often limited, and class sizes – even for tutorials – are frequently rather large compared with the number of tutorial support staff available. These factors pose major challenges for both students and academic staff.

There have been many attempts to address such issues. Since the 1960s and 70s, academics teaching Mathematics to Engineering students investigated the potential of using "Programmed Learning" approaches in course materials and textbooks. In this methodology, theoretical concepts were introduced in conjunction with a large number of worked examples, broken down into small individual "steps" or "tasks" and arranged in a way such that students could "cover up" the latter parts of the model answer, try each task themselves, then reveal the next part of the model solution and compare that with their attempt. This approach led to several highly successful textbooks, including the very popular volumes by Ken Stroud (Stroud, 1970) from Coventry – Stroud's "Engineering Mathematics" is now in its 7th edition (Stroud and Booth, 2013) and popular as ever with students.

Some higher educational establishments have gone for an approach which restores the more traditional face-to-face approach to tutorial support, but in a more flexible way. An example of this is the system of "MathsAid" drop-in clinics held at Kingston University and some of its affiliated colleges. At each campus, drop-in sessions are held most weekdays (usually for 2 or 3 hours around lunchtimes) run by either a member of academic staff, postgraduate student or senior, trained undergraduate student. These sessions are designed to provide support to students of all disciplines who are having difficulties with mathematical and/or statistical concepts and exercises which they encounter during their studies, no matter what their main area of study is. Hence, in addition to people taking mathematics or statistics as their principal subjects, many students from chemistry, biology, economics, business, or engineering subjects attend these sessions in order to resolve issues and problems. However, it is always emphasized that these sessions are not there to give students answers to assessed exercises (Atkins et al., 2005). Moreover, these sessions are not substitutes for their regular tutorial sessions on their courses

More recently, some computer-based and on-line resources and exercises have been developed to address these issues for teaching and learning mathematics. A few example of these are the CATAM (Harding, 1974), CALMAT (Booth, 1998, Marjoram, 2004, Gorman et al., 2009) Mathletics system (Greenhow, 2008), MathDox (Cohen et al., 2006), GeoGebra (Hohenwarter et al., 2008), MyMathLab and STACK (Sangwin, 2013, Sangwin and Grove 2006), and there are also commercial systems such as MapleT.A. (Maplesoft, 2015) and MyMathLab (Pearson Education, 2015). Each of these systems has some good and some poor aspects, and only a few of them, such as STACK, provide any feedback on the students' answers. CALMAT is primarily targeted at senior high school students and the core material it covers is more at a rather elementary level. Additionally, it only permits multiple choice and short answer questions and, although these questions may include an option to reveal a "hint" to students in the middle of an attempt at solving them, the restricted type of responses which may be entered do not permit checking of a student's answer for algebraic consistency with the model solution, nor do they offer meaningful feedback to students. Similarly, the "Mathletics" system developed at Brunel University in London (Greenhow, 2008) only offers a limited range of question types and only very basic feedback to students on their answers, which is not tailored to the specific nature of the student's mistakes. Furthermore, MapleTA and MyMathLab are proprietary systems, with subscription payable. Although the MathDox system developed at the Technical University of Eindhoven (Cohen et al., 2006) covers a more comprehensive and advanced range of mathematical topics, and is interfaced with a Computer Algebra System (CAS) which enables the checking of students answers and working for consistency, most of the resources devolped for it are currently only available in Dutch !

In many institutions, a Virtual Learning Environment (VLE), such as BlackBoard or Moodle, is used as a tool for formative and/or summative assessment. The method of assessment normally uses very short answer or multiple–choice questions (MCQs) that are not well suited for the assessment of mathematical topics. Additionally, this form of assessment tool fails to provide a useful and detailed feedback.

However, appropriate on-line resources and exercises should provide additional "virtual tutorial" support at any time and location, which is consistent with the expectations of modern students. Hence, the use of suitable e-materials is expected to improve the quality of student's learning as these materials would provide a platform which will allow students instant access at anytime and anywhere. Furthermore, use of structured exercises will allow the students to develop and test their own knowledge and understanding of mathematical topics, concepts & methods. Linking the system to a Computer Algebra System (CAS) will also enable checking of the student's answers for mathematical/algebraic *consistency* with, rather than requiring an exact match to, the model answer,

allowing answers to be expressed in different, but equivalent and possibly equally correct, forms and still being marked as “correct”. Furthermore, use of this CAS, in conjunction with hand-crafted rules encoded in XML, allow the detection of “common errors” in solutions, and the possibility of offering constructive feedback specifically tailored to the error(s) the student has made.

Identifying all the above issues has helped us to create an on-line system, called CalculEng, covering a good range of intermediate level calculus topics, allowing students to carry out a series of exercises, and offering hints and meaningful feedback, dependent on the student’s answers. These resources not only have the benefits of both computer-based and face-to-face tutorial support but also, by breaking exercises into small stages, they should assist students to learn and master the essentials of differential and integral calculus.

The aim of this project is to evaluate these on-line materials by using both qualitative (e.g. via questionnaires) and quantitative (via students’ performance in formative assessments) perspectives.

2. CalculEng Resources

As noted above, CalculEng offers students structured exercises on elementary differential and integral calculus (including applications of both of these), such that the students can enter their answer to each section as a mathematical expression, typed-in typing using an ASCII-based mathematical format (and example of this is shown in Figure 1 below), rather than just making a selection, as is the case for multiple-choice questions, or just entering a numerical value. Although the general nature and structure of each question is pre-defined, specific parameters and coeficients in formulae and equations can be selected randomly (within limits or ranges pre-specified by the tutor, e.g. in order to ensure that a quadratic equation involved in the solution has real roots) automatically by the system. The system then checks the student’s response for alegrebaic consistency with the “model answer”, using the MAXIMA computer algebra system (Schelter, 1998) and whether the format of the response is correct (e.g. has the student simplified his/her answer sufficiently ?). Furthemore, a set of rules, encoded in XML, for each question, allow the student’s answer to be checked against a list of perceived “common errors” for that type of problem (e.g. has the student differentiated a function which the question required him/her to integrate ?), and then provide feedback tailored to the particular type of mistake made. This is expected to assist students with understanding and mastering mathematical concepts, and ultimately should help them in problem-solving situations in their main subject of study. In multi-section questions, detailed feedback will be revealed to the student in a step-by-step process. This feedback facility, customised to the precise question and student answer, is a particularly powerful feature of CalculEng, as it can be used for both formative and summative assessment.

Expression as rendered on-screen : $7e^{(3z-2)}\cos(5z-6)$

ASCII Format : 7*e^(3*z-2)*cos(5*z-6) or 7e^(3z-2)*cos(5z-6)

Figure 1 : ASCII-based mathematical format for a student entering a given mathematical expression

2.1 Development of CalculEng Resources

The calculus exericses used in CalculEng were originally selected from conventional paper-based tutorial exercises which two of us (MD & GH) had developed over many years of teaching calculus at foundation and first year university level. However, some further, primarily engineering application-based, exercises were put forward by another of us (JD), who is currently completing his MEng degree in Engineering at Kingston. Model solutions were prepared by the person who had originally set each question, and some anticipated “common student mistakes” (e.g. the student forgot to include the correct multiplying factor in a differentiation exercise using the “chain rule”) identified for each one. Appropriate constructive feedback comments were devised for each anticipated answer – correct, consistent but not simplified, “common mistake” or “otherwise incorrect” - and the question and each of the expected attempts at solutions encoded using QTI XML code (Neve et al., 2012)

2.2 Example CalculEng Exercises

A basic example question, with the student's response and feedback, is shown in Figure 2. In addition, students are able to use the *Show Hint* button, which provides useful formulae and rules to assist them with solving that particular mathematical problem. CalculEng will also provide full worked solutions, which are generated according to the question for study and analysis by the individual student (see Figure 2). CalculEng can be accessed through a VLE system, which enables re-use of the materials: copying questions between modules, setting formative and summative assessments and automatically grading the assessments.

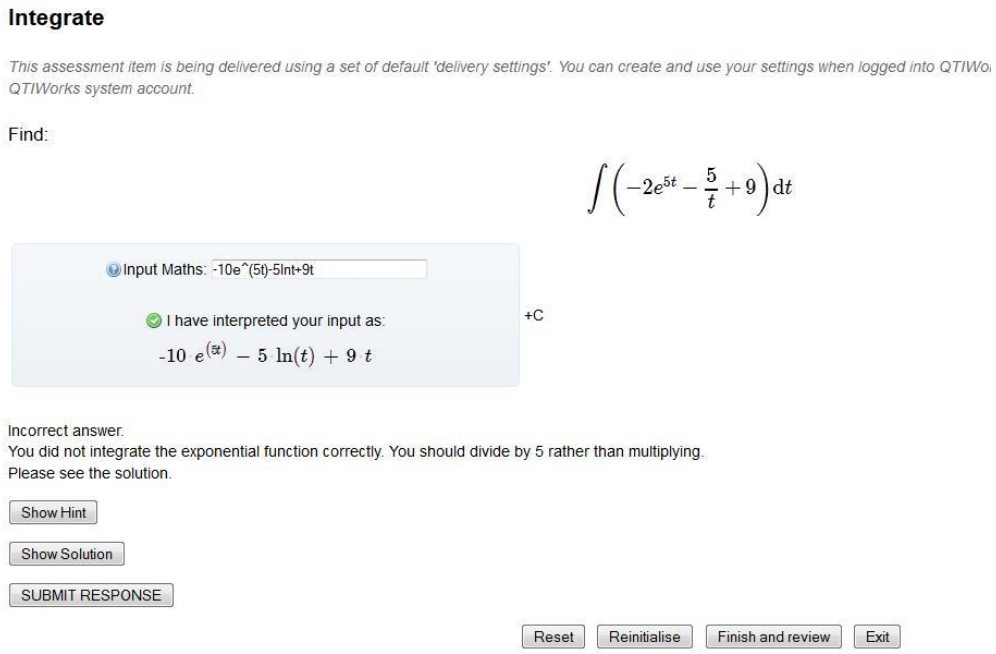

Figure 2 : Example CalculEng question, with feedback on student's answer

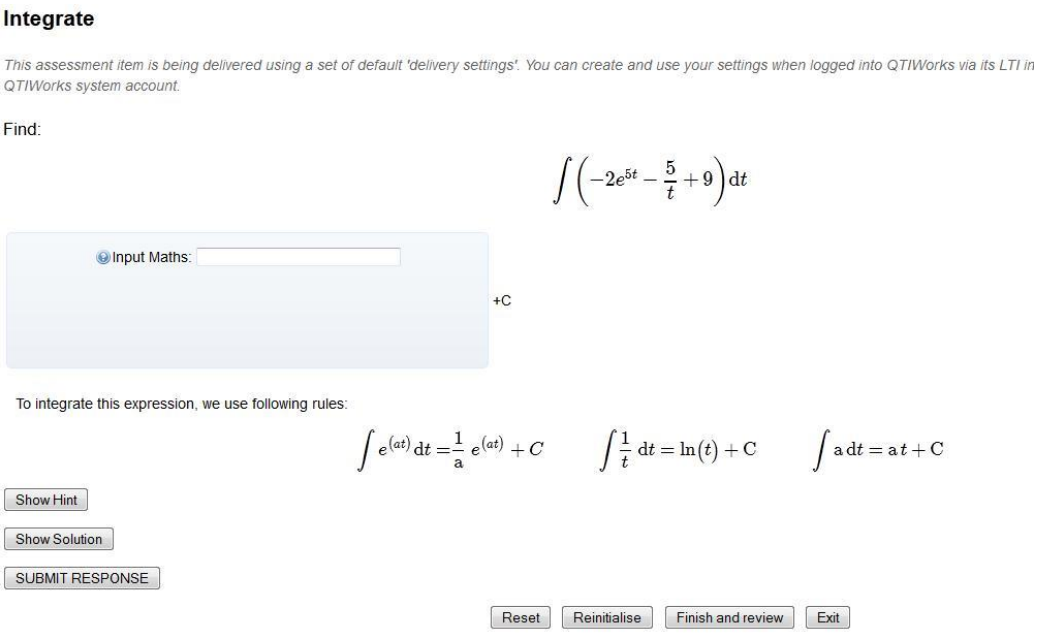

Figure 3 : Example CalculEng question, with hint provided to the student

These materials are expected to make learning mathematical skills more engaging and encourage students to develop more conceptually-based understanding of mathematics and how to use it in problem solving situations in their main subject of study.

3. Pilot Study - Evaluation of CalculEng by First Year Undergraduate Students

It was originally planned to evaluate CalculEng in use by having it used by several groups of first year undergraduate Engineering degree students studying Mathematics, including both differential and integral calculus, as part of their curriculum. However, delays in getting the resources ready, the scheduling of parts of some modules across only parts of the academic year, plus a local restriction on surveys of students due to National and intra-institutional Student Surveys, resulted in the initial evaluation only being performed on one group of approximately 40 first year BEng Civil Engineering students, towards the end of the Spring term 2015. This period coincided with a large number of other deadlines for their in-course assessments, with the consequence that only 13 students completed the evaluation survey. The survey took place after the students had been given the opportunity of using the CalculEng system during three 2 hour supervised practical sessions. They were then given a questionnaire regarding their views on their mathematical studies, their confidence with mathematical topics and their opinions on their experience of using CalculEng. The questionnaire they were asked to complete is given in the Appendix.

3.1 Results of Preliminary Evaluation

During informal discussion, several students indicated that they believed that solving exercises on CalculEng could prove useful to them during their revision and preparation for their exams in May. They also liked the way in which CalculEng presented model solutions in a logical, step-by-step manner, and found the feedback on their answers very helpful. Quantitative results obtained from the students' questionnaire responses are shown in Figures 4 to 6 below.

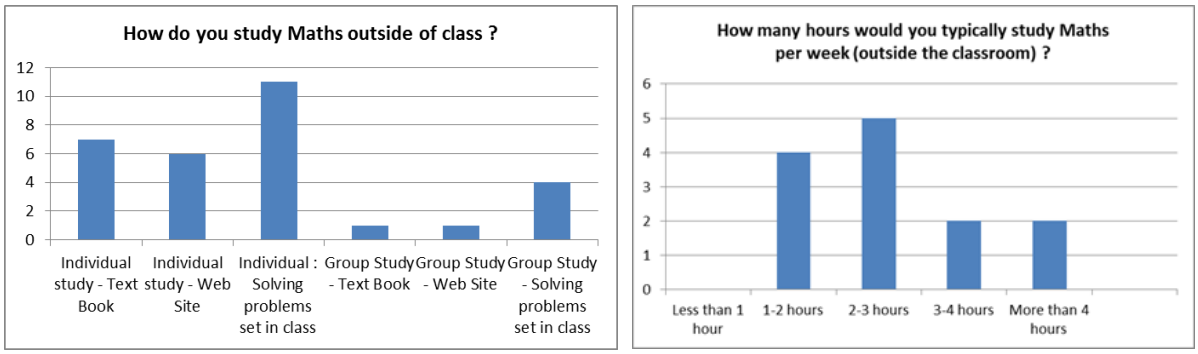

Figure 4: Student respondents' attitutes to studying Mathematics outside of class

As can be seen from the results in the figures, only 23% of the respondents claimed they regularly used web resources for their out-of-class mathematical studies. 51% mostly practiced exercises set in class, either individually or in groups of peers, with 26% preferring to use textbooks. The majority of the students (71%) carried out between 1 and 3 hours of individual study for the module per week, with only 15% spending 4 or more hours per week studying for this module outside of class. Since this 30 credit module has approximately 100 hours of scheduled class time, this figure contrasts with the University's expectation that students put in 200 hours of individual study for the module, equivalent to approximately 8 hours per week !

Students' opinions on CalculEng were rather divided – roughly equal numbers believed that it could help their understanding of theoretical topics in calculus as did not consider that to be the case, and similarly for whether CalculEng could help them improve their time management skills when solving mathematical problems. However, slightly more students stated that they found CalculEng easy to use than disagreed with that statement.

It should be born in mind that these survey results were based on a very small sample of students, and came after those students had only had a limited opportunity to use the system. It is intended to review the on-line tutorial materials provided in CalculEng, and the questions asked to the students, before carrying-out a more comprehensive evaluation of the system during the 2015-16 academic

year. This follow-up study should also allow the students more time to become familiar with CalculEng and the facilities it offers before they are expected to evaluate them.

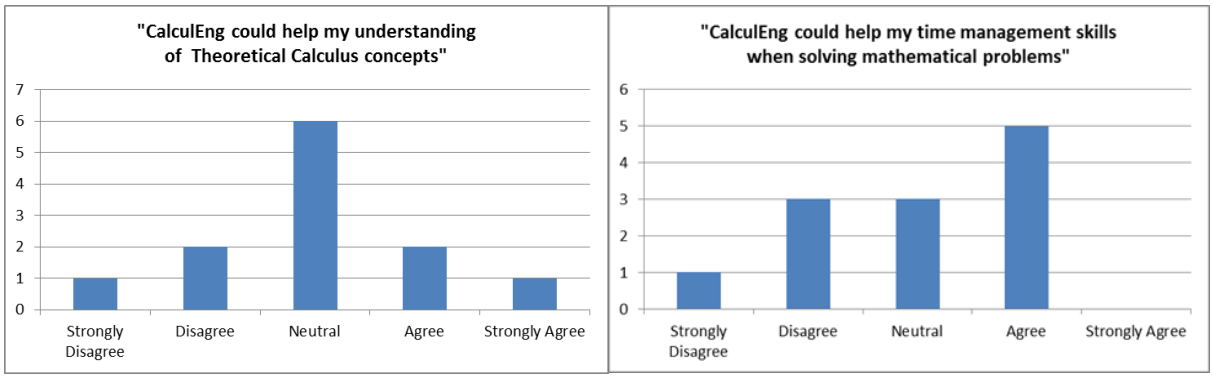

Figure 5: Student respondents' opinons on whether CalculEng could help their understanding of mathematics, and time management for solving exercises.

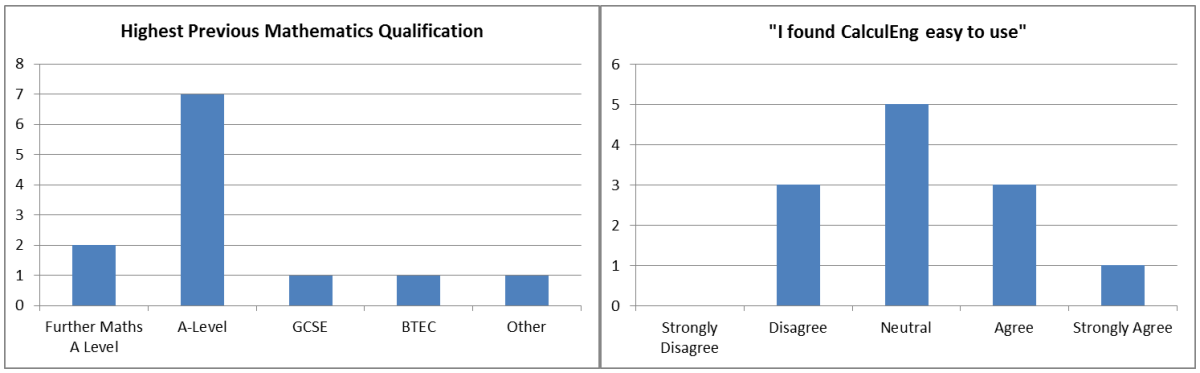

Figure 6: Student respondents' mathematical background & views on how easy CalculEng was to use

4. Conclusions

In this paper, we have motivated the need for high quality, easy to use, on-line tutorial resources for mathematical topics at University level, particularly for students from "non-traditional" academic backgrounds, including disabled and mature students, who may have problems attending classes due to mobility, working or family commitments, and in accessing traditional support resources (James & Gardner, 1995). We have described our attempt at developing an on-line system, called CalculEng, for providing such resources, including exercises with appropriate feedback to the user's answers, for students learning differential and integral calculus. We have performed an initial "pilot study" evaluation of our system on a small group of first year undergraduate Engineering degree students, and presented an discussed the results of this evaluation.

We will use the results of this evaluation to refine and improve our materials, and scrutinize the questions asked in the evaluation questionnaire, in an attempt to remove any ambiguities therein. We intend to carry out an more extensive evaluation of the revised CalculEng system with a larger and more diverse range of students next academic year. We will also extend the number of exercises and the range of topics which they cover and, once our resources have been thoroughly tested and evaluated, we intend to make these available to the wider community.

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Appendix – Questionnaire given to Students after they had worked with CalculEng

SADRAS Maths Project Questionnaire:

This short Questionnaire has been developed with the aim of gaining a grasp on student learning styles and related information in order to further develop the CalculEng system for on-line tutorial support for Mathematics. Your feedback is extremely valuable to this research and will be much appreciated.

Please choose the appropriate answers for you, for each of the short list of questions below:

(1) Your ideal method(s) of studying, outside of class, to prepare for a Maths test would be:
(More than one answer may be ticked , if appropriate)

- Individual Study from a Text Book
- Individual Study through Websites online
- Individual Study solving Examples given by Class Lecturers
- Group Study using Text Books
- Group Study Using Online Resources
- Group Study solving Examples given by Class Lecturers

(2) How many hours would you typically engage with studying Maths in a week (outside the classroom)?

- Less than 1 hour
- 1 – 2 hours
- 2 – 3 hours
- 3 – 4 hours
- More than 4 hours

(3) During a typical Maths exercise, how long (at a single stretch) would the topic typically hold your interest and attention before your concentration begins to waver?

- Less than 30 minutes
- 30 minutes to 1 hour
- 1 – 2 hours
- 2 – 3 hours
- More than 3 hours

(4) How confident (on a scale of 1 – 6, with 1 being "Not at all confident" and 6 being "Extremely confident") do you generally feel while attempting THEORETICAL Calculus Questions (e.g. on a problem sheet given to you by your lecturer) ?

(5) How confident (on a scale of 1 – 6, with 1 being "Not at all confident" and 6 being "Extremely confident") do you generally feel while attempting PRACTICAL Calculus Questions (e.g. those applied to Engineering Disciplines, such as Thermodynamics, Stress Analysis or Fluid Mechanics)?

(6) To what extent do you agree (or disagree) with the following statement ?

"CalculEng has improved my understanding and expertise of concepts of Calculus, in terms of being able to solve Theoretical Questions" (Indicate on the Bar below)

Definitely Disagree-----Disagree-----Neutral (No major Change)-----Agree----Strongly Agree

(7) To what extent do you agree (or disagree) with the following statement ?

"CalculEng has improved my understanding and expertise concepts of Calculus, in terms of solving Practical Applied (Engineering) Questions" (Indicate on the Bar below)

Definitely Disagree-----Disagree----- Neutral (No major Change)------Agree----Strongly Agree

(8) To what extent do you agree (or disagree) with the following statement ?

"CalculEng has improved my Time Management skills in terms of solving Theoretical and/or Practical Applied (Engineering) Questions involving Calculus" (Indicate on the Bar below)

Definitely Disagree-----Disagree----- Neutral (No major Change)-----Agree----Strongly Agree

(9) How easy did you find CalculEng to use ? Please indicate on the bar below

Very difficult-----A bit difficult----- O.K.------Fairly easy-----Very easy

(10) Thank you for using CalculEng and taking time to complete this study. Please state any additional features or changes to the software that you would like to see in future

(11) Please mark below the most recent/highest Mathematics qualification you hold, along with Grade/Mark you obtained in the subject:
(This information is purely for reference and will be kept fully confidential and anonymous.)