Abstract: Successful educational achievement generally leads people to better career and life opportunities throughout the world, and success in Mathematics (at least at an elementary level) is an essential pre-requisite for both further study and career success in many fields. However, Mathematics is a subject which many students find problematic. Creating and editing mathematical text in electronic documents is difficult even for experts, and all the more so for novices or for people with many disabilities – especially the blind and those with severe visual impairments – since traditional mathematical notation uses specialised symbols arranged in a complicated layout, and fine details or small errors can alter meaning, correctness or interpretation. Correctly reading, interpreting and understanding such symbolic expressions therefore becomes quite difficult for such disabled people. Mathematics tends to use uncommon language for names of symbols, functions, etc. and this, coupled with the rather complicated layout of mathematical equations and formulae then to make reading them aloud (either by a person or by a machine using synthetic speech) in a clear, easy to understand yet unambiguous way, difficult (Österholm, 2006). All of these features greatly complicate reading, writing, typing and editing mathematics for people who are blind or visually impaired people, or have limited or no use of their hands or arms. This suggests that alternative types of interfaces are needed for these type of people to access mathematical content effectively.

Keywords: Accessibility, usability, mathematical e-text, speech synthesis, speech recognition
In this paper, we briefly discuss some alternative modalities to make mathematical text more accessible to disabled people – and the blind and visually-impaired in particular. We then briefly describe our system, TalkMaths, which allows the creation and editing of mathematical expressions in electronic documents via dictated spoken input in relatively natural language – an easy to learn subset of English. These mathematical expressions can subsequently be parsed, rendered into conventional mathematical notation, and further processed, e.g. by a computer algebra system. We discuss some evaluations of the usability of our system, both as a tool for the creation & editing of mathematical text, and as a tool for assisting the learning of mathematical concepts. Although most of the participants in the evaluation studies did not have any noted disability, in the former case one participant was disabled and his performance using and views on TalkMaths were particularly interesting. The potential benefits and limitations of TalkMaths and similar systems for visually-impaired people are discussed, and the paper ends with our conclusions and suggestions for future work.

2. Modalities for Communication of Mathematics for the Blind and Visually-Impaired

Braille has been used successfully as a means of written communication for the blind for around 200 years, and has more recently been adapted for use with computers and to describe relatively sophisticated mathematical concepts (e.g. British Math Braille code (Braille Authority of the U.K., 1987)). Through the use of such mathematical codes, some people have developed Braille-based mathematical interfaces for computers. DotsPlus (Gardner, 2003) is a two-dimensional Braille system for representing complex mathematical expressions, based on the 6-dot Braille system, but uses graphic symbols rather than Braille for punctuation marks. It is designed for people who lose their sight later in childhood or as an adult and can be taught and learnt relatively easily by people who are already familiar with mathematical notation. UMA (Karshmer et al, 2004) is a system which allows conversion from Nemeth or Marburg Braille to LaTeX, MathML or OpenMath, and vice versa, also providing synthetic speech output. However, particularly as more blind and visually-impaired children are now being educated in mainstream (rather than specialist) schools and colleges, a smaller proportion of such students are now proficient in Braille, and (with the exception of teachers in specialist schools), very few teachers master this medium, leading to a serious dearth of Braille resources for teaching & learning – particularly for subjects such as Mathematics.

Several specialised mathematical text editors have been produced for blind and severely visually impaired people, who cannot use conventional screen-based editors. These include Infty (Komada et al, 2006), WinTriangle (Gardner, 2005), BlindMath (Pepino et al, 2005).

Some alternative output modalities have been proposed, notably LAMBDA (Linear Access to Mathematics for Braille Devices and Audio Synthesis, Edwards et al, 2006) and various screen readers (e.g. JAWS and Window-Eyes) and audio description systems (e.g. AsTeR (Audio Systems for Technical Readings, Raman 1998), MathGenie (Jacobs 2006), and AudioMath (Ferreira & Freitas, 2004)) but these either do not allow, or do not make, the input of mathematical expressions, equations or formulate easy for the user. Although there have been some previous commercial (e.g. MathTalk™ (Metroplex Voice Computing, 2007), Math Speak & Write (Guy et al, 2004)) and research prototype (e.g. Bernareggi & Brigatti, 2008, Hanakovič & Nagy, 2006) software applications that aim at allowing input of Mathematical text via speech, in a way easily-usable by disabled people, most of these have been seriously limited, or only work with a proprietary editor. In next section, we briefly describe our TalkMaths system, a speech user interface allowing the user to dictate mathematical expressions, in a relatively natural, easy to learn way, before going on to discuss some evaluations of TalkMaths by pilot groups of users.

3. The TalkMaths System

The latest version of our TalkMaths system (Attanayake et al 2013) is now a web-based system, using a client-server architecture, for creating and editing mathematical content within electronic documents. (An earlier version (Wigmore et al., 2009) was PC-based.) It allows both input of mathematical expressions via dictation (thanks to it being interfaced to an Automatic Speech Recognition (ASR) system) or through keyboard input, using relatively natural language, “Spoken Mathematics”, which is easy to learn and only prescriptive in as far as is necessary to avoid ambiguity. This is in contrast to conventional mark-up languages for Mathematics, such as LaTeX and MathML.
which are complicated for novices to learn, and not easy to dictate using ASR systems. For example, consider the trigonometric identity shown, as rendered by TalkMaths, in Figure 1. This should be dictated into TalkMaths as “Sine fraction theta over two end fraction equals plus or minus the square root of fraction one minus cosine theta over two end square root”. Compare this with the corresponding LaTeX code: \[ \sin \frac{\theta}{2} = \pm \sqrt{\frac{1-\cos \theta}{2}} \], or the even less user-friendly (or “speaker/listener friendly”) equivalent in MathML! Further examples of quite complicated expressions being dictates in a relatively natural way are given in Attanayake et al (2013), where comprehensive technical details of the most recent version of the system can also be found.

Figure 1 : Screen shot of TalkMaths, showing rendered output for the trigonometric identity for the sine of a half-angle.

TalkMaths is also designed to make editing of mathematical expressions via speech relatively straightforward. Figure 2 shows an example where another trigonometric identity has been dictated and rendered on the screen, but where the user wishes to edit the expression as it currently stands. Following the user making a few simple speech commands, the expression is decomposed into constituent parts, shown by coloured boxes which are indexed by numbers. The user can, via speech, select one of these numbered boxed to specify which part of the expression he/she wishes to edit. Although this feature should be very useful to people without visual impairments (including those with non-visual disabilities), in its present form this would not be of major benefit to people with no, or very limited sight, although the possibility exists for some kind of audio or Braille description to replace the labelled boxes.

Figure 2 : Example of the TalkMaths screen display, showing the colour-coded labelled boxes to indicate sub-expressions and their constituent components, resulting from a trigonometric identity being dictated and then rendered on the screen.

Further discussion of editing paradigms in TalkMaths is given in Wigmore (2011) and Attanayake (2014).
3.1 Usability of TalkMaths by Disabled and Non-Disabled Students

TalkMaths was originally designed to be used by people who had limited (or no) use of their hands or arms, rather than by the blind and severely visually-impaired, and thus may need further development (particularly regarding rendering and other output modalities, e.g. by Braille or other tactile interfaces, or via audio description) to be of full benefit to such groups. Furthermore, due to the numbers of seriously disabled students currently studying Scientific and Technical subjects in Higher Education in the UK being relatively small, the majority of people who have tested and evaluated TalkMaths to date have not had any major disability, nor significant experience of using Automatic Speech Recognition (ASR) systems.

An evaluation of the first, PC-based, version of TalkMaths was carried out by Wigmore (2011), with preliminary results being reported in Wigmore et al (2010a,b). In that study, 7 male and 11 female participants, aged between 19 and 36, carried out various Mathematical text creation and editing tasks using both TalkMaths and MathType™ (Design Science 1996, 2015). All the group were either native, or near-native level bilingual, English speakers, and all had passed mathematics examinations, at least up to U.K. GCSE (intermediate high school) level. Of this group, only one had a minor disability (due to an arm injury), all had used conventional keyboard & mouse based mathematical equation editors before, but none had any previous exposure to TalkMaths, nor significant experience of using ASR systems. Half the group first carried out some tasks using TalkMaths, and then some similar tasks using MathType. The other half did the tasks using MathType first, followed by those based on TalkMaths. The time they took to complete each task, along with the numbers of speech recognition errors, “user errors”, “assists” by the supervising tutor, and uses of the built-in “Help” facility, were recorded for each participant. As expected, these participants were faster using MathType than TalkMaths for doing comparable jobs (see Figure 3) - probably since MathType made use of a modality more familiar to them. Similarly, for a given type of task, users tended to make more errors, use the Help facilities more and require more “assists” from the supervisor when using TalkMaths than when using MathType (Wigmore 2011). In questionnaires completed after the tests, they mostly stated that they had found MathType easier to use, but that they had enjoyed using TalkMaths, and would be willing to try using it again (Wigmore et al 2010a).

![Figure 3 The mean time taken by participants to complete the given tasks using TalkMaths and MathType. Tasks with the same numerical index were similar in nature and difficulty. Tasks with the second index “a” were carried out using TalkMaths, and those indexed “b” were done using MathType.](image)

However, the one seriously disabled student (who suffered from Duchenne Muscular Dystrophy, but had normal speech & vision) who did have an opportunity to test the system found it enjoyable to use, and easier for him than doing the equivalent task using the keyboard & mouse. This student made fewer errors in carrying-out specified equation editing tasks, and was often faster, when using TalkMaths than when using the more conventional equation editor (which required the user to “type & click” with the keyboard & mouse – tasks difficult for this disabled user) and also outperformed most non-disabled students doing the same tasks using TalkMaths (Wigmore 2011). This shows the potential for TalkMaths to be of real practical value as an equation editor for some types of disabled students, and other people with physical disabilities who need to edit mathematical expressions.
Table 1: Times in minutes for users to complete various equation creation and editing tasks. MD stands for the student with Muscular Dystrophy, Able represents the average times for non-disabled students to complete that task and Expert the time for an expert user of both TalkMaths (TM) and MathType (MT) to carry out the same tasks. Adapted from Wigmore (2011).

3.2 Potential Use of TalkMaths as a Tool to Aid Students’ Understanding of Mathematics

A small trial was also conducted to investigate a potential application of TalkMaths as an educational tool to assist students in their learning and understanding of mathematical concepts (Attanayake et al. 2013). We developed a set of classroom mathematical materials and learning tasks for undergraduate Life-Science students who were taking a basic mathematics module at Kingston University. Each of the volunteer student participants were allocated to one of two groups randomly. The first group (A) used a conventional editor (Microsoft Word Equation Editor) while the other group (B) used our prototype web-based version of TalkMaths. Each participant was asked to complete three questionnaires. The first questionnaire was about the participant’s own perception of their mathematical competence at the start of the experiment. The second was a diagnostic test related to the tasks they were about to carry out and the final questionnaire, given at the end of the exercise, was similar to the second, in order to assess improvements to the participants’ understanding, but also included questions concerning their experience of using whichever system they were allocated. After completing the first two questionnaires, participants were required to undertake three tasks creating and editing mathematical expressions involving fractions, functions and square roots respectively. Each task required them to create a specified equation and then carry out a minor modification to this using the editor they were assigned. Details of the tasks, and the questionnaires, can be found in Attanayake et al. (2013). The regular teaching staff and demonstrators for the module supervised the participants carrying out the tasks of the experiment, without actually instructing them. The students were required to learn by themselves, with the only resources available being the instructions given in the worksheet and the Help facility of whichever tool they were assigned to.

Figure 4 presents the results of the post task feedback from the participants regarding any improvements they perceived in their understanding of the mathematical concepts involved in the task after using the appropriate tool they were allocated to perform specified equation creation and editing tasks, relevant to the elementary “Mathematics for Biologists” module they were studying. On the whole, more students believed TalkMaths to have helped their learning and understanding of the mathematical concepts than was the case for MathType. We also evaluated the students’ performance on knowledge of relevant mathematical terminology, both before and after the tasks, to investigate whether the exercise, possibly including use of the Help facility of whichever tool they were allocated, had improved this knowledge. We gave a score of +1 whenever the participant correctly gave the prescribed answer to one of these “mathematical knowledge” questions, or a score of +0.5 for an answer we considered to be a relevant “near miss”. If the answer given was incorrect or the student did not respond, a score of 0 was given for that question. We observed the change to each student’s score between the pre- and post-task questionnaires.

Although, in both groups, most individuals gave the same response to any given question in both pre- and post-task questionnaires, indicating no change to their knowledge, some participants in group B (TalkMaths) did show a modest improvement in their overall “score”, and one participant in group A (MS Word Equation Editor) actually did worse the second time, indicating a decline in understanding! Although the observed effects were small, some were statistically significant, and these, together with the students’ perceptions summarised in Figure 4, suggested that TalkMaths could, in conjunction with appropriate teaching & learning resources and exercises, potentially aid students’ learning & understanding of mathematical concepts.

<table>
<thead>
<tr>
<th></th>
<th>Task 1</th>
<th>Task 2</th>
<th>Task 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MD</td>
<td>Able</td>
<td>Expert</td>
</tr>
<tr>
<td>TM</td>
<td>4.1</td>
<td>8.49</td>
<td>0.30</td>
</tr>
<tr>
<td>MT</td>
<td>1.8</td>
<td>1.28</td>
<td>0.25</td>
</tr>
<tr>
<td>Comments</td>
<td>MD better with TM than Able.</td>
<td>MD better with TM than Able.</td>
<td>MD better with TM than Able, better than with MT!</td>
</tr>
</tbody>
</table>
Figure 4: Post-task feedback on the perceived effect of using TalkMaths and MS EquationEditor on the participants’ learning and understanding of elementary mathematical concepts. Adapted from Attanayake (2014)

However, it was evident that the participants found the Microsoft Equation editor tool fairly easy to use, compared to the web-based TalkMaths prototype (see Figure 5). This was expected, as the participants all already had some experience using the MS Equation Editor prior to taking part in the user trial, but none of them had used TalkMaths previously.

Figure 5: Post-task feedback on the ease of use of TalkMaths and MS EquationEditor for given tasks

4. Conclusions and Future Work

From the studies described in this paper, we do have some evidence, albeit on the experience of a single disabled user, that TalkMaths could be of real benefit to students and other people with disabilities who need to be able to create and/or edit mathematical equations and formulae in electronic documents. Although the spoken input modality allowed by TalkMaths could also be of significant benefit to blind and other visually-impaired users, with the exception of a “read back what was dictated” facility included (Wigmore et al. 2010a), at present the system does not provide output in a form easily usable by people with those types of disability. This is something we hope to address in the future. Further developments to TalkMaths are being planned – such as incorporating output via Braille or other tactile interfaces, and/or audio descriptions of Mathematical expressions using synthesised speech – in order to address the outstanding issues and problems discussed above. Another possible development is to optimise the MathML/XHTML output currently used to render the parsed mathematical expressions on a display screen, in order that it is as suitable as possible for being “read out” by commonly-available screen reader software. This may require customisation dependent on which screen reader is to be used. Alternatively it may be possible for the transcribed spoken input, or MathML/XHTML output, to be converted into a format such as SAMPA (Wells, 1997, 2005) appropriate for use with Text To Speech (TTS) synthesiser software.
Further evaluations of *TalkMaths* are also being planned, incorporating some usability testing of the system by disabled students, including some with severe visual impairments, in order to investigate how useful *TalkMaths* could be to them, and to suggest how it could be further improved and enhanced to maximise its potential benefits and ease of use.

The TalkMaths system is still under development, and enhancements currently being investigated include the use of Statistical Language Models and edit distance metrics to allow predictive and/or (semi-)automatic correction of input (Attanayake et al 2013), and a more user-friendly interface for incorporating new commands and keywords into the vocabulary and/or new operators and rules into the grammar. Through these additions, it should be possible to increase the range of mathematical notation with which the system can be used, and improve the ways in which text can be edited and corrected. Novel methodologies for parsing the spoken input - based on a Generalised Operator Precedence Parser, a type of LR parser (Aho et al 1986) - to allow greater flexibility, including the ability to deal with incomplete or potentially ill-formed spoken mathematical input (Attanayake 2014, Attanayake et al 2014) without causing the system to fail, are also being developed and implemented.

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Acknowledgements: This work was partly supported by the Research Informed Teaching initiative at Kingston University (KU), and one of us (Dilaksha Attanayake) was partly funded by KU SEC Faculty research bursary, and another (Angela Wigmore) by an EPSRC Research Studentship. We would like to thank Dr Nigel Atkins, Dr Mastaneh Davis, Dr Vincent Lau and Dr Maryam Hajiesmaeili for allowing us to run the tests in their classes and assisting with supervising the tests, and all the student volunteers for their participation in the study.

Appendices

Equations Created/Edited by Participants in First User Trial (Wigmore et al 2010a, Wigmore 2011)

**TalkMaths Task 1a**

\[ f(x) = x^2 - 4x + 3 \]

**MathType Task 1b**

\[ f(x) = x^2 - 6x + 14 \]

**TalkMaths Task 2a**

\[ HP = \frac{40 \times \sin 105^\circ}{\sin 22^\circ} \]

**MathType Task 2b**

\[ \sin c = \frac{9 \times \sin 24^\circ}{5} \]

**TalkMaths Task 3a**

\[ a = \sqrt{3.5^2 + 6.06^2 - 2 \times 3.5 \times 6.06 \times \cos 30^\circ} \]

**MathType Task 3b**

\[ c = \sqrt{3.5^2 + 3.5^2 - 2 \times 3.5^2 \times \cos 120^\circ} \]

Expressions Created/Edited by Participants in Second User Trial (Attanayake et al 2013)

For both TalkMaths and Microsoft Word Equation Editor:

\[ a + b = \frac{2 + c}{x - y} \] \hspace{1cm} -(1) \hspace{1cm} \[ a + f(2x - 5) \] \hspace{1cm} -(2) \hspace{1cm} \[ a + \sqrt{x + 2y} \] \hspace{1cm} -(3)