Hazard and Risk Modelling for Weapon Danger Areas

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Abstract
When using probabilistic methodology to determine a Weapon Danger Area (WDA) there are additional requirements over deterministic methods and methods for weapons effectiveness calculations. These additional requirements manifest in three ways. Firstly, in effectiveness calculations the probability of failures are accounted for but the subsequent effects are not modelled e.g. what happens when a fuze functions early or not at all. Secondly analysis only considers events up to the target whereas a WDA has to take into account all events up until a weapon and all its components come to rest. Finally, simplifying assumptions are sometimes used because only close in effects are considered e.g. when considering weapon performance at function, detonation in the case of an explosive filled warhead, the probabilistic method requires detailed models of the fragmentation flight as it has to take into account the full trajectory of fragments.

This paper describes the current approach when considering the probabilistic methodology for the calculation of a WDA. This covers the different measures of hazard and risk that are used. It provides details for two models with additional requirements – the modelling of terrain, which requires more detail than in effectiveness calculations, and the modelling of impact and post-impact, which is not considered at all in effectiveness calculations.

1. Introduction
A Weapon Danger Area (WDA) is that area associated with firing a weapon where the risk of death or injury exceeds some threshold. The risk outside the WDA does not exceed this threshold and hence the risk to people outside the WDA is acceptable or tolerable. A Weapon Danger Zone (WDZ) extends this into three dimensions. Traditionally, WDA have been developed using deterministic methodology and WDA are extended into WDZ by using a constant height above the WDA. In both cases the level of risk associated with the area or zone has been assessed as acceptable or tolerable, but has not been explicitly quantified. In order to quantify the levels of risk we have to use a probabilistic methodology.

The general principles of the probabilistic methodology are described in Reference 1. Common models and the application of the methodology to unguided weapon systems are described in Reference 2. The application of the methodology to guided weapon systems is described in Reference 3. Applications are concerned with the distribution of trajectories in the same way that estimates of effectiveness are. The estimation of effectiveness assumes that weapons work as they should, with for instance models for dispersion about the optimal trajectory. In the development of WDA/Z additional allowances have to be made for faults in performance and the models are generalizations of those used for the estimation of effectiveness. Some examples are (a) the firer does not necessarily aim at the correct target, (b) the tracer in small arms ammunition fails to ignite and burn, (c) fuzes function between arming and the intended point of functioning and (d) fuzes fail to function at the target.

With the probabilistic methodology a direct estimate of risk is attempted. There are a number of measures of risk that are used in developing WDA/Z (a) probability of escape, which is related to the distribution of the final resting places of projectiles or their components, (b) individual risk of death, which is related to the locations where projectiles would be lethal and where people are present and (c) measures related to individual risk of death, where for instance only hit, or injury can be considered.

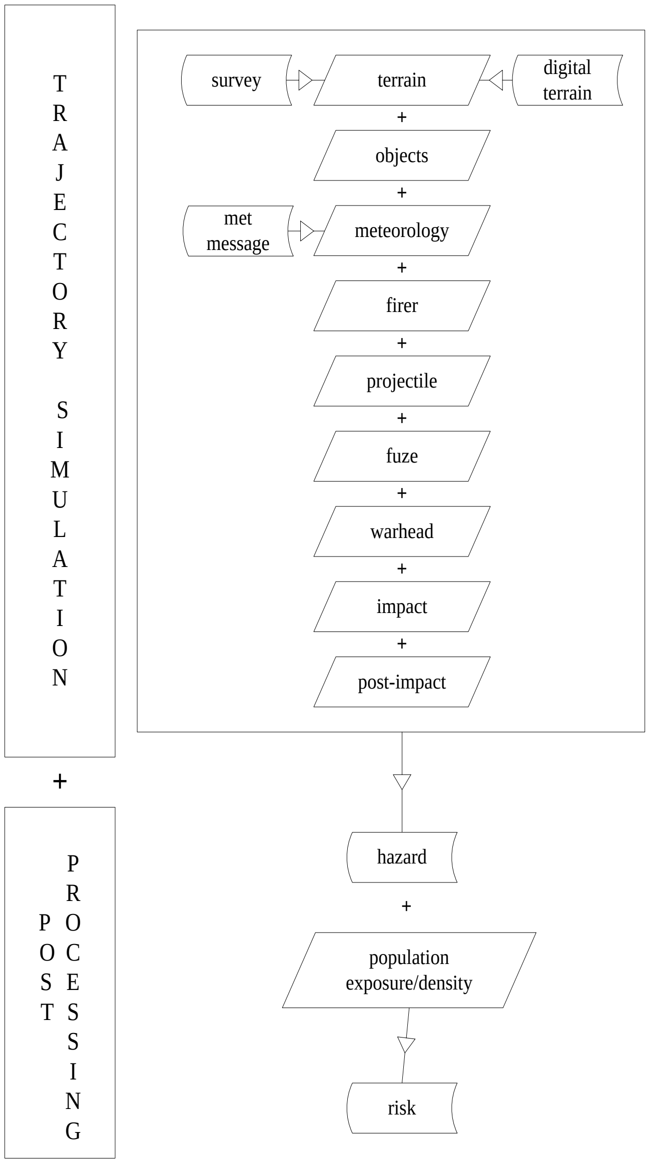
In this paper we describe the different measures of hazard and risk that may be used and focus on examples of models with additional requirements over that for effectiveness calculations. These are models for terrain, impact and post-impact (i.e. ricochet and post-ricochet flight).

A simulation of the use of a weapon system is carried out to produce representations of the estimates of the distributions of interest such as the final resting places or probability of hit.

Representations of the population exposure, or the probability of being present at a particular location, both of which are related to population density, may be created.

The representations may be combined to produce new histograms that represent estimates of the distributions of individual risk of hit, injury, or death. Note that these histogram representations may themselves be combined again to produce new histogram representations corresponding to ever more complex scenarios.

The final representations may be processed to develop WDA/Z. This usually involves the calculation of contours in the distribution of interest or calculations of the risk associated with given WDA/Z.



2. Hazard and Risk

Here we describe some measures of risk that are in common use for developing WDA/Z. The simplest is probability of escape. The most general is frequency of death from which a range of similar measures can be derived.

Traditionally probability of escape was used with weapon systems for developing WDA/Z. When we assume that the weapon system fires a single projectile that remains in one piece and a single projectile does not kill more than one person the result is a probability. Where multiple projectiles result from a single firing, e.g. for projectiles that break up on impact with the terrain, or projectiles that fragment, the result is a frequency as a single firing can result in multiple escapes.

This is a hazard (i.e. not a risk) as the escape from the WDA/Z is not necessarily an adverse consequence. However, it can be converted to a risk by including additional terms, which are then assumed to have value 1. For example, if we include two terms for hits per escape and deaths per hit we obtain:

$$\begin{array}{ccccccc} P_{D/R} & = & P_{D/H} & \times & P_{H/E} & \times & P_{E/R} \\ \dfrac{\text{deaths}}{\text{round}} & = & \dfrac{\text{deaths}}{\text{hit}} & \times & \dfrac{\text{hits}}{\text{escape}} & \times & \dfrac{\text{escapes}}{\text{round}} \end{array} \tag{1}$$

When worst case values of 1 are taken for the additional terms it is seen that this places a bound on the risk of death in terms of deaths per round:

$$\begin{array}{ccccccc} P_{D/R} & \leq & 1 & \times & 1 & \times & P_{E/R} \\ \dfrac{\text{deaths}}{\text{round}} & = & \dfrac{\text{deaths}}{\text{hit}} & \times & \dfrac{\text{hits}}{\text{escape}} & \times & \dfrac{\text{escapes}}{\text{round}} \end{array} \tag{2}$$

Probability of escape and any measures of risk derived from it are properties of the WDA/Z. The levels of hazard or risk apply to the exterior region between the WDA/Z boundary and the boundary of the zero energy area/zone. They are only indirectly functions of position — generally the risk decreases from the boundary of the WDA/Z to zero at the boundary of the zero energy area/zone but the calculation of probability of escape cannot demonstrate this.

Frequency of death in deaths per person year, known as annual individual risk of death (IR) (Reference 4) is the most common measure of risk of death used in compiling national statistics, where it is calculated on an actuarial basis as the ratio of deaths from a particular activity in a year divided by the number of people that participated in that activity in that year.

It is the accepted measure of risk for accidental death whilst at work and is the preferred measure used by UK Government Departments for setting safety standards (References 5 and 6) and is the measure of risk specified in the policy for range safety in the UK (Reference 7).

The calculation for the frequency of deaths per person year is derived from probability of escape by including two additional terms: (1) the frequency, i.e. number, of rounds fired in a year, and (2) the exposure i.e. the proportion of time that people are present when the weapon system is in use:

$$\begin{array}{ccccccccc} F_{D/PY} & = & F_{R/Y} & \times & P_{D/H} & \times & P_{H/R} & \times & E_{Y/PY} \\ \dfrac{\text{deaths}}{\text{person}\cdot\text{year}} & = & \dfrac{\text{rounds}}{\text{year}} & \times & \dfrac{\text{deaths}}{\text{hit}} & \times & \dfrac{\text{hits}}{\text{round}} & \times & \dfrac{\text{years}}{\text{person}\cdot\text{year}} \end{array} \tag{3}$$

This is more complicated than probability of escape. To calculate probability of hit a more complicated trajectory calculation is required. Where probability of escape only requires the intersection of the trajectory with the terrain to be found, probability of hit requires those parts of the trajectory where the projectile would hit someone standing on the terrain. To calculate individual risk population data is also required.

3. Terrain Modelling

Most effectiveness calculations use a simple model of terrain, which even if it is not flat has no other properties other than height. For WDA/Z the impact and post-impact models depend on the angle of incidence between trajectory and the terrain and the combination of projectile and terrain material properties. Hence the terrain model has to provide height, slopes, and terrain material. In addition the impact and post-impact models are sensitive to slopes and the model has to be able to take into account uncertainty in the terrain slopes.

A blend of digital terrain data and a triangulation created from a survey is used. Trajectories are calculated in a local Cartesian coordinate system with the y axis aligned with the local vertical with its origin at local sea level. Both the digital terrain data and survey are initially created in geographic or geodetic coordinate systems and are transformed to the local Cartesian coordinate system before use in trajectory calculations.

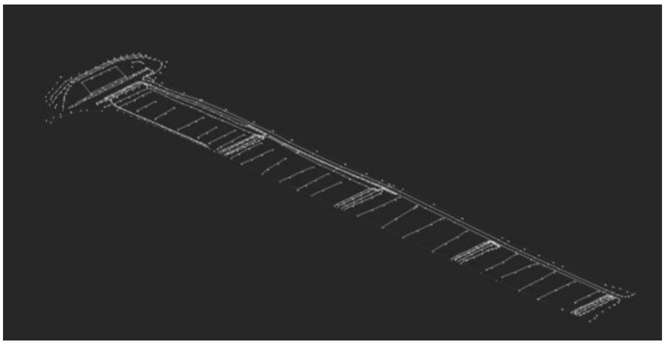


Figure 1 – Survey points and polyline constraints for a triangulation.

The survey produces a geo referenced irregular grid of points on the terrain surface with these chosen so that areas with different materials will be distinct in the final model. The points are joined together using 3D polylines, and these are used as constraints for the triangulation. An example is shown in Figure 1.

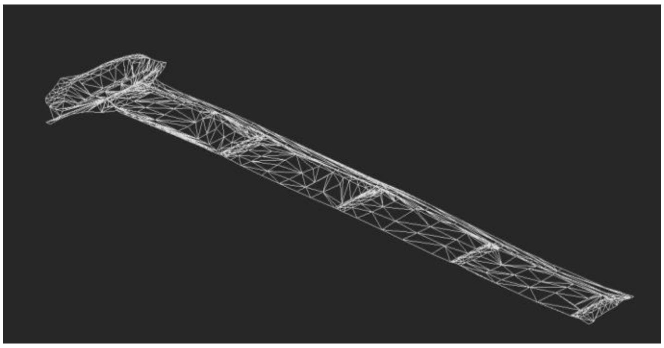


Figure 2 – Constrained triangulation representing the model topography.

The constrained triangulation creates the model topography with each triangle having a specified surface type and therefore specific ricochet characteristics. An example is shown in Figure 2. The final range model should be a smooth surface and accurate digital depiction of the reality of the range floor.

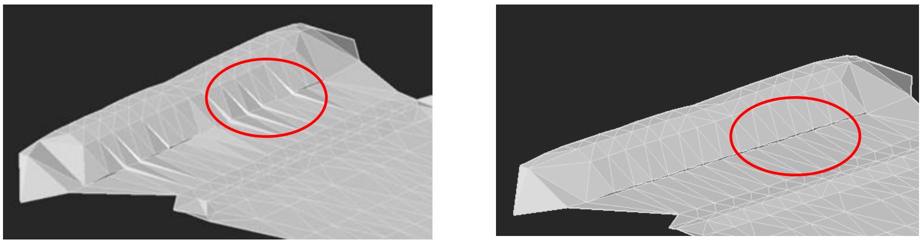


Figure 3 – Unconstrained triangulation (left) and constrained triangulation (right).

Care must be taken when defining the 3D polylines as they act as constraint boundaries for the generation of the triangles. The detail shown in Figure 3 illustrates how incorrect placement of the polyline causes the model to triangulate between incorrect points and therefore create peaks in the terrain topography which are not a true representation of the range terrain.

Once the terrain model has been produced it can then be combined with a digital terrain model. When the combined models are being used the survey model takes precedence and the background model is only used if the point (x,z) is not in the survey model. The terrain model returns the height, the slopes and also surface type and roughness which are used in the impact and post-impact models for specified location:

$$y = y(x,z) \quad y_x = \frac{\partial y}{\partial x}(x,z) \quad y_z = \frac{\partial y}{\partial z}(x,z) \quad R = R(x,z) \quad S = S(x,z) \qquad (4)$$

4. Impact Modelling

Impact models allow the outcome of an impact, between a projectile and the terrain, to be determined as a function of some of the impact parameters. Traditionally such models have been described as ricochet models. The more general term has been adopted as we consider projectile pieces and projectile break-up as well as ricochet. Impact is a complex problem and depends on many parameters of the terrain, the projectile and its trajectory. In order to produce models that are not purely theoretical and can be populated using trials data many simplifications have been made.

The models used are dependent on the terrain material type, the projectile, the impact angle, and the impact velocity only. A different model and/or different model parameters may be used for each combination of material type and projectile. Where a projectile has multiple impacts all impacts are modelled in the same way. At impact the terrain is modelled as a plane tangential to the terrain surface. This means that, for example, the possibility that projectiles could perforate ridges is ignored. Impact has no effect on the terrain i.e. any change in shape and/or material behavior that may occur in real impacts is ignored.

The models are illustrated using data from small calibre ammunition, 5.56 mm ball and 7.62 mm tracer, impacting on damp sand at a range of 100 metres.

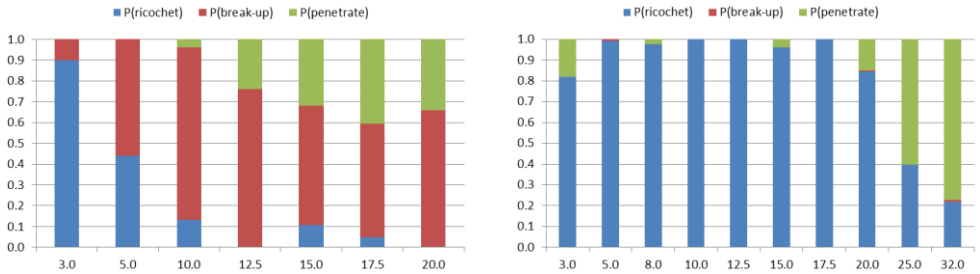


Figure 4 – Probabilities of ricochet, break-up and penetration against impact angle for 5.56 mm ball (left) and 7.62 mm tracer (right).

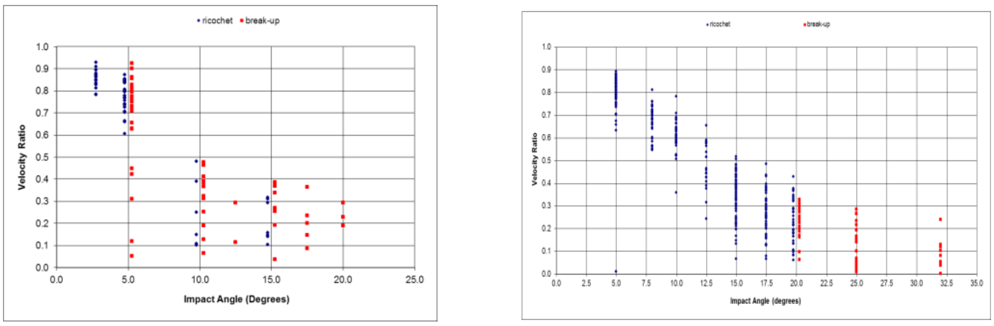


Figure 5 – Speed ratio against impact angle for 5.56 mm ball (left) and 7.62 mm tracer (right) for ricochet (blue) and break-up (red).

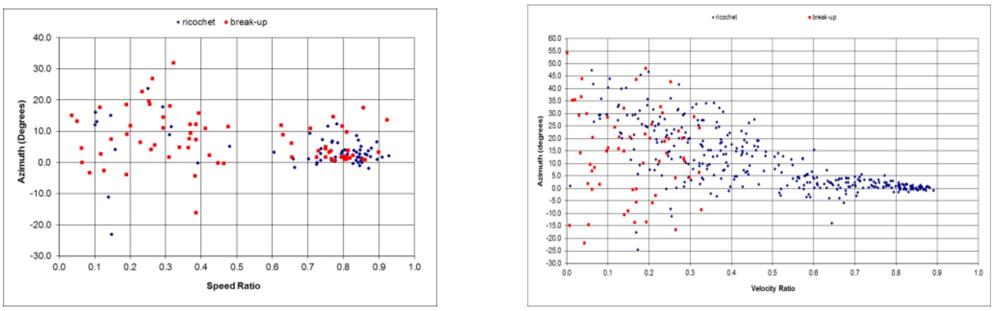


Figure 6 – Exit azimuth angle against speed ratio for 5.56 mm ball (left) and 7.62 mm tracer (right) for ricochet (blue) and break-up (red).

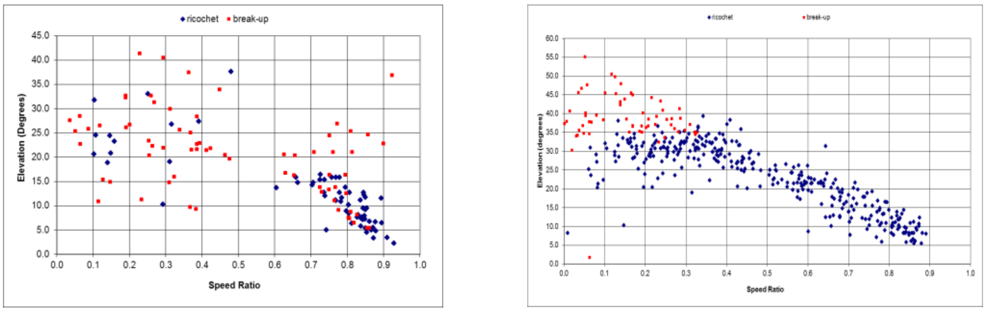


Figure 7 – Exit elevation angle against speed ratio for 5.56 mm ball (left) and 7.62 mm tracer (right) for ricochet (blue) and break-up (red).

There are a number of features of the behaviour evident in the diagrams. The 7.62 mm tracer is much more robust than the 5.56 mm ball with the 7.62 mm tracer hardly ever breaking up but the 5.56 mm ball breaking up almost all the time as the impact angles get above 10 degrees. The speed ratio decreases with increasing impact angle. The exit elevation increases with decreasing speed ratio and the angles for break-up have much more spread than for ricochet.

The impact is modelled as follows. The impact angle, the acute angle between the tangent to the trajectory and the plane tangential to the terrain surface, is calculated. A homogeneous transformation

$$\boldsymbol{T} = \left[\begin{array}{ccc|c} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & \sin\psi & 0 \\ 0 & -\sin\psi & \cos\psi & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{5}$$

is applied to the augmented velocity vector $\boldsymbol{v} = \left[u \quad v \quad w \mid 1\right]^{\mathrm{T}}$ where

$$\psi = \arctan\left[\frac{-y_z(x,z)}{1}\right] \qquad \theta = \arctan\left[\frac{-y_x(x,z)}{\cos\psi - \sin\psi \cdot y_z(x,z)}\right] \tag{6}$$

and $y_x(x,z)$, $y_z(x,z)$ are the terrain slopes at the impact point. This transformation would make the tangent plane to the terrain surface parallel to the horizontal plane with its normal pointing upwards and hence the impact angle is given by

$$I = \left| \arctan\left(-v' \Big/ \sqrt{u'^2 + w'^2} \right) \right| \quad (7)$$

where $[u', v', w']^{\mathrm{T}}$ is the transformed velocity. The slopes have a random adjustment made to it $I' = N(0, R/5)$ where R is the surface roughness at the impact point.

On impact the projectile ricochets, breaks up, or penetrates the terrain and comes to rest. These three outcomes are modelled as a multinomial distribution with the probability of ricochet P_{R} and the probability of break-up P_{B} given as functions of impact angle I and velocity $V_I = \sqrt{u^2 + v^2 + w^2}$. The probability of penetration P_{P} is simply the complement of the sum of these two probabilities i.e.

$$\begin{aligned} &0 \le P_{\mathrm{R}}(I,V) \le 1 \\ &0 \le P_{\mathrm{B}}(I,V) \le 1 \\ &0 \le P_{\mathrm{P}}(I,V) = 1 - \left[P_{\mathrm{R}}(I,V) + P_{\mathrm{B}}(I,V) \right] \le 1 \end{aligned} \quad (8)$$

When the projectile ricochets the models should produce the velocity V_R, elevation E_R, and azimuth A_R. The elevation and azimuth are initially specified relative to the tangent plane to the terrain surface. The velocity vector in this coordinate system is

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix}_{\mathrm{R}} = V_{\mathrm{R}} \begin{bmatrix} \cos E_{\mathrm{R}} \cos A_{\mathrm{R}} \\ \sin E_{\mathrm{R}} \\ \cos E_{\mathrm{R}} \sin A_{\mathrm{R}} \end{bmatrix} \quad (9)$$

The velocity vector is transformed into the true coordinate system by applying the inverse of the homogeneous transformation used in obtaining the impact angle to the augmented velocity vector $[u', v', w']_{\mathrm{R}}^{\mathrm{T}}$ i.e.

$$\begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}_{\mathrm{R}} = \boldsymbol{T}^{-1} \begin{bmatrix} u' \\ v' \\ w' \\ 1 \end{bmatrix}_{\mathrm{R}} = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi & 0 \\ 0 & \sin\psi & \cos\psi & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|c} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} u' \\ v' \\ w' \\ 1 \end{bmatrix}_{\mathrm{R}} \quad (10)$$

A similar calculation is made for projectiles that break up.

5. Post-Impact Modelling

Post-impact models allow the aerodynamic behaviour of projectiles or projectile pieces to be determined as a function of both the impact parameters and the outcome of the impact models. As with impact, the post-impact behaviour is complex and depends on many parameters of the impact and projectile. Although some theoretical models exist (Reference 8) they are not practical for the large scale simulations involved in this probabilistic methodology. Empirical models, derived from analysis of trials data, have been developed.

The models are dependent on the terrain or object material type, the projectile, the impact angle, the impact velocity, and the angle of turn only. A different model and/or different model parameters may be used for each combination of material type and projectile. Where a projectile has multiple impacts all impacts and subsequent post-impact flights are modelled in the same way. The post-impact aerodynamic behaviour is the principal reason for limiting the number of impacts that are modelled for each projectile — in nearly all cases where projectiles have been found after ricochet the projectiles are lying on top of surface of the terrain. This suggests that the projectiles do not fly in a stable manner and thus subsequent impacts should not be modelled in the same way as the first impact. The post-impact model results in some projectiles having high constant drag. As this is suitable for the pieces that arise when a projectile breaks up there is usually no need to deal with the aerodynamic behaviour of fragments in a separate model. One significant departure from this is required for dealing with armour piercing fin-stabilised (APFSDS) projectiles. Break up will produce a number of cylindrical pieces of the original projectile and an aerodynamic model for tumbling cylinders is required but this is not covered here.

On impact a projectile may be damaged and on exit the projectile or pieces of it will almost certainly tumble. Projectile damage and tumbling will both increase drag, but while tumbling may be a transient phenomenon, increased drag due to projectile damage is likely to apply over the whole trajectory. The long term drag is modelled using a "damage" factor D, which defines the limiting, steady state drag of a projectile post-impact. The transient behaviour is modelled using a "tumble" factor T, which is multiplied by an exponential term that allows for decay to the long term drag. The rate at which drag reduces to the long term value may be controlled by a "re-stabilization" factor R. The post-impact drag coefficient is obtained by multiplying the free flight drag coefficient by a combination of factors:

$$C_{DPI}(t) = D\left\{1 + T\exp\left[-R(t - t_I)\right]\right\}C_{DFF}. \qquad (11)$$

Empirical distributions of these factors are constructed from trials data. The distribution of the damage factor is a function of turn angle β and impact velocity V_I. The distribution of the initial drag factor, which is the product of the damage factor and the tumble factor, is a function of impact angle and impact velocity. The tumble factor is derived from the initial drag and the damage factor. The distribution of the re-stabilization factor is a function of tumble factor and impact velocity. These relationships may be summarized by the following

$$\begin{aligned}
\beta &= \arccos\left(u_I u_P + v_I v_P + w_I w_P\right) = \arccos\left(u_I' u_P' + v_I' v_P' + w_I' w_P'\right)\\
D &\equiv D(\beta, V_I)\\
D_0 &\equiv D_0(I, V_I) = D \cdot T \qquad (12)\\
T &= D_0/D - 1\\
R &\equiv R(T, V_I)
\end{aligned}$$

where $\mathbf{v}_I = \left[u_I \quad v_I \quad w_I\right]^T, \mathbf{v}_P = \left[u_P \quad v_P \quad w_P\right]^T$ are the impact and post-impact velocity vectors in the trajectory coordinate system and $\mathbf{v}_I' = \left[u_I' \quad v_I' \quad w_I'\right]^T, \mathbf{v}_P' = \left[u_P' \quad v_P' \quad w_P'\right]^T$ are the same velocity vectors relative to the tangent plane to the impact surface.

As with the impact models the post-impact model is illustrated using data from small calibre ammunition, 5.56 mm ball and 7.62 mm tracer, impacting on damp sand at a range of 100 metres.

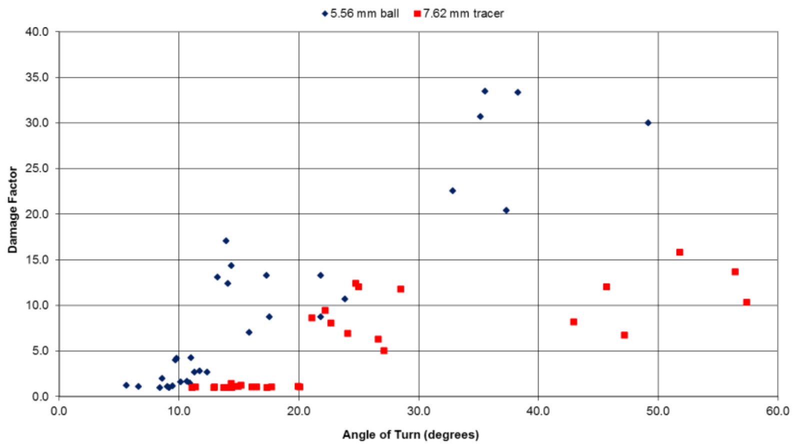


Figure 8 – Damage factor D against angle of turn β for 5.56 mm ball (blue) and 7.62 mm tracer (red) for ricochet.

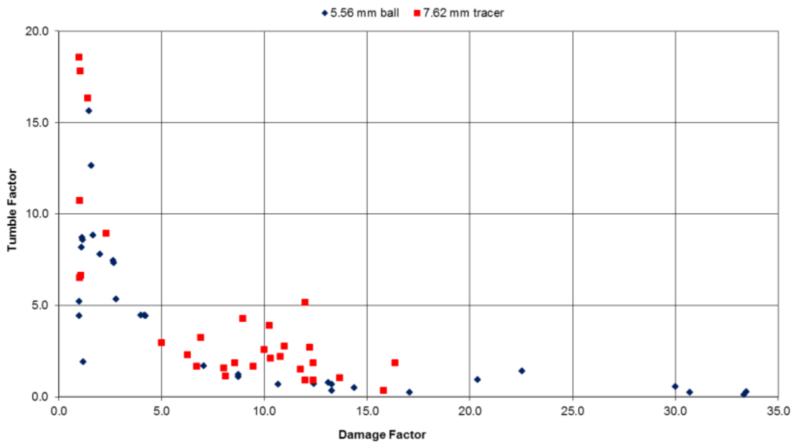


Figure 9 – Tumble factor T against damage factor D for 5.56 mm ball (blue) and 7.62 mm tracer (red) for ricochet.

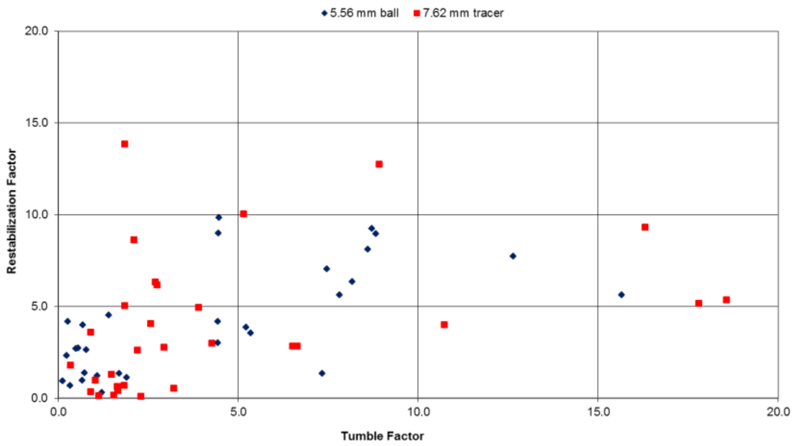


Figure 10 – Restabilization factor R against tumble factor T for 5.56 mm ball (blue) and 7.62 mm tracer (red) for ricochet.

There are fewer data for post-impact flight than for impact. This is because the early trials only used witness screens with the exit azimuth and elevation angles calculated from straight lines between impact locations on the target and the holes in the witness screens. Later trials also used radars and it became evident that the trajectories were not straight. This became apparent from comparing the exit azimuth and elevation angles obtained from the witness screens with those from analysis of the radar signals.

There are discontinuities in damage factor for both types of ammunition with, for example for 7.62 mm tracer the damage factors for angles of turn greater than 20 degrees being greater than 1. Those for angles of turn at 20 degrees or less are equal to 1 signifying that the long term drag post-impact, after restabilization, is the same as that for free flight. Where projectiles have been recovered during trials those exhibiting this behaviour were undamaged. Conversely those with damage factors above 1 were damaged, with the amount of damage increasing with increasing damage factor.

High tumble factors are seen where the damage factor is low with the tumble factors decreasing as the damage factors increase. The largest damage factors are presumed to correspond to tumbling fragments where the aerodynamic forces are independent of orientation.

6. Conclusions

In this paper we described the current approach when considering the probabilistic methodology for the calculation of a WDA. We briefly outlined the simulation process and covered the different measures of hazard and risk that are used. Full details, including specific data requirements, are provided in Reference 1.

We also provided details for two models with additional requirements – the modelling of terrain, which requires more detail than in effectiveness calculations, and the modelling of impact and post-impact, which is not considered at all in effectiveness calculations. There are other additional models used for calculating WDAs and most models require additional considerations. Further details covering these points are given in References 2 and 3.

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