Solution 2:

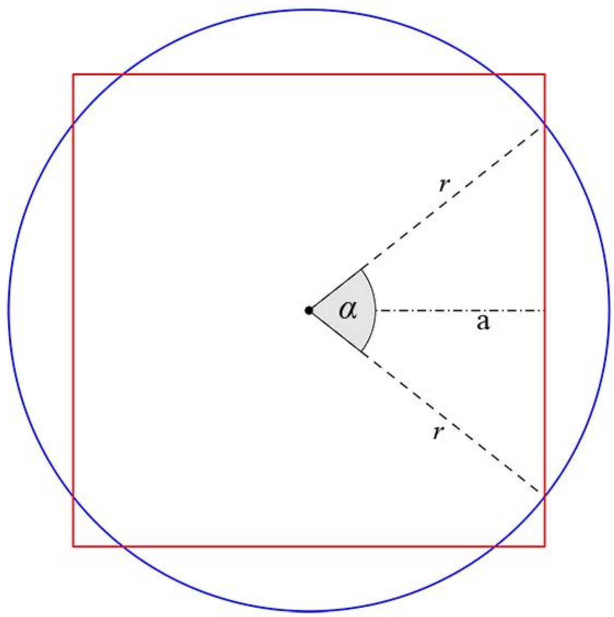
Area of circle outside of the square

First, we should derive a relationship between the radius of the circle and the side length of the square. Let the square have sides of length $2\mathrm{a}$, and the circle have a radius of length r.

Using $C = 2\pi r$, $$8\mathrm{a} = 2\pi r$$

Rearranging, $$\frac{\mathrm{a}}{r} = \frac{\pi}{4} \qquad (1)$$

Now that we have this relationship, we can build a general case, and use trigonometry to find the important areas.



Drawing the shapes in this way, we can build a right-angled triangle with hypotenuse r and side a, and use this to find the angle α, and the area of the circle segment outside the square.

Using $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $$\cos\frac{\alpha}{2} = \frac{\mathrm{a}}{r}$$

Rearranging, $$\alpha = 2\cos^{-1}\frac{\mathrm{a}}{r}$$

Using equation (1), $$\alpha = 2\cos^{-1}\frac{\pi}{4} \qquad (2)$$

To find the area of the segment, $\mathrm{S_g}$ outside of the square, we use the area of the triangle, T, subtracted from the area of the sector, $\mathrm{S_c}$.

Sector Area

Using $\text{Area of Sector} = \frac{1}{2}r^2\theta$ $$\mathrm{S_c} = \frac{r^2\alpha}{2}$$

Triangle Area

Using $\text{Area of Triangle} = \frac{1}{2}ab\sin C$ $$\mathrm{T} = \frac{r^2\sin\alpha}{2}$$

Segment Area

Using $\mathrm{S_g} = \mathrm{S_c} - \mathrm{T}$ $$\mathrm{S_g} = \frac{r^2(\alpha - \sin\alpha)}{2}$$

We multiply this value by four to find the total area outside of the square, and divide the resulting value by the entire circle area. This gives the fraction of the circle area that lies outside of the square.

Multiply by 4: $$4\mathrm{S_g} = \frac{4r^2(\alpha - \sin\alpha)}{2} = 2r^2(\alpha - \sin\alpha$$

Divide by circle area: $$\frac{4\mathrm{S_g}}{\mathrm{C}} = \frac{2r^2(\alpha - \sin\alpha)}{\pi r^2} = \frac{2(\alpha - \sin\alpha)}{\pi} \;)$$

Using equation (2), $$\frac{4\mathrm{S_g}}{\mathrm{C}} = \frac{2(2\cos^{-1}\frac{\pi}{4} - \sin[2\cos^{-1}\frac{\pi}{4}])}{\pi}$$

Simplifying, $$\frac{4\mathrm{S_g}}{\mathrm{C}} = \frac{4\cos^{-1}\frac{\pi}{4} - 2\sin[2\cos^{-1}\frac{\pi}{4}]}{\pi}$$

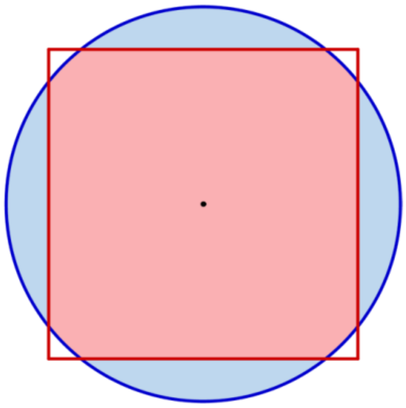
So we find that the fraction of the circle area that lies outside the square is:

$$\frac{4\cos^{-1}\frac{\pi}{4} - 2\sin[2\cos^{-1}\frac{\pi}{4}]}{\pi} = 0.2308 \text{ (4 s.f.)}$$

Area of square outside of the circle

We can find the corner areas using the Segment area we have already found. The area of the square *inside* the circle (red) is found by subtracting the segment areas (blue) from the area of the circle (see diagram).

We then subtract this area from the square to find the area *outside*.



So, $$\text{Area Inside Circle} = \text{Circle Area} - \text{Segment Areas}$$

Subbing in, $$\text{Area Inside} = \pi r^2 - 2r^2(\alpha - \sin\alpha)$$

Subtracting from square, $$\text{Area Outside} = \text{Square Area} - [\pi r^2 - 2r^2(\alpha - \sin\alpha)]$$

Simplifying, $$\text{Area Outside} = (2\mathrm{a})^2 - \pi r^2 + 2r^2(\alpha - \sin\alpha)$$

$$\text{Area Outside} = 4\mathrm{a}^2 - \pi r^2 + 2r^2(\alpha - \sin\alpha)$$

We now divide by the area of the square to find the desired fraction.

$$\frac{\text{Area outside}}{\text{Total Area}} = \frac{4\mathrm{a}^2 - \pi r^2 + 2r^2(\alpha - \sin\alpha)}{4\mathrm{a}^2} = 1 + \frac{2r^2(\alpha - \sin\alpha) - \pi r^2}{4\mathrm{a}^2}$$

Factorise $\left(\frac{r^2}{\mathrm{a}^2}\right)$, $$\frac{\text{Area outside}}{\text{Total Area}} = 1 + \left(\frac{r^2}{\mathrm{a}^2}\right) . \frac{2(\alpha - \sin\alpha) - \pi}{4}$$

Then, using equation (1), $$\frac{\text{Area outside}}{\text{Total Area}} = 1 + \left(\frac{4^2}{\pi^2}\right) . \frac{2(\alpha - \sin\alpha) - \pi}{4}$$

We can then use equation (2) to put this in terms of known values, and calculate to find the proportion:

$$\text{Area of square outside of circle} = 0.02068 \ _{(4\text{ s.f.})}$$