The Sultan's Daughters

Question

A sultan has 100 daughters. A commoner may be given a chance to marry one of the daughters, but he must first pass a test. He will be presented with the daughters one at a time. As each one comes before him she will tell him the size of her dowry, and he must then decide whether to accept or reject her (he is not allowed to return to a previously rejected daughter). However, the sultan will only allow the marriage to take place if the commoner chooses the daughter with the highest dowry. If he gets it wrong he will be executed!   The commoner knows nothing about the distribution of dowries. What strategy should he adopt?

Solution

One strategy the commoner could adopt is simply to pick a daughter at random. This would give him a 1/100 chance of getting the correct daughter.   Can he do better?

In order to do better he needs more information, and the only way to get more information is to take a sample of the dowries before making a choice. Because of the conditions imposed by the Sultan he doesn't have much freedom to determine his sample. What he *can* do is to use the first n daughters to get a sample of dowries. The best strategy is then to choose the first succeeding daughter who comes with a dowry larger than any of those he's seen so far.

The trick is to determine the value of n that maximises his chances of success. If n is too small the largest dowry he samples will probably be too small to make a difference. If n is too high he is likely to pass by the highest dowry.

The expression for calculating the probability that the commoner will succeed in choosing the right daughter, as a function of n is given, rather simply, as:

$$p\left(n\right) = \frac{n}{100} \sum_{k=n}^{99} \frac{1}{k}$$

We can plot this as a function of n:

Probability of getting highest dowry after rejecting the first n
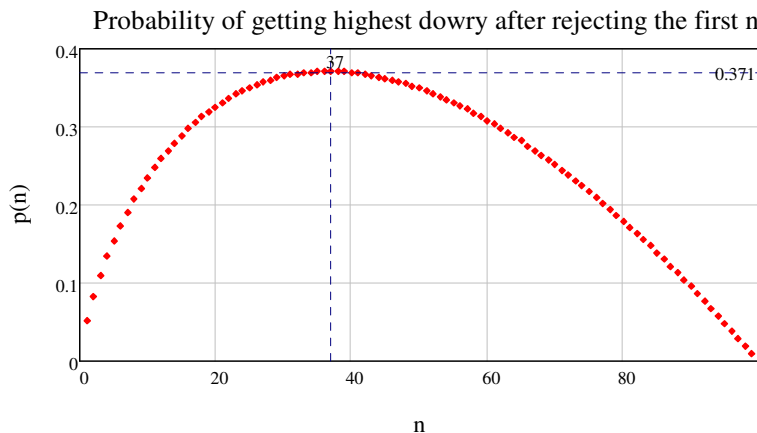
If we look at the actual values predicted by the above expression we find that the probability of the commoner accepting the daughter with the highest dowry is about 37% if he rejects the first 37 daughters and then chooses the next one whose dowry is greater than any he's seen so far. This is a fraction 1/e of the total number of daughters (rounded to the nearest integer) and is significantly better than just choosing at random!

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