# Editorial

**One of the most enjoyable aspects** of being a mathematician in academia is encountering diverse and exciting challenges. One such incident occurred recently when a refrigeration engineer brought two tubes into my office and plonked them on my desk. 'Can you model intrinsic weight gain and predict time to saturation?' she asked. As so often happens, my mouth uttered 'Yes' with little input from my brain and despite my ignorance.

These right annular cylinders were 127 mm long and sealed at both ends, with external diameter 124 mm and bore 70 mm, as illustrated in Figure 1. One of them rattled ominously, though my visitor explained that they were sections of pipe lagging and that the noise was caused by a desiccant. Cold liquid refrigerants pass through the pipes, with the adverse effect of causing costly condensation damage. The engineer had executed the following well designed experiment, which was repeated many times for tubes of various materials and dimensions, in order to address this problem.

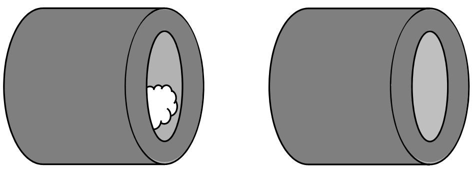

*Figure 1: Refrigeration pipe lagging, test tube and blank tube.*

Both tubes are initially dry and then placed in a humid environment for several weeks during which they are weighed regularly. The intrinsic weight gain (IWG) at any time point is the difference between the masses of the test tube (with desiccant) and the blank tube (without desiccant). If the tubes were saturated, IWG would increase linearly with time. However, the saturation point for this experiment occurs only asymptotically, so we consider approximate saturation that corresponds to near linearity of IWG increase.

Our aim is to model and predict approximate saturation points for these and other tubes, ideally without the need for further experimentation. To develop a mathematical model for IWG (grams), define parameter $\rho$ as the rate (grams per day) of inflow to a tube when dry, parameter $\mu$ as the mass (grams) of water that the tube holds when saturated and function $w(t)$ as the mass (grams) of water in the tube at time $t$ (days).

For a test tube at time $t$, the rate of outflow to the internal desiccant is $\rho w(t)/\mu$ and the rate of inflow from the external environment is $\rho\{1 - w(t)/\mu\}$. Hence, the rate of water absorption by the tube (inflow minus outflow) is

$$w'(t) = \rho\left\{1 - \frac{2w(t)}{\mu}\right\}$$

and we can solve this linear, first-order ordinary differential equation to give

$$w(t) = \frac{\mu}{2}\left(1 - \mathrm{e}^{-2\rho t/\mu}\right). \tag{1}$$

The mass of water in the desiccant at time $t$ is equal to the accumulated outflow

$$\int_0^t \frac{\rho w(s)}{\mu}\mathrm{d}s = \frac{\rho t}{2} - \frac{\mu}{4}\left(1 - \mathrm{e}^{-2\rho t/\mu}\right). \tag{2}$$

For a blank tube at time $t$, the rate of water absorption by the tube (inflow only) is

$$w'(t) = \rho\left\{1 - \frac{w(t)}{\mu}\right\},$$

which we can solve to give

$$w(t) = \mu\left(1 - \mathrm{e}^{-\rho t/\mu}\right). \tag{3}$$

The IWG $f(t)$ at time $t$ is then determined by adding equations (1) and (2) and subtracting equation (3), which simplifies to give

$$f(t) = \frac{\rho t}{2} - \frac{\mu}{4}\left(3 - 4\mathrm{e}^{-\rho t/\mu} + \mathrm{e}^{-2\rho t/\mu}\right). \tag{4}$$

Note that $f(0) = 0$ and $\lim_{t\to\infty} f(t)/g(t) = 1$ where $g(t) = \rho t/2 - 3\mu/4$ is a linear function of time as expected. Now to check whether this theory has any practical value. My colleague presented me with many data sets and I fitted model (4) to some of these, using the method of least squares for convenience. Figure 2 displays one of the scatter plots with fitted curve and asymptotic line.

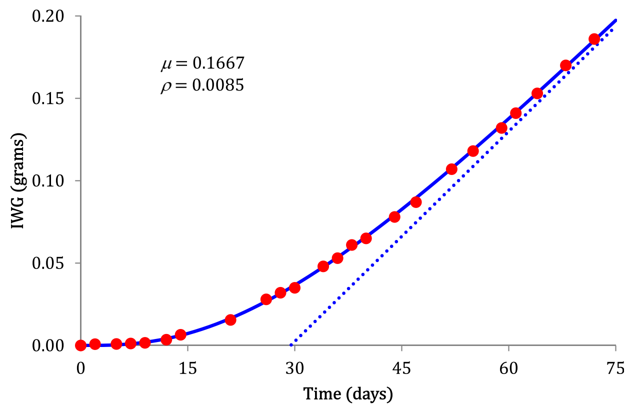


*Figure 2: Sample IWG data with fitted curve $f(t)$ and asymptotic line $g(t)$.*

The next task is to predict the approximate time to saturation, $t^*$. A reasonable, dimensionless measure of asymptotic behaviour at any time point $t$ is the relative discrepancy

$$d(t) = \frac{f(t) - g(t)}{g(t)}$$

and another might be the second logarithmic derivative of $f(t)$. It is a challenging exercise to prove that $d(t)$ is a strictly decreasing function of time, in which case we can determine the time to near saturation by solving $d(t) = \delta$ for specified tolerance $\delta$. This leads to the nonlinear equation

$$\frac{\mu\mathrm{e}^{-\rho t/\mu}}{2\rho t - 3\mu}\left(4 - \mathrm{e}^{-\rho t/\mu}\right) = \delta, \tag{5}$$

which we solve numerically for $t$ to evaluate $t^*$. Setting $\mu = 0.1667$ and $\rho = 0.0085$ as above and $\delta = 0.05$ arbitrarily, we obtain a predicted time to near saturation of $t^* = 62$ days for the experiment displayed in Figure 2.

Finally, there are two principal methods for predicting the approximate time to saturation for a specific lagging that is to be used in future applications. The first approach is based on a regression model, which requires no specific experimentation and relies on the tube's characteristics, such as material type and wall thickness. However, this needs an extensive set of training data including observed times to near saturation, in order to estimate regression parameters well and avoid extrapolating to accommodate new materials and dimensions. The second approach is based on fitting model (4) to IWG data collected for several days, to determine the parameters $\mu$ and $\rho$, and then solving equation (5) to evaluate $t^*$. Although this approach requires some specific experimentation for the new lagging, it needs no training data and readily accommodates new materials and dimensions.

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As you might imagine, I was pretty pleased with the overall success of this analysis. Although I readily admit that some of my research fails miserably, this was a satisfying outcome. Perhaps we should consider the matter of publication bias another day. Just like the variety of unexpected challenges that spring up in my profession, April's issue of *Mathematics Today* contains a similarly diverse and exciting range of feature articles, which I hope will be of considerable interest and inspire you to put finger to keypad. For sure, the content reveals just how relevant mathematics is to everybody and leaves us in no doubt about its ubiquity and importance. Of course, maths is good fun too.

In particular, we have two *Maths Matters* papers that should be compulsory reading for everyone. The entertaining topic of *Small Worlds by Design* discusses clinking glasses at parties with an underlying discrete optimisation problem that affects us all. Similarly, *A Short Monograph on Exposition and the Emotive Nature of Research and Publishing* offers some astute observations with a slight element of provocation for our amusement. We also have an excellent article on *Generalised Functions and Differential Equations*, along with a comparison of traditional and IT-based statistics modules in higher education, and the ever-popular *Historical Notes*.

Anonymous authorship of *Urban Maths* articles generally ceases after this issue, though the feature will continue under invited authorship. It is with great pleasure and gratitude that I can now reveal the identity of the legendary 'A. Townie' who contributed so many of these excellent articles in past issues. This is former Vice President (Communications), Dr Alan Stevens, ably assisted by Rob Ashmore and Ellis-Fauve Cresswell, so thank you all. I would also like to express my sincere appreciation to the prolific author of the *A Doctor Writes* articles, who modestly prefers to remain incognito. Happy reading!

**David F. Percy CMath CSci FIMA**