Historical Notes: A Mathematical Inscription from Ancient Pergamon

Richard Simpson MIMA, BSHM

During the latter part of the 18th century, the wonderfully named polymath Comte Marie-Gabriel-Florent-Auguste de Choiseul-Gouffier, French ambassador to the Ottoman Empire based in Constantinople, embarked on an extensive series of travels around the Aegean and Asia Minor. Writing up his journeys and observations in the similarly wonderfully named *Voyage Pittoresque de la Grèce* [1], he copied the details of a particular inscription, written in Greek, found at Pergamon and dating from the 2nd century CE during the Roman Antonine period. This inscription is the only ancient one, known to me, that contains any mathematics. Sadly the stone itself would now appear to be lost.

The stone was erected by a Roman, Aelius Nikon, described as an architect and believed to be the father of the Roman medical doctor, Galen. Choiseul-Gouffier noted the inscription, not initially because of its mathematical content, but because it was an example of 'isopsephy' – also known in other contexts as 'Gematria' – where each line of the inscription is designed to add up numerically to the same total. It should be remembered that ancient Greek contained no number signs and that numerals were denoted by letters making this addition a straightforward process. As an aside, the requirement for the even then archaic Greek letters digamma and sampi to make the number system work is interesting in itself. Luz [2] gives an account of isopsephy, and this inscription in particular. Unfortunately this book is only available in German. The long inscription is abridged in Figure 1.

ΑΚΑΚΙΑΔΕΕΠΙΧΡΗΜΑΤΙΣΜΟΣ	ΆΨΚϚ
ΙΔΙΑΔΗΔΙΑΤΟΥΔΩΡ	ΆΨΚϚ
ΘΕΖΕΙΕΙΣΑΠΑΙΩΝΟΣ	ΆΨΚϚ
ΚΑΙΛΑΒΡΟΝΑΜΑΕΙΣΕΝΚΟΣΜΩ	ΆΨΚϚ
ΕΠΑΓΑΘΑΤΟΙΣΤΕΧΝΙΤΑΙΣ	ΆΒΡΝϚ

.
.
41 lines in total
.
.

ΜΥΣΩΝΑΡΞΕΙΓΕΩΜΕΤΡΙΑ	΄Γ

Figure 1: Part of the inscription. See [3] for the complete inscription in miniscule.

The top lines add to 1726, the second set to 2156 and the remainder to 3000. Naming himself and his profession in the top stanzas, Nikon goes on to describe his admiration for Archimedes' famous theorem on the sphere and cylinder where the volumes and surface areas are both in the ratio 2:3 – and he also introduces the cone into the set of solids, although Archimedes did not do this directly, to give ratios 1:2:3 for volumes. Then, at lines 23 onwards, Nikon submits his observation that if the sphere and cylinder are surrounded by a cube, the relevant ratios for both volume and surface area become 22:33:42 – somewhat astonishing! 42 is of course an approximation because the cube does not contain π but it is clear that to find these values Nikon used 22/7 as his value for the constant, consistent with the upper bound found by Archimedes. As a piece of serendipity, the final line translates as the mantra 'Geometry exerts its power over all things' – would that that is inscribed over every classroom in the country!

ΑΚΑΚΙΑΔΕΕΠΙΧΡΗΜΑΤΙΣΜΟΣ

1+20+1+20+10+1+4+5+5+80+10+600
+100+8+40+1+300+10+200+40
+70+200 = 1726

ΑΨΚϚ

1000+700+20+6 =1726

Figure 2: Isopsephy applied to the first line of the inscription.

Choiseul-Gouffier, undaunted, translated the inscription himself, and realising its potential mathematical importance, asked his friend, the French astronomer, Jean Bapiste Delambre to make some observations about it. These are also detailed in [1] and make interested reading, particularly the seemingly convoluted way Delambre set about proving Nikon's assertion, compared to the way we might perhaps direct a year 9 (aged 13–14) class to do it today.

A later traveller and British diplomat, William Turner, also noted the inscription in 1820 [4] and this appears to be the last time the inscription was actually recorded.

Ancient Pergamon lies in Western Turkey about 10 km from the coast. The modern town of Bergama is built around, and in some places on top of, the ruins. Initially a Lydian city, Pergamon fell to Alexander the Great in 334 BCE, became essentially Greek in nature and benefitted from a number of rulers of some longevity, thus giving the city stability. The last ruler Attalos III died without issue in 133 CE and, in order to avoid a civil war, left the entire kingdom to the Roman Empire in his will. Particularly during the time of Trajan, which incidentally is probably the era of the inscription, the city saw great expansion.

Geographically the city is spectacular. The acropolis sits atop a large hill and the Roman town falls down one side and onto the plain below. The main buildings are the temple and library of Trajan, second only to that at Alexandria; one of the largest and most vertiginous Greek theatres of antiquity built into the side of the citadel (see title photo); and the so-termed Temple of the Egyptian Gods, or Red Hall, a huge complex within the modern town on the plain below. About 2 km away is the fascinating Asklepieion (see Figure 3) or medical centre, one of three in antiquity, where the young Galen must have learnt his trade. Finally a very extensive and impressive aqueduct and piping system brings in water from 20 km away. Since the 1870s, the site has been, and still is being, excavated by German archaeologists. The famous 'Altar of Zeus' with panels rivalling the Parthenon, was removed to Berlin and re-erected to form the centre piece of the Pergamon Museum in that city.

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Figure 3: The Asklepieion with the footprint of a building Nikon may have built in the foreground.

The whereabouts of the inscription remains an enigma. Before 1890 the site was almost completely covered in earth and debris. Early sketches in [1] and [4] attest to this as do excavation photographs taken in the 1900s. It is possible that the inscription was originally mounted in the Lower Agora but Turner said that the inscription was seen built into the wall of the garden of the 'Greek Church' where he attended a mass baptism. This building should be easily found but the events surrounding the relationship between Turkey and Greece in both the 1822 Greek War of Independence and tragic invasion of 1922, probably led to the church's destruction or conversion to a mosque. I have visited Pergamon twice with this in mind and explored a little of the town but failed to find anything. Nor are there any references to the stone in German excavation records, and my conclusion, supported by others, is that it has been long lost.

Aelius Nikon clearly enjoyed writing these kinds of enigmatic verses. One of them is mounted on the wall outside the Bergama Museum. A perhaps surprising and complete record of his known inscriptions appears as an appendix in Thomas [5]. Further, Thomas conjectures in the text what type of buildings Nikon might have designed.

Finally, [1] and [4] make fascinating reading in themselves as travelogues and can been found online. One might conclude that, for travellers, some things have not improved much since 1800.

Acknowledgements

REFERENCES

1 Choiseul-Gouffier, M.G.F.A., Comte de (1809) *Voyage Pittoresque de la Grèce, Vol. 2*, Paris, in French, pp. 167 et seq.

2 Luz, C. (2010) *Technopaignia, Formspiele in der griechischen Dichtung* (Mnemosyne, Supplements), in German.

3 Frankel, M. (1895) *Die Inschriften Von Pergamon: Romische Zeit; Inschriften Auf Thon, Aus: Altertumer Von Pergamon, Bd. 8,2*, Berlin, in German, Inscription no. 333, pp. 244 et seq. (The inscription can also be found in all the standard classical inscription collections and databases.)

4 Turner, W. (1820) *Journal of a Tour in the Levant, Vol. 3*, John Murray, London, pp. 271 et seq.

5 Thomas, E. (2007) *Monumentality and the Roman Empire: Architecture in the Antonine Age*, OUP.