**Developing Intellectual Rigour Through Peer Assessment**

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Abstract

Graduates in mathematics should possess intellectual rigour and reasoning skills, and the ability to present mathematical arguments with accuracy and clarity. Undergraduate students appear to be mainly concerned with the correctness of their arguments, and pay little attention to the need for their arguments to be properly expressed. We propose that students can learn to analyse and evaluate mathematical arguments through structured peer assessment, before applying the same principles to their own work. In this paper we make the case for implementing peer assessment in undergraduate mathematics programmes, and present a structured scheme which involves students working in teams and taking part in collective marking sessions. We also discuss the benefits that the scheme offers to both students and tutors, and present some rather weak statistical evidence which suggests that the scheme can lead to enhanced outcomes for students.

1. The Case for Peer Assessment

According to the subject benchmark statement for mathematics, statistics and operational research, graduates in these subjects should possess intellectual rigour and reasoning skills, and the ability to present mathematical arguments with accuracy and clarity (QAA, 2015). These are not independent: one cannot properly express mathematical arguments without intellectual rigour, and striving to express arguments with accuracy and clarity serves to enhance intellectual rigour.

A basic tenet of academic discourse is that statements should be supported by facts and argument. The primary criterion by which to assess mathematical arguments is their correctness: premises must lead to conclusions by a rigorous process of reasoning and logical deduction. A beautifully expressed but logically flawed argument has no permanent value in mathematical terms, but the value of a logically correct but poorly expressed argument is also questionable if nobody can understand it. Students often appear to be exclusively concerned with the correctness of their arguments, and pay little attention to the need for arguments to be properly expressed. The situation is not helped by the fact that most university tutors are also research mathematicians, and are themselves far more concerned with the correctness of mathematical arguments than their correct expression.

Assessment in mathematics is often concerned with testing students' understanding of certain mathematical ideas, but not their ability to express these ideas. How should one assess work which demonstrates that a student understands the main ideas, but which fails to define its terms, contains ambiguous statements, lacks narrative content and fails to stand as a valid argument in its own right? If we are to promote intellectual rigour and effective reasoning by demanding that mathematical arguments are properly expressed, factors related to correct expression should be included explicitly in assessment criteria. This stands in contrast to more traditional assessment schemes in mathematics, which typically involve awarding a fixed number of marks for various steps in a deductive process, with no marks reserved for how the argument is expressed overall.

Students are continually exposed to well-crafted mathematical arguments, during lectures or in textbooks for example, but this does not appear to greatly enhance their own abilities to express mathematical ideas. Students who are not able to discuss and evaluate the relative merits of different mathematical arguments will inevitably find it difficult to evaluate the quality of their own work. Based on the principle that it is easier to criticise other people's work than it is to criticise your own, we propose that students can better learn to analyse and evaluate mathematical arguments through structured peer assessment, before learning to apply the same processes when preparing written work of their own.

2. Collective Marking Sessions

We present a highly structured scheme for peer assessment, which for the past two years has been implemented as a weekly cycle of collective marking sessions, as part of the second-year *Foundations of Probability and Statistics* module delivered at the School of Mathematics, Cardiff University. The method is decidedly low-tech, and relies only on a single one-page marking form, shown in Table 1. This form can be used in any context where exercises are set as formative assessments. Assessment criteria are shown in Table 2, which students encouraged to interpret for themselves.

A collective marking session is implemented as follows.

1. Teams of students prepare written solutions to exercise questions set by the tutor, which they bring to a collective marking session. A standard marking form (Table 1) is stapled to the front of each script, which are then exchanged between teams.
2. The tutor sketches solutions on the board while marking teams simultaneously assess the quality of their peers' arguments against five broad criteria, namely that good mathematical writing should be *clear*, *concise*, *complete*, *coherent* and *correct* (Table 2).
3. For each question, marking teams write appropriate feedback on their peers' script, then write summary feedback and record a grade for the solution on the marking form. Once all solutions have been assessed, marking teams also write summary feedback and record a grade for the submission as a whole.
4. The marked scripts are then returned to their authors, who collectively reflect on the feedback provided by their peers and identify potential areas for improvement, before recording their responses on the marking form.
5. At the end of the session, the tutor collects all scripts to verify that appropriate feedback has been provided, include additional feedback where necessary, and comment on whether any grades awarded are too generous, or too severe.

The following remarks are based on the author's experience of running collective marking sessions over the past two years.

* It is important to allow sufficient time for students to discuss and negotiate with each other. This means that rather less content can be covered than would be the case in traditional examples classes, where tutors simply run through solutions on the board. The lesson plan shown in Table 3 allocates only 15 minutes for this activity, with the remaining time dedicated to group discussions where the tutor circulates and contributes to discussions as appropriate.
* To encourage students to think for themselves and discourage learning by mimicry, tutors should be careful not to write model answers on the board, but instead provide only sketch solutions which cover the main technical details.
* For students to develop confidence in their critical abilities, tutors should neither dismiss nor correct feedback provided by marking teams, except on matters of fact. Instead, tutors should simply indicate that they disagree, and provide an alternative viewpoint.
* It should be made clear to students that standards for mathematical writing are more stringent than standards for written material provided as part of an oral presentation, with the latter typically being rather terse, with context and detail provided verbally. Students should not necessarily award high grades for work that closely matches what is written on the board, but instead for work that meets the assessment criteria.
* For legibility, teams should write up their work using only blue or black ink, the marking team should write their comments and feedback using red ink, and the tutor should then write additional comments and feedback using a different colour. Tutors should consider taking a stapler and some spare red pens to the sessions.
* At the end of the marking session, students can be encouraged to identify aspects of the subject matter that remain unclear, and record these on the marking form. Common areas of difficulty can then be addressed during subsequent lectures, with additional exercises set as appropriate.
* Completed marking forms can be photocopied or scanned, with copies sent to every student involved with either preparing or marking the submission. A copy can also be kept as a record of engagement.
* Students can be encouraged to cite the sources of their information. Not only is this good academic practice but it might also provide insight into how students learn, and reveal opportunities for enhancing those learning processes.
* Rather than presenting solutions on the board, tutors can provide sketch solutions in printed form and dedicate the entire session to group discussions.

3. Discussion

As well has developing their ability to construct, critique and communicate mathematical arguments, the peer assessment scheme described above has several other benefits for students.

* Students receive prompt and regular feedback on their work.
* Working in groups provides mutual support and encouragement.
* Encouraging students to interpret assessment criteria for themselves helps to develop academic independence and autonomy. To interpret vague criteria is also often a key part of practical problem solving.
* Through awarding grades and providing feedback, students come to better understand the processes involved in assessment, the need for academic judgement and the nature of academic standards.
* Marking sessions provide an opportunity for revision and consolidation, and help to identifying difficult areas.

The collective marking sessions also help to develop students' employability skills.

* Communication skills are developed through learning to properly express mathematical arguments, and through writing constructive feedback in an appropriate manner.
* Collaboration and negotiation skills are developed through working in teams.
* Leadership skills are developed by taking responsibility for assessing the work of others.
* Self-management is developed by reflecting on feedback and planning improvements.

The scheme also has several benefits for tutors, especially where large classes are taught by relatively few people.

* Students working in teams reduces the number of scripts to be checked.
* Checking scripts is quick because most of the marking has already been done.
* Topics that students find difficult can be identified and addressed.
* The scheme provides a non-invasive method of embedding employability skills into existing modules.

We have no statistically significant results to show that taking part in collective marking sessions has resulted in increased learning gain for students. We do however have some evidence of improved student satisfaction in several areas. Over the past four years, the author has delivered the *Foundations of Probability and Statistics* module to second-year students at the School of Mathematics, Cardiff University. In 2013/14 and 2014/15, individual assignments were collected and marked in the traditional way. This was replaced by the collective marking sessions described above during 2015/16 and 2016/17. Table 4 shows the results of the annual Module Evaluation survey for the module over the period. The data indicate that the collective marking sessions have contributed to improved student satisfaction in terms of feedback, employability and overall satisfaction with the module.

The introduction of collective marking sessions in 2015/16 was part of a wider overhaul of the module. The module was previously delivered over two semesters, with 2.5 contact hours per week and a long break over Christmas, but since 2015/16 has been delivered over a single semester with twice the number of contact hours per week, resulting in a more focused learning environment. Video technology for lecture capture was also introduced in 2015/16 and has proved immensely popular with students. Both factors have undoubtedly contributed to the increase in overall student satisfaction over the last two years.

Finally we include some feedback from students regarding the collective marking sessions.

* *The marking sessions improved my way of writing answers and justifying them.*
* *The two-stage marking system by peers and the lecturer is great.*
* *Marking sessions and group homework were especially beneficial.*
* *The peer review sessions forced me to pick out the important points of a question.*
* *I really liked the feedback lessons and how the lecturer checked the work.*

4. Conclusion

We have presented a practical peer assessment scheme mathematics teaching. The scheme has been designed to develop students' intellectual rigour and reasoning skills, their ability to present mathematical arguments with accuracy and clarity, and to enhance their employability. Tutors' workloads should not be significantly increased by implementing the scheme, and may even be reduced. Although we are not able to provide statistically significant results to demonstrate that taking part in collective marking sessions leads to enhanced outcomes for students, we have attempted to justify the method by appealing to sound pedagogic principles together with anecdotal evidence.

References

Quality Assurance Agency for Higher Education (2015) *Subject Benchmark Statement: Mathematics, Statistics and Operational Research*.

Su, F. E. (2015) Some Guidelines for Good Mathematical Writing. MAA FOCUS, Vol. 35 (No. 4):20-22.

Table 1 Marking form for peer assessment.

|  |  |
| --- | --- |
| **Marking Form** | |
| Module Code: .......................................... | Date of Class: ......................................... |
| Team (2-4 students)  1. ..........................................................  2. ..........................................................  3. ..........................................................  4. .......................................................... | Markers (2-4 students)  1. ..........................................................  2. ..........................................................  3. ..........................................................  4. .......................................................... |
| **Instructions**  **Team**: Please staple this form to the front of your script and enter the relevant question numbers in the table below. All work should be written neatly in either **blue** or **black** ink.  **Markers**: Please write your feedback on the script in **red** ink, and enter a grade for each question in the table below along with an overall grade for the submission as a whole. | |
| **Assessment criteria**  Good mathematical writing should be **correct**, **clear**, **concise, coherent** and **complete**. Please choose from among the following grades. There is no Pass or Fail.  A: Very Good B: Good C: Moderate D: Poor E: Very Poor U: Unclassified | |
| |  |  |  | | --- | --- | --- | | Question No. | Grade | Comments | |  |  |  | |  |  |  | |  |  |  | | Overall Grade |  |  | | |
| **Markers' Feedback** | |
| **Team's Response**:  Do you accept the overall grade awarded? Yes ☐ No ☐  How do you plan to improve in view of the feedback received?  Do any of the mathematical aspects of the material remain unclear? | |
| **Tutor's Comments** | |
| *All scripts must be handed in at the end of the class.* | |

Table 2 Assessment criteria for collective marking sessions

|  |  |
| --- | --- |
| Clear | Statements are explicit and unambiguous |
| Coherent | An appropriate narrative is provided |
| Concise | No irrelevant details are included |
| Complete | All relevant details are included |
| Correct | Arguments are precise and logically sound |

Table 3 Lesson plan for a 50-minute peer marking session with three questions. Tutors should ensure that students have sufficient time for discussion and reflection.

|  |  |
| --- | --- |
| Time | Activity |
| 00 - 05 | Staple coversheets, swap scripts, review assessment criteria. |
| 05 - 10 | Tutor presents solution to first question. |
| 10 - 15 | Markers discuss and seek clarification. Tutor circulates. |
| 15 - 20 | Tutor presents solution to second question. |
| 20 - 25 | Markers discuss and seek clarification. Tutor circulates. |
| 25 - 30 | Tutor presents solution to third question. |
| 30 - 35 | Markers discuss and seek clarification. Tutor circulates. |
| 35 - 40 | Markers discuss and write summary feedback. |
| 40 - 45 | Team reflects on feedback received. |
| 45 - 50 | Team records action points in response to feedback. |
|  | Scripts handed in at the end of the session. |

Table 4 Module Evaluation Results 2013/14 - 2016/17. Each entry represents the percentage of students who either *definitely agreed* or *mostly agreed* with the corresponding statement during that year.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Tutor Assessment** | | **Peer Assessment** | |
| **Statement** | **2013/14** | **2014/15** | **2015/16** | **2016/17** |
| The module has helped my personal development by improving my employability skills | 29% | 31% | 48% | 81% |
| I feel confident in communicating the knowledge I have gained on the module | 21% | 26% | 64% | 73% |
| I had a clear sense of what was required of me in the assessments for this module | 64% | 55% | 83% | 95% |
| Feedback was useful to help me to clarify things I did not understand | 73% | 52% | 80% | 91% |
| Overall I am satisfied with the quality of this module | 50% | 52% | 80% | 92% |
| **Number of respondents** | **28** | **27** | **51** | **37** |