Mathematics in Transport Editorial

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Transportation is often described as an *umbrella discipline* because transport professionals come from many and varied backgrounds. Within academia transport groups are often located within either civil engineering or geography departments due to early work in road building and town planning and more recently within business schools due to the need for economics in transport. There are also transport psychologists, sociologists, environmentalists etc., and of course mathematics and mathematical modelling permeate throughout all fields within transportation studies.

A special edition of Mathematics in Transport could include a vast range of possible application areas – often *transport modelling* is assumed to be the main location of mathematics in transport, but whilst this term is understood in a certain context by transportation engineers and planners, mathematicians generally have a broader understanding of what traffic modelling may encompass. Within the transportation profession of government officers and consultants *traffic/transport modelling* is normally taken to mean the application of a software package to perform some form of *assignment,* that is the generation and distribution of travellers to different modes (e.g. car, bus, train) and then to different roads and streets throughout a network. Such models may then be used to answer questions regarding network management and future network planning, for example the benefit of building a new road or piece of infrastructure can be assessed, traffic management schemes can be appraised and adverse environmental emissions estimated.

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Models are generally commissioned by government bodies and may have small geographical scope with a high level of detail such as a single street or alternatively large geographical scope with less fine detail; for example trans-European models including air and maritime links in addition to road modes, which typically show only primary routes.

Increased computing capabilities have facilitated a class of models termed *microsimulation* where each vehicle is modelled as a single unit and network flow is simulated based on certain underlying rules, whereas previously traffic models were developed based on modelling traffic as *fluid flow* and allowing it to reach *equilibrium.* From a mathematical perspective the *traffic model* of whichever type is a collection of mathematical models which are used to define and represent vehicle movements. A classical model will consist of models to generate and then distribute demand between a set of origins and destinations, apply modal split to allocate users to travel mode and then to allocate or *assign* traffic to roads.

Ben Heydecker describes the mathematics implicit in the assignment/routing part of such a classical model and presents the concepts of *user equilibrium* where drivers simultaneously decide their route based on network congestion and minimise their own travel time, and the *system optimal* assignment that maximises aggregate user benefit. He uses Braess' paradox to illustrate how road building may result in poorer network performance; thus demonstrating that network planning is essential and more roads do not necessarily imply improved traffic flow. Ben further discusses the implications for traffic management, including economic mechanisms such as road pricing/congestion charging.

Richard Connors also utilises the classic traffic network model to explain the *price of anarchy*. In recent history motor vehicle manufacturers have marketed cars by appealing to a driver's sense of autonomy – adverts feature driving on open roads in appealing countryside rather than in congested cities and new drivers are encouraged to purchase a car to give themselves more freedom as opposed to being subject to public transport timetable restrictions. The reality however is that in most urban areas congested networks impose restraints on drivers' ability to make choices; many will avoid congestion or routes with many signalised junctions if other options are available. Allowing drivers, within legal constraints, to select their own preferred routes or departure times results in traffic flows across networks which are suboptimal and more efficient traffic flows can result from a traffic management system that routes traffic according to *system optimality*. The difference in overall travel time between system optimal routing and routing according to user equilibrium is termed the *price of anarchy*.

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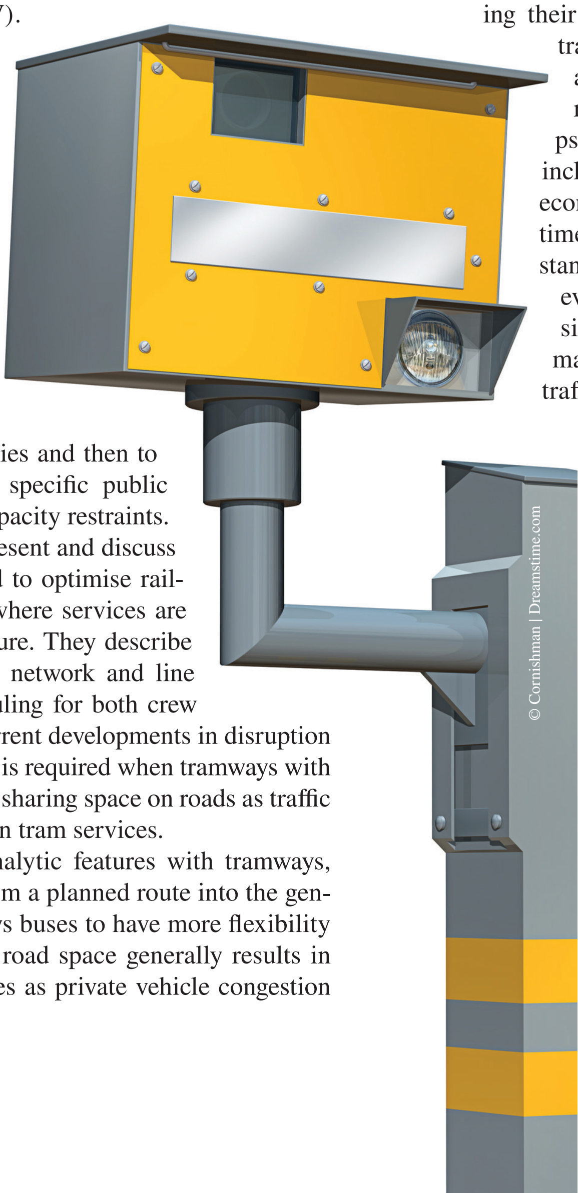
A microsimulation model will also include models to describe vehicle dynamics and will have statistical mechanisms to release traffic into the modelled network in a pseudo-random way. Included are stochastic vehicle release models, car following models, overtaking models, gap-acceptance models etc., in addition to formulating sets of rules to govern junction behaviour and desired speeds. The above relate primarily to *traffic* modelling, which refers to road traffic which performs independent route choice (e.g. car, Taxi, HGV). Transport modelling describes a broader remit also including public transport where the routes are predetermined; private vehicles are then assigned to routes with the public transport movements having been pre-allocated. Transit assignment has a rather different meaning though, and refers to the allocation of public transport vehicles to routes/lines which can involve the creation of timetables or service frequencies and then to allocate potential passengers to specific public transport services allowing for capacity restraints.

Janny Leung and colleagues present and discuss the constituent elements required to optimise railway/light rail/tramway services where services are restrained by physical infrastructure. They describe the models required to optimise network and line planning, timetabling and scheduling for both crew and vehicles, and then discuss current developments in disruption management. Additional analysis is required when tramways with fixed rails run with general traffic sharing space on roads as traffic congestion adversely impacts upon tram services.

Bus operations share some analytic features with tramways, but unlike a tram may deviate from a planned route into the general road space. Whilst this allows buses to have more flexibility in theory, in practice sharing of road space generally results in delay for public transport services as private vehicle congestion impacts upon their schedules.



Andy Chow describes how bus operations are modelled in conjunction with road traffic. A problem in public transport service provision is that bus bunching can adversely affect service reliability. The age-old complaint that *buses arrive in threes* is a well understood consequence of bus bunching, where a small delay to the front bus results in more passengers waiting at each stop, boarding/disembarkation times increasing and hence the bus delay increasing, whilst the converse of fewer passengers waiting for the next buses results in them catching up with the ones in front. Improvement in communication technology can result in improved control strategies that speed-up late running buses by extending greens at traffic signals and slow-down early arriving buses by holding them at control points. In addition to explaining the theory of bus bunching, Andy presents a case study demonstrating how control systems can help keep buses on a regular service frequency.

The above cover some of the key theoretical issues that concern transport mathematicians that relate to the issues of traffic assignment and optimisation and modelling of public transport. However whilst the articles in this special edition show mathematical methods across a range of transport modes, there are a large number of theoretical areas which are outside the scope of this edition. Dynamic traffic assignment, where time is modelled explicitly and drivers may optimise their route choice by altering their departure time is an active research area; transport logistics as related to freight and air traffic movements comprises a separate research field within OR; almost all social/psychological aspects of transport studies include substantial use of statistical methods; economics of transport including valuation of time require discrete choice methods as well as standard mathematics of economics; pedestrian evacuation uses dynamics of attraction/repulsion; big data analysis is required to utilise the many and varied data sources that can inform traffic/transport analysis.

However the fifth article in this special edition covers an application area which cannot be overlooked in any discussion of traffic/transportation – that of road safety analysis. Mike Maher presents an overview of the modelling of road accidents which describes the art of predictive accident modelling and thereafter presents two examples; an analysis and discussion regarding the effectiveness of fixed speed cameras and the modelling of two-car crashes. Mike also raises some of the current issues discussed in road safety analysis such as the impact of graduated licence schemes, speed limit changes on motorways and issues of fairness in motor insurance.

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Guest Editor