Windows on Advanced Mathematics

If you are reading this editorial, the chances are that you take it as read that to think mathematically affords a powerful means to understand and control one's social and physical reality. Yet despite some twelve or so years of compulsory mathematical education, most adults in the developed world leave school with only limited access to mathematical ideas, or without much affinity with the idea of taking a mathematical point of view. In many cultures – the UK is particularly guilty – it is quite acceptable to admit ignorance of things mathematical in ways which would be inexcusable in relation to art, literature or music.

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From the individual's point of view, remoteness from mathematical ways of thinking may be limiting but, on the face of it, merely an inconvenience. At a slightly more urgent level, it may lead to a closing of employment opportunities (see for example the Presidential address on pages 265–271 of this Special Issue). An inability to engage with the world mathematically does, of course, imply a corresponding limit to the appreciation of scientific things, although the illiterate scientist has as much to lose in terms of his or her humanity as the innumerate artist.

But the real impoverishment is cultural not individual. For the social and material world is becoming increasingly mathematised, a world in which – in Weizenbaum's [1] memorable phrase – judgement is increasingly replaced by calculation. It is often argued that increasing mathematisation leads to a rising profile of mathematical ideas within the culture. In fact, the reverse is true. Mathematics is becoming all-pervasive: but it is invisible mathematics which lies deep inside the machines which govern our existence. For example the invisible mathematics that lies dormant in the electronic circuits of our washing machines, or buried within the communication networks which so powerfully structure our view of the world. It is mathematics produced by the few, designed for the consumption of the many. From this imbalanced state emerges a danger that citizens are losing their ability to understand the systems which govern them. The realm of social and cultural life, as much as the quality of life of the individual, is impoverished by the absence of mathematical windows to understand and interpret the world, and to change it (these ideas are elaborated in [2]).

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For this special issue we invited a range of scholars from across the world to write about the challenge of rethinking some 'advanced' mathematics in the interests of its learnability, which might include broadening who might study mathematics as well as how it might be studied. To align with our research, we were particularly interested in contributions that involved the use of digital technologies as a way to rethink mathematics and how it was represented. It is easy to cast the computer in the role of the problem. But paradoxically, it can also be part of the solution because a computer screen is a public object that can open new windows for an observer to infer what, say, a programmer is thinking. Computing leaves a trace of thinking in ways that are simply impossible – or at least inaccessible – without it. It can afford us insights into the evolution of learners' mathematical meanings and provide a means for expressing mathematical relationships that would be impossible for the novice who does not already have a grasp of mathematical language.

In this Special Edition, many papers – but not all – take the computer as an integral component of what it means to learn mathematics. All of the papers offer (we hope) new insights into mathematical practice and how it might be promoted. All authors acknowledge that the readership of *Mathematics Today* is diverse, consisting of professional mathematicians in academia and industry, commerce, teaching, and the mathematically interested 'general public'. It is this diversity that gives us an opportunity to think about the windows metaphor in productive ways.

Some of the papers focus on learnability tackling hitherto rather inaccessible or little explored mathematical ideas (Kenderov and colleagues; Rila). Others are concerned with the nature of mathematical knowledge itself – a relatively under-researched issue: see for example, work on the relationship between research mathematics and teaching (Cuoco). More specifically using digital technologies, Rasmussen et al., discuss the design of computational applets that help students create fundamental concepts and methods that are needed to sensibly use more sophisticated software tools. Sangwin argues that the development of computer algebra systems requires people to make use of complex numbers earlier and more thoroughly than before. The paper by Muller and his colleagues indicates how both pedagogy and epistemology can be radically mediated by exploiting the computer in productive ways at undergraduate level, and Nardi's paper shows us how mathematics and mathematics education can be viewed as parts of a single system to the mutual benefit of both.

We hope you find reading the contributions to this special issue as rewarding as we have found the process of putting it together.

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References

1 Weizenbaum, J. (1984) *Computer Power and Human Reason: From Judgement to Calculation*, Harmondsworth: Penguin Books.

2 Noss, R. and Hoyles, C. (1996) *Windows on Mathematical Meanings: Learning Cultures and Computers*, Dordrecht: Kluwer.