Pursuit of a Moving Ground Target on a Graph Using Partial Information

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Abstract

The optimal control of a "blind" pursuer searching for an evader moving on a road network toward a set of goal vertices is considered. The evader's speed is time-varying and not known to the pursuer; although upper and lower bounds on the speed are assumed known. To aid the pursuer, certain roads in the network have been instrumented with Unattended Ground Sensors (UGSs) that detect the evader's passage. When the pursuer arrives at an instrumented node, the UGS therein informs the pursuer if and when the evader visited the node. The pursuer's motion is not restricted to the road network. In addition, the pursuer can choose to wait/loiter at any UGS location/node. At time 0, the evader passes by an entry node and proceeds towards one of the exit nodes. The pursuer also arrives at this entry node after some delay and is thus informed about the presence of the intruder/evader in the network, whereupon the chase is on. Capture entails the pursuer and evader being co-located at an UGS location. If this happens, the UGS is triggered and this information is instantaneously relayed to the pursuer, thereby enabling capture. On the other hand, if the evader reaches one of the exit nodes without being captured, he is deemed to have escaped. We provide an algorithm that computes the maximum initial delay at the entry node for which capture is guaranteed and the corresponding optimal pursuit policy.

1. Introduction

We are concerned with capture of a moving ground target on a road network. The operational scenario is as follows. The access road network to a restricted (protected) zone is instrumented with Unattended Ground Sensors (UGSs), placed at critical locations. As the target, referred to as the "evader", passes by an UGS, the UGS is triggered. A triggered UGS turns, say, from *green* to *red* and records the evader's time of passage. The UGSs are placed on certain edges of the graph. We assume that the layout of the road network and the placement of the UGSs is known to the pursuer. The speed of the evader v can be time-varying and is unknown to the pursuer, but it is lower and upper bounded by v_L and v_U respectively, where $v_L > 0$ and $v_U < \infty$ are both known to the pursuer. When the pursuer arrives at an UGS location, the information stored by the UGS is uploaded to the pursuer, namely, the green/red status of the UGS and, if the UGS is red, the time elapsed (delay) since the evader's passage. The evader can be captured in one of two ways: either the evader and pursuer synchronously arrive at an UGS location, or the pursuer is already loitering/waiting at an UGS location when the evader arrives there. The decision problem for the pursuer is to select which UGS to visit next, including possibly staying at the current UGS location awaiting the arrival of the evader. The decisions are made by the pursuer at discrete time instants after arriving at an UGS and collecting the information therein. This is a deterministic pursuit-evasion game where
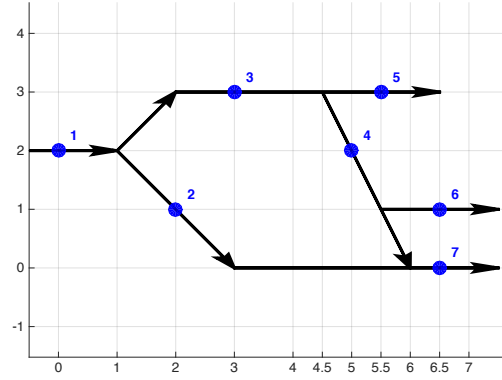
FIGURE 1. Example Road Network with UGSs

the evader's strategy is open-loop control and the pursuer has partial information. Such a game was previously considered in Kalyanam *et al.* (2013a), Kalyanam *et al.* (2013b), where the highly structured graph considered therein, was a Manhattan grid. Due to the pursuer's information pattern, which is restricted to partial observations of the physical state of the dynamic game, we are running into difficulties brought about by the *dual control* effect Başar (2001), where the current information state determines the pursuer's optimal control while at the same time the information that will become available to the pursuer will be in part determined by his current control.

Fig. 1 depicts an example road network, with a grid in the background, to highlight the (x, y) coordinates of nodes and distances along edges. The roads are shown in black (arrows indicate allowed direction of travel) and the UGSs are shown as numbered blue circles. The pursuer is able to make decisions when it gains new information at an UGS. For this reason, we focus on the embedded graph, $G(\mathcal{U}, E)$, where the m UGSs are the vertices, i.e., $\mathcal{U} = \{1, \ldots, m\}$. We make the critical assumption that G is a directed acyclic graph. Any directed edge $e_{i,j} \in E$ represents a path from node i to node j with a weight equaling the distance (along the path) between the two nodes. For each $j \in \mathcal{U}$, let $C(j) \subset \mathcal{U}$ indicate the set of child nodes that the evader can get to from j. Let $\mathcal{G} = \{j : j \in \mathcal{U} \text{ and } C(j) = \emptyset\}$ indicate the set of exit/goal nodes that the evader is heading towards. In Fig. 1, $\mathcal{G} = \{5, 6, 7\}$. Without loss of generality, we assume that, upon entering the network, the evader first visits node 1 at time 0 and furthermore, $1 \notin \mathcal{G}$. Let there be n (≥ 1) possible evader paths emanating from node 1 and terminating at an exit node. If node j can be reached from node i by traveling along path k on the network, we let $d_e(i, j; k)$ indicate the distance between the two nodes. Else, we set $d_e(i, j; k) = \infty$. We simply use $d_e(i, j)$ when the path index is apparent. The pursuer's travel distance from node i to node j is given by a metric, $d_p(i, j)$ that satisfies:

$$d_p(i, j) \leq d_p(i, s) + d_p(s, j), \tag{1.1}$$

for any $i, j, s \in \mathcal{U}$ and $d_p(j, j) = 0$, $\forall j \in \mathcal{U}$. We assume that the pursuer travels at unit speed. Moreover, the pursuer's travel time between any two nodes is strictly less than the evader's travel time between the two, i.e.,

$$d_p(i, j) < \frac{d_e(i, j; k)}{v_U}, \ \forall i, j \in \mathcal{U}, \ \forall k \in \{1, \ldots, n\}. \tag{1.2}$$

Let the length of each path k be given by $P_k = d_e(1, x_k; k)$, where x_k is the exit node along path k. We also define the set, $\mathbb{P}_j$, $j = 1, \ldots, m$, to be the set of paths that go through

node j i.e., $\mathbb{P}_j = \{k : d_e(1, j, k) < \infty, \ k = 1, \ldots, n\}$. We define the uncertainty in evader state information available to the purser to be $\mathcal{I} = (\mathcal{P}; (\underline{n}, \underline{t}))$. It has two components: the evader path information, $\mathcal{P} \subseteq \mathbb{P}_1 = \{1, \ldots, n\}$ and the last known evader (UGS) location $\underline{n}$ and time of visit, $\underline{t}$. The initial path uncertainty is given by $\mathcal{P}_0 = \{1, \ldots, n\}$ and $\mathcal{I} = (\mathcal{P}_0; (1, 0))$. This is so because the evader was at node 1 at time 0 and he could have taken any one of the n paths emanating from 1. The second component of the information is crucial in determining future pursuer wait times at nodes, where capture is likely. This definition of path uncertainty, meaning, the uncertainty about which of the n paths the evader is actually traveling on, results in a significant simplification of the underlying coupled estimation and control problem. Hereafter, we shall use the words uncertainty and information interchangeably.

1.1. Evolution of System State

Let the pursuer position at decision time t be specified by the UGS index, $p \in \{1, \ldots, m\}$. The decision variable, $u \in \{1, \ldots, m\}$ indicates the UGS location that the pursuer should visit next. Even though the pursuer and evader motion evolve in continuous time, decisions are made (by the pursuer only) at discrete time steps. The pursuer makes a decision at time t upon obtaining the measurement y at UGS location p: $y = -1$ for "green", or $y = d$ for "red" + delay $d \geq 0$. Suppose the evader state uncertainty information available to the pursuer at time t is $\mathcal{I} = (\mathcal{P}; (\underline{n}, \underline{t}))$, where $\mathcal{P}$ is the path information and $(\underline{n}, \underline{t})$ is the most recent evader position (UGS location) and time of visit known to the pursuer. The control action u is dependent on the current time, pursuer position and most recent information: $u = \mathcal{F}(t, p, \mathcal{I})$, where the mapping $\mathcal{F}$ is to be determined by an optimality principle - see (2.2) in the sequel. We shall refer to the tuple $(t, p, \mathcal{P}, \underline{n}, \underline{t})$ as the system state. The pursuer's position and decision time evolve according to:

$$p^+ = u,$$
$$t^+ = \begin{cases} t + d_p(p, u), & \text{if } u \neq p, \\ \underline{t} + \frac{1}{v_U} \min_{k \in \mathbb{P}_u \cap \mathcal{P}} d_e(\underline{n}, p; k), & \text{if } u = p, \end{cases} \qquad (1.3)$$

So, if the pursuer decides to stay put at the current location, the next decision epoch is the earliest possible time at which new information becomes available at the current UGS p. Indeed the pursuer need stay at the current location only if $\mathbb{P}_p \cap \mathcal{P} \neq \emptyset$ i.e., there is a likelihood of capturing the evader there. While waiting, one of two things will happen: either the evader will show up, leading to capture or the UGS will remain green long enough for the pursuer to determine that the evader has taken a path $k \notin \mathbb{P}_p$. The pursuer need never wait at (or revisit) an UGS location after receiving a "red" measurement from it - since the evader never revisits any UGS location and so the pursuer gains nothing new by doing so. Upon moving to u, the information/ uncertainty set at time t^+ is updated for the two possible observations at u as follows:

Red $(y^+ = d \geq 0)$: This implies that the evader was at node u at time $t^+ - d$. If this evader location information is more recent, i.e., $t^+ - d > \underline{t}$, the path information at time t^+ is updated to:

$$\mathcal{P}^+(u, d) = \left\{ k : k \in \mathbb{P}_u \cap \mathcal{P}, \ \frac{d_e(\underline{n}, u; k)}{v_U} \leq t^+ - d - \underline{t} \leq \frac{d_e(\underline{n}, u; k)}{v_L} \right\}. \qquad (1.4)$$

So we only retain those paths that the evader can take to arrive at u at time $t^+ - d$. Accordingly, the information state is updated to:

$$\mathcal{I}^+(u, d) = \begin{cases} (\mathcal{P}^+(u, d); (u, t^+ - d)), & \text{if } t^+ - d > \underline{t}, \\ \mathcal{I}, & \text{otherwise.} \end{cases} \qquad (1.5)$$

Green $(y^+ = -1)$: This implies that the evader has not visited u thus far. Therefore, the path information update is given by:

$$\mathcal{P}^+(u, -1) = \mathcal{P}\backslash Q, \ Q = \left\{ k : k \in \mathbb{P}_u, \ \underline{t} + \frac{d_e(\underline{n}, u; k)}{v_L} \leq t^+ \right\}. \tag{1.6}$$

So we remove all the paths that the evader can take to arrive at u no later than t^+. Accordingly, the information state is updated to:

$$\mathcal{I}^+(u, -1) = \left\{ \begin{array}{l} (\mathcal{P}^+(u, -1); (\underline{n}, \underline{t})), \ \text{if } Q \neq \emptyset, \\ \mathcal{I}, \ \text{otherwise}. \end{array} \right. \tag{1.7}$$

2. Problem Statement

The evader passes by the source node 1 at time 0. The pursuer arrives for the 1st time at node 1 at time $t_0 > 0$ and is tasked with capturing the evader. Let $\mathcal{T}(1|\mathcal{I}_0) > 0$ be the *latest* time that the pursuer can arrive at node 1 and still capture the evader, knowing that the evader could have taken any one of n paths. Here, time is measured relative to time 0. The information available to the pursuer at node 1 is given by $\mathcal{I}_0 = \{1, \ldots, n; (1, 0)\}$. In a similar fashion, for any UGS, $j = 1, \ldots, m$, we define $\mathcal{T}(j|\mathcal{I})$ to be the latest time the pursuer can arrive at node j and guarantee capture, armed with the information $\mathcal{I} = (\mathcal{P}; (\underline{n}, \underline{t}))$. If the pursuer arrives at node j at time $t > 0$ and $t \leq \mathcal{T}(j|\mathcal{I})$, let $\mu(j|\mathcal{I}) \in \{1, \ldots, m\}$ be the corresponding UGS index to which the pursuer should head towards next, to enable capture. Before we address the question of how to compute $\mathcal{T}(1|\mathcal{I}_0)$, we establish a sufficient condition for guaranteed capture.

LEMMA 1. *A sufficient condition for guaranteed capture is that the system state* $(t, p, \mathcal{I})$ *satisfy* $\mathcal{P} \subseteq \mathbb{P}_u$ *for some* $u \in \mathcal{U}$ *and* $t + d_p(p, u) \leq \underline{t} + \frac{1}{v_U} \min_{k \in \mathcal{P}} d_e(\underline{n}, u; k)$.

Proof. Since $\mathcal{P} \subseteq \mathbb{P}_u$, all feasible evader paths go through node u. So, if the pursuer can get to node u no later than the earliest time that the evader can get there by taking any path, $k \in \mathcal{P}$, capture is guaranteed. □

Given the above sufficient condition, we can establish the following lower bound.

COROLLARY 1. *If the information state* $\mathcal{I} = (\mathcal{P}; (\underline{n}, \underline{t}))$ *satisfies* $\mathcal{P} \subseteq \mathbb{P}_u$ *for some* $u \in \mathcal{U}$, *then:*

$$\mathcal{T}(u|\mathcal{I}) \geq \underline{t} + \frac{1}{v_U} \min_{k \in \mathcal{P}} d_e(\underline{n}, u; k). \tag{2.1}$$

Since every $\mathcal{P} \subseteq \mathbb{P}_1$, we have the trivial lower bound, $\mathcal{T}(1|(\mathcal{P}, (1, 0))) \geq 0$.

2.1. *Max-Min Optimization*

Suppose the pursuer is at UGS index p at time t with information $\mathcal{I} = (\mathcal{P}; (\underline{n}, \underline{t}))$ and decides to visit u next. Upon reaching u, the information will change to: $\mathcal{I}^+(u, y)$, where y is the observation that the pursuer makes at u. Recall that $\mathcal{I}^+(u, y)$ is defined according to (1.5) and (1.7) for the red and green UGS observations respectively. By definition, $\mathcal{T}(u|\mathcal{I}^+(u, y))$ is the latest time at which, armed with the new information $\mathcal{I}^+(u, y)$, the pursuer can leave u and still guarantee capture of the evader. So, the latest time that the pursuer can leave p and still capture the evader satisfies the Recursive Equation (RE):

$$\mathcal{T}(p|\mathcal{I}) = \max_{u \in \mathcal{U}} \left[\min_{y \geq -1} \mathcal{T}(u|\mathcal{I}^+(u, y)) - d_p(p, u) \right]. \tag{2.2}$$

Per our convention, the corresponding optimal control, $\mu(p|\mathcal{I}) = u^*$, where u^* is the maximizing control in (2.2). We wish to use RE (2.2) to compute $\mathcal{T}(1|\mathcal{I}_0)$. At first glance, RE (2.2) looks like a typical Bellman equation, that is amenable to Dynamic Programming (DP). Unfortunately, this is not the case especially given that the evader's speed is uncertain. Moreover the solution to RE (2.2), ostensibly requires the knowledge of $\mathcal{T}(u|\mathcal{I}^+(u,y))$ for all possible u and y at the next decision epoch. This is complicated by the fact that the likely observation y is a function of the path chosen by the evader and his speed profile, $v(t); t > \underline{t}$. Fortunately, we can enforce a restriction to the search space $\mathcal{U}$, without any loss in optimality, as shown below.

LEMMA 2. *The optimal* (u^*, y^*) *to* (2.2) *are such that either:*
a) $\mathcal{P}^+(u^*, y^*) \subset \mathcal{P}$ *or*
b) $y^* \geq 0$ *and* $t^+ - y^* > \underline{t}$.

Proof. Suppose neither a) or b) hold. Then, it follows from (1.5) and (1.7) that $\mathcal{I}^+(u^*, y^*) = \mathcal{I}$ and also, capture does not occur at u^*. Hence, at the next decision epoch, the pursuer again has to make a decision on where to go with the same information as before. Let the subsequent optimal control be $\bar{u}$ and the corresponding optimal observation information state at $\bar{u}$ be $\mathcal{I}_1$. So, the optimal exit time at $\bar{u}$ is given by $\mathcal{T}(\bar{u}|\mathcal{I}_1)$. From (2.2), we have:

$$
\begin{aligned}
\mathcal{T}(p|\mathcal{I}) &= \mathcal{T}(u^*|\mathcal{I}) - d_p(p, u^*), \\
&= [\mathcal{T}(\bar{u}|\mathcal{I}_1) - d_p(u^*, \bar{u})] - d_p(p, u^*), \\
&\leq \mathcal{T}(\bar{u}|\mathcal{I}_1) - d_p(p, \bar{u}),
\end{aligned} \tag{2.3}
$$

where the last inequality follows from (1.1). This implies that it is no less optimal to go directly from p to $\bar{u}$. The only question that remains is: what if $\mathcal{I}_1 = \mathcal{I}$ also and capture doesn't happen at $\bar{u}$? If so, we can repeat the arguments above and show that $\bar{u}$ can be skipped as well and the pursuer can directly go to the subsequent optimal node. Note that this cannot continue indefinitely, since the pursuer must observe $y = 0$ in a finite number of moves, to guarantee capture. $\square$

Lemma 2 tells us that under optimal action, either the path uncertainty is reduced or a more recent evader location is observed at the next decision epoch. In lieu of Lemma 1, the former is obviously beneficial in capturing the evader. In the latter case, the more recent evader location (and time), is beneficial in reducing future wait times. We shall elaborate on this point in the next section.

2.2. *Optimal Reduction in Search Space*

For any information set $\mathcal{I} = (\mathcal{P}, (\underline{n}, \underline{t}))$, we define the restrictions $\mathcal{B}(\mathcal{I})$ and $\mathcal{E}(\mathcal{I})$ as follows. A node $u \in \mathcal{E}(\mathcal{I})$ if the following condition holds:

$\mathbb{P}_u = \mathcal{P}$ and $u \in C^i(\underline{n})$, for some $i > 0$, where $C^1(j)$ denotes the set of all child nodes of j, $C^2(j)$ denotes the set of all grand child nodes of j and so on.

To clarify, the above condition enforces that a (more recent) red UGS will be encountered on visiting u. This includes the possibility of capture at u. On the other hand, a node $u \in \mathcal{B}(\mathcal{I})$ if the following condition holds:

$\mathbb{P}_u \subset \mathcal{P}$ and $\exists \bar{k} \in \mathbb{P}_u$ such that $\mathcal{T}(u, \mathcal{I}^+(u, -1)) \geq \underline{t} + \frac{1}{v_L} d_e(\underline{n}, u; \bar{k})$, where $\mathcal{P}^+(u, -1) = \mathcal{P} \backslash Q_{u,\bar{k}}$, $Q_{u,\bar{k}} = \{k : k \in \mathbb{P}_u, \ d_e(\underline{n}, u; k) \leq d_e(\underline{n}, u; \bar{k})\}$ and as per (1.7) $\mathcal{I}^+(u, -1) = (\mathcal{P}^+(u, -1), (\underline{n}, \underline{t}))$.

To clarify, the above condition enforces that the path uncertainty is reduced upon visiting u. Moreover, the latest pursuer exit time for a green observation at u must be no earlier

than the time until which the pursuer needs to stay at u to verify that paths in $Q_{u,\bar{k}}$ were not taken. By definition, $Q_{u,\bar{k}}$ is the set of all paths connecting $\underline{n}$ and u that are no greater (in length) than $\bar{k}$. Note that if $\mathbb{P}_u \subset \mathcal{P}$ and no such $\bar{k}$ exists, capture cannot be guaranteed at u.

Having introduced the above restrictions to the search space, we can now state the main result.

THEOREM 1. *The RE* (2.2) *to compute* $\mathcal{T}(j|\mathcal{I})$ *can be re-written as follows:*

$$\mathcal{T}(j|\mathcal{I}) = \max_{u \in \mathcal{B}(\mathcal{I}) \cup \mathcal{E}(\mathcal{I})} \min_{y \geq -1} \left[\mathcal{T}(u|\mathcal{I}^+(u,y)) - d_p(j,u) \right]. \tag{2.4}$$

Proof. From Lemma 2, we know that the optimal control u^* is such that either $u^* \in \mathcal{B}(\mathcal{I})$ or $u^* \in \mathcal{E}(\mathcal{I})$. Hence, the restriction of the control space to $\mathcal{B}(\mathcal{I}) \cup \mathcal{E}(\mathcal{I})$ is optimal. $\square$

At first glance, (2.4) appears no easier to solve than the original recursion, (2.2). But this is misleading. We note that under the restriction, $u \in \mathcal{B}(\mathcal{I}) \cup \mathcal{E}(\mathcal{I})$, the uncertainty is always reduced at every (optimal) pursuer move. Either, the path uncertainty is reduced (at least by one) or the last known evader location is updated to a more recent observation. The former obviously helps in capturing the evader. The latter plays a more subtle role in reducing potential waiting times in the future. We illustrate this point by going over a simple scenario involving the example problem considered earlier (see Fig. 1). In either case, the restriction helps prune the original tree search. Indeed, for any given $(\underline{n}, \underline{t})$, the search need only involve the sub-tree that emanates from $\underline{n}$. Furthermore, when the size of the path uncertainty is 1, i.e., the evader's path is known to the pursuer, the computation is straightforward as shown below.

Suppose $\mathcal{P} = \{k\}$, i.e., the pursuer knows the path $k \in \mathbb{P}_{\underline{n}}$ that the evader has taken. In this scenario, the latest time that the pursuer can arrive at the exit node x_k and still guarantee capture, is clearly the earliest time at which the evader can reach x_k from his last known position $\underline{n}$. So, we have:

$$\mathcal{T}(x_k|(k;(\underline{n},\underline{t}))) = \underline{t} + \frac{1}{v_U} d_e(\underline{n}, x_k). \tag{2.5}$$

As a consequence, the optimal control $\mu(x_k|(k;(\underline{n},\underline{t}))) = x_k$. Since the evader's speed is uncertain, the pursuer may have to wait at x_k for a while. Indeed, his strategy (until capture) is to wait at x_k until time, $\underline{t} + \frac{1}{v_L} d_e(\underline{n}, x_k)$. In light of the triangle inequality (1.1) and speed advantage (1.2) assumptions, we can extend the above result to all nodes $j \in \mathcal{U}$.

LEMMA 3. *If the path information is the singleton* $\{k\}$, *then* $\forall j \in \mathcal{U}$, *we have:*

$$\mathcal{T}(j|(k;(\underline{n},\underline{t}))) = \underline{t} + \frac{1}{v_U} d_e(\underline{n}, x_k) - d_p(j, x_k), \ \forall k \in \mathbb{P}_{\underline{n}}. \tag{2.6}$$

In essence, the pursuer reaches the exit node x_k of path k from node j, just in time to intercept the evader. The corresponding optimal control is given by, $\mu(j|(k;(\underline{n},\underline{t}))) = x_k$.

2.3. *Role of Uncertainty in Evader Speed*

In Fig. 2, we show the four different evader paths (ordered from left to right) along with the distances to nodes (in parentheses) from the entry node along each path. Suppose the pursuer knows that the evader has taken either path 2 or 3 i.e., $\mathcal{P} = \{2, 3\}$. Since $\mathcal{P} = \mathbb{P}_4 \subset \mathbb{P}_3$, the evader can be intercepted at node 4 or earlier at node 3. But, we are
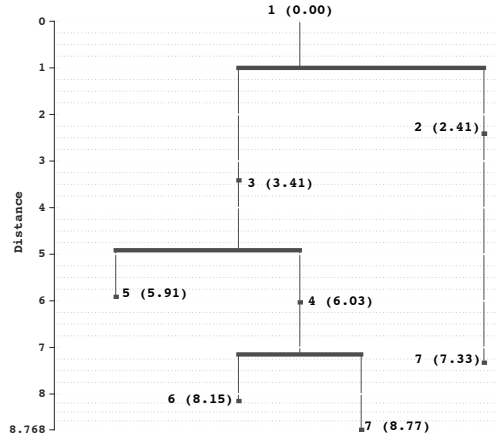
FIGURE 2. Evader Paths showing nodes and corresponding evader travel distances in parentheses

interested in maximizing the initial delay with a capture guarantee and so, the following policy makes sense. The pursuer gets to node 6 no later than $\underline{t} + \frac{d_e(\underline{n},6)}{v_U}$. Upon getting there, the pursuer must wait at node 6 for the evader. Either capture occurs or the time exceeds $\underline{t} + \frac{d_e(\underline{n},6)}{v_L}$. At this time, the pursuer knows that the evader has taken path 3 and so he proceeds to node 7. But he must arrive at 7 no later than $\underline{t} + \frac{d_e(\underline{n},7)}{v_U}$ to guarantee capture. Hence, we need:

$$\frac{d_e(\underline{n},6)}{v_L} + d_p(6,7) \leq \frac{d_e(\underline{n},7)}{v_U}. \tag{2.7}$$

We see the interplay between the lower and upper bounds of the evader's speed and the pursuer's (unit) speed in the above relation. We will illustrate this point with a numerical example in the next section.

2.4. Numerical Example

For purposes of illustration, we reconsider the example problem (see Fig. 1) and the following evader parameters: $v_L = 0.498$, $v_U = 0.51$. We assume that at time 0, the evader passes UGS 3 and proceeds towards the goal nodes. So, the initial information state available to the pursuer is $\mathcal{I}_0 = (\mathcal{P}_0; (3,0))$, where the path information, $\mathcal{P}_0 = \{1,2,3\}$. We employ the recursive equation (2.4) to compute the maximal delay $\mathcal{T}(3|\mathcal{I}_0)$ and the corresponding pursuit policy. From Fig. 1, we note that $d_p(6,7) = 1, d_e(3,4) = 1.5 + \sqrt{1.25}, d_e(4,6) = 1 + \sqrt{1.25}$ and $d_e(4,7) = 1 + \sqrt{5}$. It can be easily verified that,

$$\frac{d_e(3,7)}{v_U} - \frac{d_e(3,6)}{v_L} \approx 0.98 < 1 \text{ and } \frac{d_e(4,7)}{v_U} - \frac{d_e(4,6)}{v_L} \approx 1.11 > 1. \tag{2.8}$$

So, (2.7) is satisfied for $\underline{n} = 4$ but not for $\underline{n} = 3$. In light of the above inequalities, the optimal pursuit policy takes the form shown in Fig. 3, which shows the decision tree for the pursuer starting with a red UGS at node 3. Fig. 3 shows (color coded) the latest pursuer exit times at future nodes visited by the pursuer, for both red and green observations. The solution dictates that:

$$\mathcal{T}(3,\mathcal{I}_0) = \frac{d_e(3,5)}{v_U} - d_p(3,5) = \frac{2.5}{v_U} - 2.5 \approx 2.4, \tag{2.9}$$
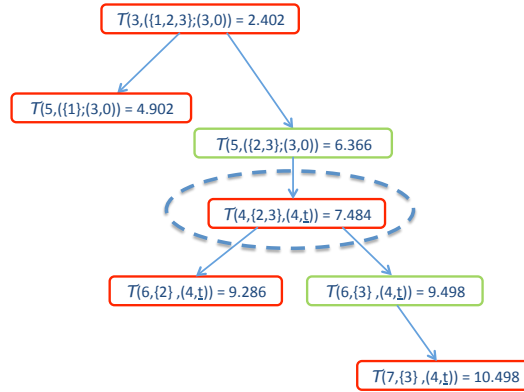
FIGURE 3. Optimal Decision Tree and Maximal Delay at UGS 3 and All Downstream UGSs

and $\mu(3, \mathcal{I}_0) = 5$. One can easily verify that if the evader's speed was a known constant $= v_U$, the resulting maximal delay at 3 would remain the same. However, the policy would differ in that the additional move to 4 is no longer required and the pursuer would directly move from node 5 to node 6. Needless to say, if the evader's speed was a known constant $= v_L$, the resulting maximal delay at 3 would increase to $\frac{d_e(3,5)}{v_L} - d_p(3,5) = \frac{2.5}{v_L} - 2.5 \approx 2.52$. Indeed, when the evader's speed is a known constant, the recursion (2.4) simplifies and allows for an algorithm that computes the maximal delays for all possible path information sets in the order of increasing cardinality (see Kalyanam *et al.* (2015)).

3. Conclusion

The optimal control of a pursuer with limited sensing capability tasked with intercepting a blind evader on a road network instrumented with UGS is considered. The pursuer is interrogating the UGSs, some of which were triggered by the passing evader, and as such has access to partial observations only; of the system's physical state. Specifically, the maximal allowable delay at an UGS s.t. a pursuit strategy exists which guarantees the evader's capture before the latter reaches his goal is calculated and the attendant pursuit strategy is obtained. Thus, a deterministic pursuit-evasion game on a directed acyclic finite graph where the evader's strategy is open-loop control and the pursuer has partial information, is solved. In the process of establishing the maximal delay at UGS 1 s.t. capture of the evader is possible, the maximal delays for guaranteed capture at all the downstream UGSs are also calculated.

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