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Non-holonomic motion planning using dynamically consistent Jacobian inverse†

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Abstract

We propose a transfer of the concept of dynamically consistent Jacobian inverse from robotic manipulators to non-holonomic robotic systems. This transfer exploits an analogy between these two classes of systems established within the endogenous configuration space approach. Similarly as for robotic manipulators, the dynamic consistency of the Jacobian inverse for non-holonomic systems distinguishes itself by two features: it does not transmit some forces from the configuration space to the operational space, and it defines a decoupling of forces acting in the configuration space into a component coming from the operational space and an internal component affecting only the motion in the configuration space. The dynamically consistent Jacobian inverse is used to solve a motion planning problem for the rolling ball.

1. Introduction

Jacobian motion planning algorithms are basic tools for solving the motion planning problem for robotic manipulators. These algorithms are defined by a dynamical system involving a right Jacobian inverse. Given a redundant manipulator, the Jacobian and the dual map to its right inverse form a pair of maps transforming velocities and forces from the configuration to the operational space. The dynamic consistency means that the forces contained in the null space of the dual Jacobian inverse do not affect the motion in the operational space. The dynamic consistency enables a decomposition of any configuration force into a component coming from the operational space and an internal force that does not affect the motion in the operational space whatsoever. The dynamically consistent Jacobian inverse for robotic manipulators was invented by O. Khatib, see Khatib (1990), Khatib (1995), and justified in the context of operational space control of manipulators, humanoid robots, and biomedical systems, Sentis, Park & Khatib (2010), Demircan & Khatib (2012).

In this paper we present a derivation of the dynamically consistent Jacobian inverse for non-holonomic robotic systems whose kinematics are represented by a driftless control system with output. This derivation is based on the analogy between the kinematics of a manipulator and the end-point map of the control system, that establishes a correspondence between manipulator's configurations and control functions steering the control system, called endogenous configurations. The endogenous configuration space approach, Tchoń & Jakubiak (2003), Janiak & Tchoń (2011), created as a methodology of analysis of the non-holonomic systems, has provided ready to use concepts and tools, including the Jacobian, the Jacobian inverse, Jacobian motion planning algorithms, regular and

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singular configurations, dexterity measures, etc. Built on this analogy, the dynamically consistent Jacobian inverse for non-holonomic systems enjoys the fundamental consistency requirement. As an illustration, the new algorithm will be applied to a motion planning problem for the rolling ball. In this paper we are following only one method of deriving the dynamically consistent Jacobian inverse, called the force method, that is due to O. Khatib. More motivation and other methods of derivation, including the optimization method, will be presented in a separate paper. A reference to the optimization as well as to the Newton method establishes a touch point of this paper with the work of Hauser (2002).

The organization of this paper is the following. Section 2 re-states the dynamically consistent Jacobian inverse for robotic manipulators. An analogous construct for non-holonomic systems is exposed in Section 3. Section 4 is devoted to a numerical example. The paper is concluded with Section 5.

2. Basic concepts

We shall begin with the kinematics of a redundant robotic manipulator described in suitable coordinates as a map

$$k : Q \longrightarrow Y, \quad k(q) = (k_1(q), \ldots, k_m(q))^T \tag{2.1}$$

from the configuration space $Q \cong R^n$ to the operational space $Y \cong R^m$, $n \geq m$. The motion planning problem of the robot consists in finding a configuration $q_d \in Q$ such that, for a desired $y_d \in Y$, $y_d = k(q_d)$, This problem can be solved numerically by means of a Jacobian motion planning algorithm. Let $J(q) = \frac{\partial k(q)}{\partial q}$ denote the robot's Jacobian, and $J^{\#}(q)$ be any right inverse of the Jacobian. Then, a solution of the motion planning problem can be computed as a limit of the trajectory of the dynamic system

$$\frac{dq(\theta)}{d\theta} = -\gamma J^{\#}(q)(k(q) - y_d), \quad q(0) = q_0, \quad \gamma > 0, \tag{2.2}$$

i.e. $q_d = \lim_{\theta \to +\infty} q(\theta)$. The dynamic system (2.2) is well defined at regular configurations of the robot, i.e. as long as the Jacobian stays full rank. By definition, the Jacobian transforms velocities from the configuration to the operational space, i.e. $J(q) : T_qQ \to T_{k(q)}Y$, T_qQ and T_yY denoting the tangent spaces. In consequence, the Jacobian inverse $J^{\#}(q) : T_{k(q)}Y \longrightarrow T_qQ$ maps operational velocities into the configuration ones. The space dual to the tangent space is called the cotangent space; covectors from the cotangent space act on tangent vectors by the dual pairing. The dual Jacobian $J^*(q) : T^*_{k(q)}Y \longrightarrow T^*_qQ$ is defined as $(J^*(q)r)v = rJ(q)v$, while the dual Jacobian inverse $J^{\#*}(q) : T^*_qQ \longrightarrow T^*_{k(q)}Y$ satisfies the identity $(J^{\#*}(q)p)w = pJ^{\#}(q)w$, where pv and rw are the pairings. In mechanics the covectors are interpreted as forces, and the pairing of a force and a velocity defines the power. It can be said that the pair of maps $(J(q), J^{\#*}(q))$ consisting of the Jacobian and the dual Jacobian inverse transforms velocities and forces from the configuration to the operational space. The request of preserving some consistency of these two maps underlies the concept of the dynamically consistent Jacobian inverse. To be more specific, we shall consider the dynamics of the manipulator. Let $M(q)$ denote the inertia matrix. This matrix defines in the configuration space a Riemannian metric

$$g_Q(q) : T_qQ \times T_qQ \longrightarrow R, \quad g_Q(q)(v_1, v_2) = v_1^T M(q) v_2.$$

For a configuration space trajectory $q(t)$, with velocity $\dot{q}(t)$, the momentum is given by $p(t) = \dot{q}^T M(q(t))$, the corresponding operational space trajectory is $y(t) = k(q(t))$, and the velocity in the operational space equals $\dot{y}(t) = J(q(t))\dot{q}(t)$. The momentum is a covector, and so is the force

$$f(t) = \dot{p}(t) = \ddot{q}^T M(q(t)) + \dot{q}^T \frac{dM(q(t))}{dt}. \tag{2.3}$$

Apparently, the last expression represents the dynamics equations

$$M(q(t))\ddot{q} + \frac{dM(q(t))}{dt}\dot{q} = f^T(t) \tag{2.4}$$

of the manipulator, without the gravity term. It follows that any right inverse of the Jacobian establishes the following decomposition of the cotangent space

$$T_q^* Q = \mathrm{Im} J^*(q) \oplus \ker J^{\#*}(q),$$

therefore the force f can be partitioned into two components

$$f^T = J^T(q)\Gamma + (I_n - J^T(q)J^{\#T}(q))f_0, \tag{2.5}$$

where $J^T(q)$ denotes the transpose Jacobian matrix, Γ^T comes from the operational space, and f_0^T acts in the configuration space. Next, let us compute the acceleration in the operational space

$$\ddot{y}(t) = J(q(t))\ddot{q} + \frac{dJ(q(t))}{dt}\dot{q}$$

and import into this identity $\ddot{q}$ from (2.4) and f^T from (2.5). The result is the following

$$\ddot{y}(t) = J(q(t))M^{-1}(q(t))\left(J^T(q(t))\Gamma + (I_n - J^T(q(t))J^{\#T}(q(t)))f_0 - \frac{dM(q(t))}{dt}\dot{q}(t)\right) + \frac{dJ(q(t))}{dt}\dot{q}. \tag{2.6}$$

The Jacobian inverse will be called dynamically consistent, if the forces from the null space of $J^{\#*}(q)$ do not affect the acceleration in the operational space. By (2.6), this will hold, provided that

$$J(q)M^{-1}(q)(I_n - J^T(q)J^{\#T}(q)) = 0$$

or, equivalently,

$$J^{\#}(q) = J^{\#\,DC}(q) = M^{-1}(q)J^T(q)(J(q)M^{-1}(q)J^T(q))^{-1}. \tag{2.7}$$

By a substitution into (2.5) of the dynamically consistent Jacobian inverse, the decomposition

$$f^T = J^T(q)\Gamma + (I_n - J^T(q)J^{\#\,DC\,T}(q))f_0,$$

provides a decoupling of a configuration space force into a force coming from the operational space, and an internal force that affects only the motion in the configuration space.

3. Non-holonomic systems

Suppose that the kinematics of a non-holonomic robotic system subject to Pfaffian motion constraints have been represented by a driftless control system with outputs

$$\begin{cases} \dot{q} = G(q)u = \sum_{i=1}^m g_i(q)u_i, \\ y = k(q) = (k_1(q), \ldots, k_r(q))^T, \end{cases} \tag{3.1}$$

where $q \in Q \cong R^n$, $u \in R^m$, and $y \in Y \cong R^r$ denote, respectively, the state, control and operational variable. The control functions are assumed to belong to a linear subspace $\mathcal{X} \subset L^2_m[0,T]$ of the Lebesgue square integrable functions from $[0,T]$ to R^m, equipped with the inner product $< u_1(\cdot), u_2(\cdot) >= \int_0^T u_1^T(t)u_2(t)dt$, such that for any initial state q_0 and any $u(\cdot) \in \mathcal{X}$, the state trajectory $q(t) = \varphi_{q_0,t}(u(\cdot))$ is defined for every $t \in [0,T]$.

With the system (3.1) a kinematics map can be associated, corresponding to the manipulator's kinematics (2.1). For fixed q_0 and T this map is the end-point map of the control system (3.1), defined as

$$K_{q_0,T} : \mathcal{X} \longrightarrow R^r, \quad \text{such that} \quad K_{q_0,T}(u(\cdot)) = k(\varphi_{q_0,T}(u(\cdot))). \tag{3.2}$$

The kinematics (3.2) assign to any control function the value at T of the operational variable. With regard to the formally analogous roles played by $q \in Q$ in (2.1) and $u(\cdot) \in \mathcal{X}$ in (3.2), the control functions will be called endogenous configurations of the non-holonomic system. The motion planning problem for the non-holonomic system (3.1) amounts to computing a control function $u_d(\cdot) \in \mathcal{X}$, such that at T the operational variable assumes the desired value y_d, so that $K_{q_0,T}(u_d(\cdot)) = y_d$. Suppose that we want to solve this problem by means of a Jacobian algorithm. In order to define the Jacobian of the system (3.1), we differentiate the kinematics, so the Jacobian $J_{q_0,T}(u(\cdot)) = \mathcal{D}\, K_{q_0,T}(u(\cdot))$. It can be proved that the Jacobian is determined by the linear approximation to (3.1) along a control-trajectory pair $(u(t), q(t))$, Sontag (1990). The procedure is the following. First, we compute the linear approximation

$$A(t) = \frac{\partial G(q(t))u(t)}{\partial q}, \; B(t) = G(q(t)), \; C(t) = \frac{\partial k(q(t))}{\partial q},$$

then, for a given control function $v(\cdot) \in \mathcal{X}$, we define

$$\xi(t) = \mathcal{D}\varphi_{q_0,t}(u(\cdot))v(\cdot), \quad \xi(0) = 0.$$

It can be shown that the trajectory $\xi(t)$ comes from a linear control system

$$\begin{cases} \dot{\xi} = A(t)\xi + B(t)v, \\ \eta(t) = C(t)\xi, \end{cases}$$

with output $\eta \in R^r$, so finally

$$J_{q_0,T}(u(\cdot))v(\cdot) = C(T)\xi(T) = C(T)\int_0^T \Phi(T,t)B(t)v(t)dt, \tag{3.3}$$

where the transition matrix $\Phi(t,s)$ satisfies the equation $\frac{\partial\Phi(t,s)}{\partial t} = A(t)\Phi(t,s)$, with the initial condition $\Phi(s,s) = I_n$. By definition, Jacobian is a linear map of tangent spaces

$$J_{q_0,T}(u(\cdot)) : T_{u(\cdot)}\mathcal{X} :\longrightarrow T_{K_{q_0,T}(u(\cdot))}Y, \quad T_{u(\cdot)}\mathcal{X} \cong \mathcal{X}, \quad T_yY \cong Y.$$

This being so, a right Jacobian inverse

$$J^{\#}_{q_0,T}(u(\cdot)) : T_{K_{q_0,T}(u(\cdot))}Y \longrightarrow T_{u(\cdot)}\mathcal{X}$$

transforms tangent vectors from the operational to the endogenous configuration space. Given a right Jacobian inverse $J^{\#}(u(\cdot))$, the Jacobian motion planning algorithm relies on the dynamic system

$$\frac{du_\theta(\cdot)}{d\theta} = -\gamma J^{\#}(u_\theta(\cdot))(K_{q_0,T}(u_\theta(\cdot)) - y), \quad u_{\theta=0}(t) = u_0(t), \tag{3.4}$$

$\gamma > 0$ denoting a convergence coefficient, whose trajectory provides a solution of the problem in the limit $\lim_{\theta\to+\infty} u_\theta(t) = u_d(t)$.

As in the previous section we shall need dual maps between dual spaces. They are: the dual Jacobian

$$J^*_{q_0,T}(u(\cdot)) : T^*_y Y \longrightarrow T^*_{u(\cdot)}\mathcal{X}, \quad (J^*_{q_0,T}(u(\cdot))r)v(\cdot) = rJ_{q_0,T}(u(\cdot))v(\cdot),$$

and the dual Jacobian inverse

$$J^{\#*}_{q_0,T}(u(\cdot)) : T^*_{u(\cdot)}\mathcal{X} \longrightarrow T^*_y Y, \quad (J^{\#*}_{q_0,T}(u(\cdot))p(\cdot))w = p(\cdot)J^{\#}_{q_0,T}(u(\cdot))w,$$

where $p(\cdot)v(\cdot)$ and rw denote the pairings between suitable covectors and vectors. By analogy to the terminology applied to robotic manipulators, tangent vectors to the endogenous configuration space will be named endogenous velocities, while the covectors are referred to as the endogenous forces. Then, the pair of maps $(J_{q_0,T}(u(\cdot)), J^{\#*}_{q_0,T}(u(\cdot))$ transforms velocities and forces from the endogenous configuration space to the operational space. The postulate of dynamic consistency of the Jacobian inverse will be expressed as a certain feature of these maps.

To unveil the consistency, we start by defining a Riemannian metric on the endogenous configuration space. A standard derivation of the dynamics equations for a non-holonomic system relies on the Lagrange-d'Alembert's Principle, see Bloch (2003), resulting in the non-holonomic inertia matrix $F(q) = G^T(q)M(q)G(q)$, where $M(q)$ denotes the inertia matrix of the unconstrained dynamics. Given $F(q)$, we take the trajectory $q(t) = \varphi_{q_0,t}(u(\cdot))$ of the system (3.1) steered by the control function $u(\cdot) \in \mathcal{X}$, and define a matrix $\mathcal{M}_{q_0}(u(\cdot))(t) = F(\varphi_{q_0,t}(u(\cdot)))$. Then, a Riemannian metric on $\mathcal{X}$ can be obtained in the following way

$$g_{\mathcal{X}}(u(\cdot))(v_1(\cdot), v_2(\cdot)) = \int_0^T v_1^T(t)\mathcal{M}_{q_0}(u(\cdot))(t)v_2(t)dt, \tag{3.5}$$

where $v_1(\cdot), v_2(\cdot) \in T_{u(\cdot)}\mathcal{X}$ are endogenous velocities. The next steps are patterned on the derivation in the previous section. Suppose that the endogenous configuration moves in $\mathcal{X}$ along a smooth curve $u_\theta(\cdot)$, with endogenous velocity $\frac{du_\theta(\cdot)}{d\theta}$. Then, the endogenous momentum $p_\theta(\cdot) = \left(\frac{du_\theta(\cdot)}{d\theta}\right)^T \mathcal{M}_{q_0}(u_\theta(\cdot))(\cdot)$. On the other hand, the curve $u_\theta(\cdot)$ gives rise to a curve $y_\theta = K_{q_0,T}(u_\theta(\cdot))$ in the operational space, and the corresponding transformation of velocities is realized by the Jacobian,

$$\frac{dy_\theta}{d\theta} = J_{q_0,T}(u_\theta(\cdot))\frac{du_\theta(\cdot)}{d\theta}.$$

Consequently, the endogenous force at θ can be computed as

$$f_\theta(\cdot) = \frac{dp_\theta(\cdot)}{d\theta} = \left(\frac{d^2u_\theta(\cdot)}{d\theta^2}\right)^T \mathcal{M}_{q_0}(u_\theta(\cdot))(\cdot) + \left(\frac{du_\theta(\cdot)}{d\theta}\right)^T \frac{d\mathcal{M}_{q_0}(u_\theta(\cdot))(\cdot)}{d\theta}. \tag{3.6}$$

Similarly as for robotic manipulators, for any right Jacobian inverse, one can make the following decomposition of the cotangent space at $u(\cdot)$

$$T^*_{u(\cdot)}\mathcal{X} = \mathrm{Im}J^*_{q_0,T}(u(\cdot)) \oplus \ker J^{\#*}_{q_0,T}(u(\cdot)),$$

allowing to represent the endogenous force $f_\theta(\cdot)$ as a sum of two components

$$f_\theta^T(\cdot) = J^*_{q_0,T}(u_\theta(\cdot))\Gamma + \left(id_{T^*_{u_\theta(\cdot)}\mathcal{X}} - J^*_{q_0,T}(u_\theta(\cdot))J^{\#*}_{q_0,T}(u_\theta(\cdot))\right) f_0(\cdot), \tag{3.7}$$

with Γ^T acting in the operational space and $f_0^T(\cdot)$ exerted in the endogenous configuration space. Now, let us compute the acceleration of the curve y_θ

$$\frac{d^2y_\theta}{d\theta^2} = J_{q_0,T}(u_\theta(\cdot))\frac{d^2u_\theta(\cdot)}{d\theta^2} + \frac{dJ_{q_0,T}(u_\theta(\cdot))}{d\theta}\frac{du_\theta(\cdot)}{d\theta}.$$

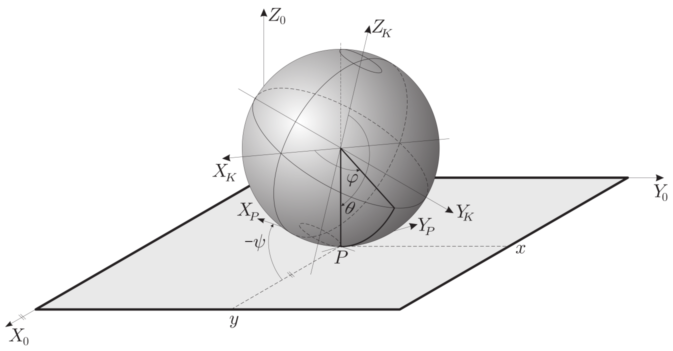

FIGURE 1. Rolling ball.

A substitution into this identity from (3.6) for $\frac{d^2u_\theta(\cdot)}{d\theta^2}$, and from (3.7) for $f_\theta(\cdot)$ yields

$$\frac{d^2y_\theta}{d\theta^2} = J_{q_0,T}(u_\theta(\cdot))\mathcal{M}_{q_0}^{-1}(u_\theta(\cdot))(\cdot)\left(J_{q_0,T}^*(u_\theta(\cdot))\Gamma + \left(id_{T^*_{u_\theta(\cdot)}\mathcal{X}} - J_{q_0,T}^*(u_\theta(\cdot))J_{q_0,T}^{\#*}(u_\theta(\cdot))\right) f_0(\cdot) - \frac{d\mathcal{M}_{q_0}(u_\theta(\cdot))(\cdot)}{d\theta}\frac{du_\theta(\cdot)}{d\theta}\right) + \frac{dJ_{q_0,T}(u_\theta(\cdot))}{d\theta}\frac{du_\theta(\cdot)}{d\theta}. \tag{3.8}$$

The dynamic consistency of the Jacobian inverse means that the endogenous forces belonging to the null space of $J^{\#*}_{q_0,T}(u_\theta(\cdot))$ do not affect the acceleration in the operational space. This will be satisfied on condition that the Jacobian inverse is defined as

$$J^{\#DC}_{q_0,T}(u(\cdot)) = \mathcal{M}_{q_0}^{-1}(u(\cdot))(\cdot)J^*_{q_0,T}(u(\cdot))\left(J_{q_0,T}(u(\cdot)\mathcal{M}_{q_0}^{-1}(u(\cdot))(\cdot)J^*_{q_0,T}(u(\cdot))\right)^{-1}.$$

Stated more explicitly and in matrix terms, this means that

$$(J^{\#DC}_{q_0,T}(u(\cdot))w)(t) = \mathcal{M}_{q_0}^{-1}(u(\cdot))(t)B^T(t)\Phi^T(T,t)C^T(T)\left(\mathcal{D}^{DC}_{q_0,T}\right)^{-1}(u(\cdot))w, \tag{3.9}$$

where $w \in T_yY$, and the mobility matrix of the non-holonomic system

$$\mathcal{D}^{DC}_{q_0,T}(u(\cdot)) = C(T)\int_0^T \Phi(T,t)B(t)\mathcal{M}_{q_0}^{-1}(u(\cdot))(t)B^T(t)\Phi^T(T,t)dtC^T(T). \tag{3.10}$$

Using the dynamically consistent Jacobian inverse, the decomposition

$$f^T_\theta(\cdot) = J^*_{q_0,T}(u_\theta(\cdot))\Gamma + \left(id_{T^*_{u_\theta(\cdot)}\mathcal{X}} - J^*_{q_0,T}(u_\theta(\cdot))J^{\#*}_{q_0,T}(u_\theta(\cdot))\right) f_0(\cdot),$$

establishes a decoupling of the endogenous forces into the forces originated in the operational space, and the internal forces affecting only the motion in the endogenous configuration space.

4. Computations

The dynamically consistent Jacobian inverse (3.9) will be applied to a motion planning problem of the rolling ball shown in Figure 1. The ball's generalized coordinates $q = (x, y, \varphi, \theta, \psi)^T$, their meaning can be easily read off from the figure. The ball has the unit mass $m = 1$, moment of inertia $I = \frac{2}{5}mR^2$, and the radius $R = 0.1$. As is well known, the kinematics of the rolling ball are represented by the driftless control system

$$\begin{cases} \dot{q}_1 = u_1 R\sin q_4 \sin q_5 + u_2 R\cos q_5, \\ \dot{q}_2 = -u_1 R\sin q_4 \cos q_5 + u_2 R\sin q_5, \\ \dot{q}_3 = u_1,\ \dot{q}_4 = u_2,\ \dot{q}_5 = -u_1\cos q_4, \\ k(q) = (q_1, q_2, q_5). \end{cases}$$

The non-holonomic inertia matrix $F(q) = (I+mR^2)diag\{\sin^2 q_4, 1\}$. Let the motion planning problem consist in rolling the ball to $y_d = (1, 1, -\frac{\pi}{4})^T$ in time $T = 2$. The initial state of the ball is $q_0 = (0, 0, 0, \frac{\pi}{4}, 0)^T$. The control functions are taken in the form of truncated trigonometric series containing the constant term and up to the 2nd order harmonics. The motion planning algorithm was initialized at $\lambda_0 = (1, 0, 0, 0, 0, 0, 0, 0, -0.2, 0, 0, 0, 0, 0)^T$ and run with $\gamma = 0.02$, until the operational space error dropped below 10^{-4}.

The motion planning problem has been solved first using the dynamically consistent Jacobian inverse, and then, for comparison, also by means of the Jacobian pseudoinverse algorithm. Results of computations are presented in Figure 2. Both these algorithms provide a solution of the problem, however, it is worth noticing that ball steered by the latter algorithm rolls through the North Pole ($q_4 = 0$), that is a representation singularity of the ball's kinematics, while for the dynamically consistent algorithm this kind of motion is not permitted.

5. Conclusion

This paper extends the concept of dynamically consistent Jacobian inverse from holonomic to non-holonomic robotic systems. This extension has been based on the analogy between the kinematics of holonomic and non-holonomic systems, and accomplished within the framework of endogenous configuration space approach. As a result, the dynamically consistent Jacobian inverse for the non-holonomic system prevents the transmission of certain forces from the system's configuration to the operational space, and establishes a decoupling of forces acting in the configuration space. It has been shown that the motion planning algorithm based on the dynamically consistent Jacobian inverse outperforms the classic Jacobian pseudoinverse algorithm. Future research should address the force control problem of non-holonomic systems.

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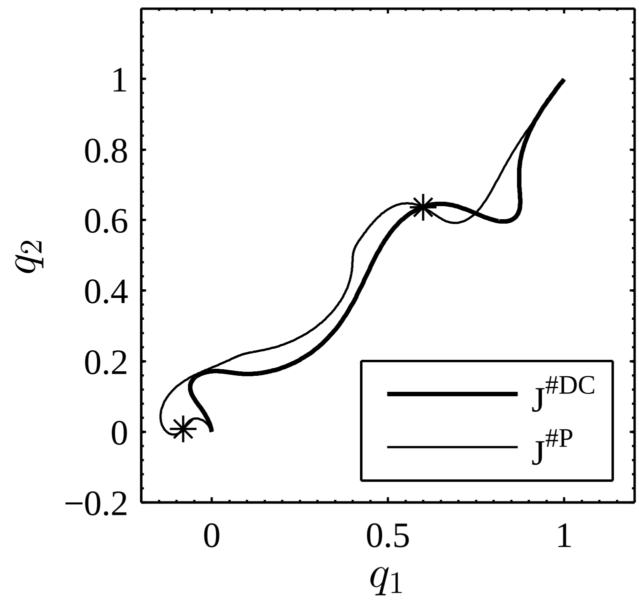

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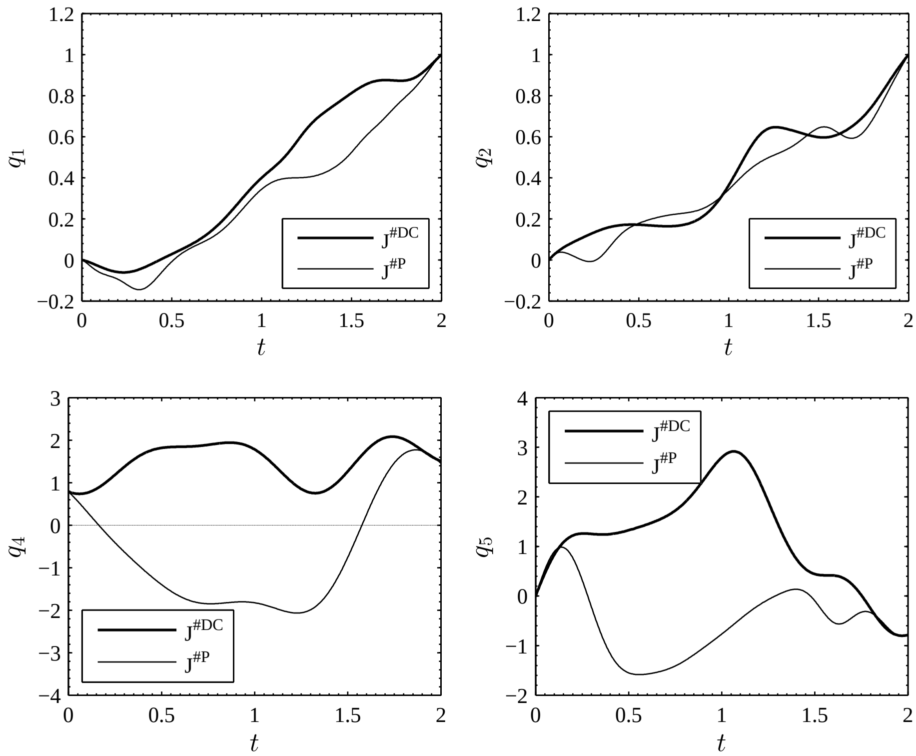

FIGURE 2. The rolling ball: paths in the XY plane (asterisks mark rolling through the North Pole), trajectories of q_1, q_2 and q_4, q_5.