Fermat's Last Theorem; a talk by Simon Singh presented to the IMA 40th Anniversary Conference

With an off-the-cuff apology for advertising, Singh begins by plugging a project for placing undergraduates in schools. The aim is to enthuse pupils and encourage them to study mathematics and science at university. He wants the collection of mathematicians arrayed before him to return to their respective departments and take this project idea with them. This said, he moves to his theme.

In 1993 Andrew Wiles, a researcher in Princeton, presented a proof of the Taniyama-Shimura conjecture and hence of Fermat's last theorem. It was big news; front page space was devoted to the story in a lot of national papers, an unusual feat for a mathematical proof. Wiles' story itself was compelling: dreaming of solving this ancient problem since he was ten years old; isolating himself for seven years to work solely on this; and then when finally he emerged to present the proof, it was discovered to have a hole. . . and then, the fact that this hole was subsequently patched by Wiles himself. Truly a story which is worthy of air time.

The BBC had intended a documentary about Wiles and Fermat's last theorem, and some level of production had started. However, when the proof was found to be flawed, the project had been dropped, only to be reinstated when the proof was patched. In the intervening period however, the original producer had been promoted and Singh was drafted in to make the feature.

Singh starts to show us clips from the programme on a large projector screen and it quickly becomes apparent that he is to talk about the documentary itself as much as Fermat's last theorem or Wiles himself. The opening scene is a trip through a house, with the light gradually increasing. Wiles provides the voice over, comparing mathematics to walking through a darkened mansion, tripping over the furniture, gradually mapping things out and eventually finding a light switch. Cut to Wiles at his desk; a stereotypical academic, wild hair, slightly balding and completely surrounded by disordered stacks of paper. He is talking about the moment of epiphany with obvious emotion, when he confesses that nothing he will ever do in the future will come close to comparing with Fermat's last theorem, he breaks down and we cut to the title sequence. Singh reveals that there is only one very brief mention of mathematics in this opening sequence, a 'random channel hopper' would simply see someone obsessed with

something, genuinely passionate about it. This is designed to hook the viewer.

There are six or seven major characters in the documentary, and although there are other mathematicians who would have been suitable, the number was kept down to allow the viewer time to properly engage with each. We focus for a while on John Conway as Singh tells us that he appears in the film for a number of reasons: firstly as an 'eyewitness' to the events—he was around Wiles in the last stages and he was present at the unveiling; secondly, he is a historian of mathematics and as such is suited to giving a history of the problem itself; and thirdly he is a great character who comes across well on screen. Conway apparently struggled with the filming process, the necessity of retakes and different camera angles which required him to repeat the same lines and actions over and over was the antithesis of his usual, natural way of speaking.

Fermat's last theorem captivates the non-mathematician because Pierre de Fermat—a seventeenth century lawyer and amateur mathematician—claimed to have a proof. He wrote a note in the margin of his copy of Diophantus' *Arithmetica* that he had a "truly marvellous proof which this margin is too small to contain". This tantalising hint at an extant proof has fuelled mathematical enquiries for centuries. The extended historical consequences of Fermat's private remark—he never intended to publish his work—had attracted the original planners of the BBC documentary. The original script provided a potted history of the many attempts to find Fermat's proof.

Although, when beginning such a project, an outline script is produced, it is generally the case that during the process of research and filming, this gets heavily modified and is often rejected entirely. In this case, when reviewing the film they had taken, Singh found that the most engaging footage was that of mathematicians talking about Wiles' outlook or their own. In response to this, the initial idea of a potted history was reduced to a twenty second sequence of pictures—with sound-bites of names overlaid—of the mathematicians who had involved themselves since Fermat's day.

One of the difficulties with making a program about mathematics for the general public is that, because mathematics has a narrow knowledge base upon which many other concepts are piled, there is a lot of defining and explaining that has to be either done or circumvented. The concept of a rigorous proof is almost essential to understanding the tribulations of attacking Fermat's last theorem and yet to really and clearly describe it in a short sequence is extremely demanding. In the end, the sequence in the film cuts between two mathematicians who seem to complete each others' sentences. There was no script for this, but the two gave descriptions of a closed and complete argument which were similar enough to allow such creative editing.

After swearing us to secrecy(!), Singh prepares to unveil a secret of the pro- gramme. He plays a 'spot the difference' game showing a short sequence of film to us several times to allow us to find any discrepancies. The sequence is of Conway sitting in his office describing why an exhaustive search—checking each number in turn—is pointless and will never provide a proof. During the recording, Conway in fact used the word 'primes', but during the editing process this was changed to 'numbers'. "He said numbers somewhere in the interview" says Singh, "so we just stole that". The justification is that, whilst maintaining the meaning and mathematical validity of the statement, this simple substitution meant there was no need to define primes or to explain why a proof covering all primes was sufficient. None of us in the audience spots the difference until it is pointed out.

Skipping much of the biography of Wiles, Singh moves us ahead to the first climax of the documentary. Whilst fast-forwarding the video, he gives us a brief run-down of the events in the middle section: Wiles' isolation for seven years whilst he worked solely on the problem; the link with the Taniyama-Shimura conjecture; and most mathematicians' lack of belief that this could be solved. When the video starts again it is Wiles sitting at his desk, speaking again about the moment of epiphany; having skipped lunch in his excitement, when he had finally wrapped it up, Wiles descended from his attic triumphantly to tell his wife that he had proved Fermat's last theorem. The emotional climax—for now—is Wiles revealing his proof to the mathematical world at a conference in Cambridge, his home town and where he spent his post-doctoral years. With an obvious eye for the theatrical, Wiles made no mention of Fermat in his title and avoided making any link with the problem throughout his mini lecture series. Rumours flew thick and fast for a few days at the conference, but only at the end of the last lecture did Wiles write up the statement of Fermat's last theorem on the board and declare that he had proved it, saying 'I think we'll leave it there'. The documentary reconstructs this event, many of those present talk about the unique atmosphere of those lectures, the drama, the tension and the anticipation. Wiles talks about the trick used to deal with the family of elliptic curves that had otherwise escaped the net. Throughout there is no narration. "There was no need", says Singh, "the momentum has built sufficiently by this point that the obvious excitement of the interviewees is all that is needed to drive the viewer forward".

But there was an error in the proof. "Fantastic", says Singh raising a laugh. It increases dramatic tension, even better is that it has a happy ending, otherwise the project would have been dropped; you can't screen a documentary about something that wasn't proved. The monster was killed, comes back and is dispatched again, for real this time. The last footage from the documentary that we see is an excited, impassioned Wiles reiterating.

As a bit of light relief, and to raise a serious point about an issue close to his heart, Singh shows us a clip from a musical—commissioned and filmed by the Clay Institute—called 'Fermat's Last Tango'. This tells the story of the proof—the collapse and subsequent reinstatement, as well as the media reaction. It is awful—painful to watch—the kind of thing that you cringe away from in embarrassed sympathy for the performers. "This is what happens when popularising maths goes wrong, badly wrong!"' says Singh. Yet he praises the Clay Institute for their millennium prizes—the seven one million dollar rewards for high-profile unsolved problems—this is good for mathematics, it means that results hit the media, the public consciousness is made aware of someone winning a million dollars when, otherwise, the publication of an obscure mathematical proof would surely be missed.

Although to suggest a moral would be overstating things, Singh leaves his audience in no doubt about his strong views on popularising mathematics. His obvious enthusiasm for the task is matched by a lack of patience with overly technical presentation, with the needless detail which immediately turns o an audience be they young, prospective mathematicians or a potentially interested member of the public.❑

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