A two-dimensional availability-based warranty policy incorporating preventive maintenance

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Abstract. A two-dimensional (2D) availability-based warranty policy is investigated from the view of the manufacturer. Under the policy, the manufacturer guarantees a negotiated availability for the products during the warranty period. In order to satisfy the availability requirement with rational cost, 2D preventive maintenance (PM) strategy is carried out, which is scheduled every K units of age or L units of usage. This study is to meet products' availability requirement with the lowest cost by optimizing 2D PM strategy. A numerical example is provided to verify the effectiveness of the proposed policy. The results show that compared with age-based or usage-based PM strategy, 2D PM strategy is superior in terms of warranty cost and operational availability.

1. Introduction

Warranty is a type of guarantee that a manufacturer makes to deal with the item which fails to fulfill its intend function throughout the warranty period (Yeh and Fang, 2015). The most common warranty policies include free repair or replacement warranty (FRW), pro-rata warranty (PRW) and renewing free replacement warranty (RFRW) (Darghouth, et al. 2017). However, for complex industrial equipment, such as wind turbine and dump truck, the above policies may not satisfy actual needs in operation and maintenance. The reason is that such devices are usually key elements in the corresponding systems and they may suffer from substantial loss or service interruption due to failure or shutdown. Hence, the availability level of such equipment should be guaranteed from the perspective of the customer, and rational warranty policies should focus on their availability requirement.

This study is to study an availability-based warranty policy for industrial equipment. Under this policy, the manufacturer should not only provide free repair or replacement upon failures, but also guarantee the availability of the warranted equipment to be over a negotiated level during the warranty period. The existing research on availability-based warranty is quite limited (Cheng, et al. 2015; Yan, et al. 2011; Iskandar, et al. 2014). Su and Wang (2016) investigated an availability-based warranty policy in the framework of one-dimensional (1D) warranty. Compared with 1D warranty, 2D warranty can not only protect manufacturers from additional damage caused by customers' heavy usage, but also provide customers with flexible options (Wang and Su, 2016). Therefore, this study investigates an availability-based warranty policy based on a 2D framework, which considers both the age and usage of the warranted products.

In the availability-based warranty policy, one of the most fundamental issues is to guarantee the warranted equipment with a negotiated minimal level of availability and at a reasonable cost. Thus, appropriate PM action is a wise choice, which can effectively reduce the possibility of failure and downtime (Iskandar, et al. 2014).

In most existing references, PM activities are usually performed by considering only one single factor, e.g., age or usage. In this paper, 2D PM strategy is considered, that is, PM actions are scheduled every K units of age or L units of usage, whichever occurs first (Wang and Su, 2016). Compared with age-based or usage-based PM strategy, 2D PM strategy is superior, or at least identical in terms of warranty cost (Su and Wang, 2016). As the major contribution, this study investigates the availability-based warranty policy in the framework of 2D warranty. Meanwhile, imperfect 2D PM strategy is carried out to achieve the prospective availability goal with the lowest cost from manufacturers' perspective.

The remaining of this paper is organized as follows. In Section 2 the mathematical models are established, including product failure, imperfect PM, warranty servicing cost and availability models. The numerical example is provided to illustrate the proposed warranty policy in Section 3. Finally, Section 4 concludes this paper and suggests some directions for future research.

2. Model Formulation

In terms of an item sold with a 2D availability-based warranty policy, the manufacture provides customers with free repair service in a rectangular region as shown in Figure 1 (Note that, this illustration ignores PM intervals). At the same time, the availability level predetermined by both sides of the transaction is guaranteed over the warranty period. The boundary of the warranty implemented by the manufacturer is age limit W or usage limit U, whichever occurs first. In addition, the manufacturer adopts a 2D PM strategy free of charge so as to satisfy the availability requirement with a lower cost, that is, the item is preventively maintained every K units of age or L units of usage, which occurs first.

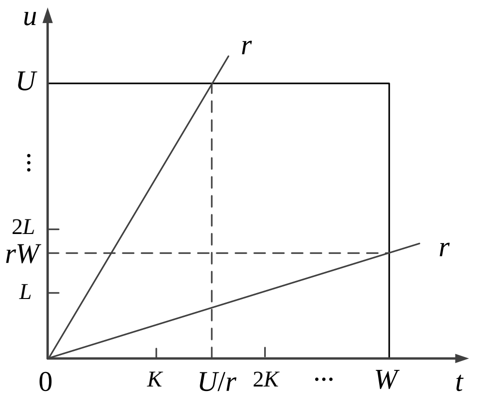

Figure 1. Two-dimensional warranty policy with PM.

In this study, it is supposed that the product is repairable and will deteriorate with both age and usage. In addition, during the warranty period, if a failure happen to the product, it will be minimally repaired so as to restore the product to working status just as that state before it failed.

Up to now, there are three kinds of methods proposed to model the product failure process under 2D warranties, including univariate method, bivariate method and composite scale method (Wu, 2014). Among them, the univariate method treats the usage as a function of the time so that 2D warranties modeling can be solved from a 1D perspective. This method is adopted to model the failures for items due to its simple and wide use.

The operating time and usage of an item can be denoted by T(t) and $U(t)$ at age t. When no failure occurs before t or all failures are repaired with negligible repair durations, $T(t) = t$ (Iskandar and Murthy, 2003). There is a fundamental assumption in the univariate method, that is, the usage rate varies randomly across the customer population but is constant over time for a specific customer. Based the above assumption, the usage rate $R = U(t)/T(t)$ can be expressed as a nonnegative variable with distribution function $G(r) = P\ (R \leq r)$, $r_{\min} \leq r \leq r_{\max}$, where $r_{\min}$ and $r_{\max}$ are the minimum and maximum values of r, respectively. Note that, the distribution $G(r)$ can be obtained from the past history of similar items or customer reports. Thus, product failures can be modeled by a stochastic process with an intensity function dependent on age and usage. Given a usage rate $R = r$, the conditional intensity function can be expressed as:

$$\lambda(t|r) = \varphi(T(t), U(t)) \tag{1}$$

where $\varphi(\bullet)$ is a non-decreasing function of $T(t)$ and $U(t)$.

Furthermore, the conditional intensity function in this work adopts the simplest first order polynomial form proposed by Iskandar and Murthy (2003), and is given by:

$$\lambda(t|r) = \theta_0 + \theta_1 T(t) + \theta_2 U(t) + \theta_3 T(t)U(t) \tag{2}$$

where θ_0, θ_1, θ_2 and θ_3 are positive constants.

Conditional on $R = r$, the usage $U(t)$ of an item equals rt at age t.

$$\lambda(t|r) = \theta_0 + \theta_1 t + \theta_2 rt + \theta_3 rt^2 \tag{3}$$

In this study, virtual age model proposed by Kijima (1989) is used to model imperfect PM effect, which means that the virtual age of the system is reduced to a certain value after PM actions. Assuming that PM program is carried out at actual age τ_1, τ_2, …, τ_j, …, with $\tau_0 = 0$. It's worth mentioning that the jth PM action only reduces the supplement of system age since the (j–1)th PM. Hence, the virtual age v_j right after the jth PM can be expressed as (Kim, et al. 2004):

$$v_j = v_{j-1} + \delta(m)(\tau_j - \tau_{j-1}) \tag{4}$$

where m is PM effort level with m = 0, 1, 2, …M. Note that, m = 0 implies that the effect of PM is as bad as old (ABAO), $m = M$ implies that the effect of PM is as good as new (AGAN), and PM is imperfect when $0 < m < M$. $\delta(m)$ is a decreasing function of m with $\delta(0) = 1$ and $\delta(M) = 0$. Here, $\delta(m)$ is modeled as an exponentially function of m, i.e., $\delta(m) = (1+m)e^{-m}$.

According to Huang and Yen (2009), Equation (4) can be recursively derived as:

$$v_j = v_0 + \delta(m)(\tau_j - \tau_0) \tag{5}$$

When $\tau_0 = 0$ and $v_0 = 0$, then

$$v_j = \delta(m)\tau_j \tag{6}$$

Note that, we assume that the level of PM effort m is constant over the warranty period (Huang and Yeh, 2009). In addition, it is supposed that the jth PM activity has the same impact on virtual usage due to the reduction of virtual age under 2D PM strategy. The above assumption is reasonable since the linear relationship $U(t) = rt$ is used in the univariate method (Wang and Su, 2016).

Let $EC(\Omega)$, $EA(\Omega)$ denote the expected warranty servicing cost and operational availability over the warranty coverage respectively, where the decision space Ω represents the parameters of 2D PM and is characterized by $\Omega = (K, L, m)$. Among them, $EC(\Omega)$ consists of the minimal repair cost and the PM cost from manufacturers' perspective and $EA(\Omega)$ is defined by mean uptime (MUT) and mean downtime (MDT) during the warranty period, that is, $EA(\Omega)$ = MUT/(MUT+MDT). In addition, faulty items are minimally repaired throughout the warranty period with cost C_f and duration T_f. Each PM action with m level is implemented at a cost of $C_p(m)$, which increases as m increases. Besides, T_p is assumed to be the expected PM duration.

Define $r_1 = U/W$, $r_2 = L/K$ and consider the following two cases under 2D PM strategy: Case a ($r_1 > r_2$) and Case b ($r_1 \le r_2$). For Case a, the following three subcases are investigated, including (1) $r \le r_2$; (2) $r_2 < r \le r_1$; (3) $r > r_1$.

(1) $r \le r_2$

In this subcase, the warranty expires at time point (W, rW). Under the 2D PM strategy, the PM actions are scheduled with interval K and when the jth PM activity ends, $\tau_j = j(K+T_p)$. Then the number of PM actions implemented during the warranty coverage can be obtained as:

$$n_1 = \max\{j \mid j(K+T_p) \le W, j \in Z\} \tag{7}$$

Note that, after the n_1th PM actions, the manufacturer would not implement additional PM action and the subsequent interval will end when the warranty terminates. In addition, the occurrence of failures between two successive PMs, which are all minimally rectified and follows a non-homogeneous Poisson process (NHPP). Thus, the conditional expected number of failures can be expressed as:

$$E[N_1] = \sum_{j=0}^{n_1-1} \int_{j\delta(m)(K+T_p)}^{j\delta(m)(K+T_p)+K} \lambda(t|r)dt + \int_{n_1\delta(m)(K+T_p)}^{n_1\delta(m)(K+T_p)+W-n_1(K+T_p)} \lambda(t|r)dt \tag{8}$$

Hence, conditional on $R = r \le r_2$, the expected warranty servicing cost $EC_1(\Omega)$ can be obtained as:

$$EC_1(\Omega) = E[N_1]C_f + n_1 C_p(m) \tag{9}$$

Furthermore, for this case MDT consists of minimal repair durations and PM durations. When $r \le r_2$, MDT_1 can be expressed as:

$$\mathrm{MDT}_1 = n_1 T_p + E[N_1] T_f \tag{10}$$

For an item with $r \le r_2$, the warranty ceases at time W. Therefore, the expected operational availability $EA_1(\Omega)$ is given by:

$$EA_1(\Omega) = \frac{W - \mathrm{MDT}_1}{W} \tag{11}$$

(2) $r_2 < r \le r_1$

For an item with $r_2 < r \le r_1$, the warranty terminates at time point (W, rW) and PM activities are carried out with interval L. Therefore, the number of PM actions and the conditional expected number of failures over the warranty period are given by:

$$n_2 = \max\{j : j(L/r + T_p) \le W\} \tag{12}$$

$$EN_2^r = \sum_{j=0}^{n_2-1} \int_{j(L/r)}^{(j+1)(L/r)} \lambda_j(t\,|\,r)\,dt + \int_{n_2(L/r)}^{W-n_2 T_p} \lambda_{n_2}(t\,|\,r)\,dt \tag{13}$$

Further, conditional on $R = r$ ($r_2 < r \le r_1$), the expected warranty cost $EC_2(\Omega)$ and operational availability $EA_2(\Omega)$ can be obtained as follows:

$$EC_2(\Omega) = EN_2^r C_f + n_2 C_p(m) \tag{14}$$

$$EA_2(\Omega) = \frac{W - (EN_2^r T_f + n_2 T_p)}{W} \tag{15}$$

(3) $r > r_1$

Similar to the above analysis, conditional on $R = r > r_1$, the expected warranty cost $EC_3(\Omega)$ and operational availability $EA_3(\Omega)$ can be obtained:

$$EC_3(\Omega) = EN_3^r C_f + n_3 C_p(m) \tag{16}$$

$$EA_3(\Omega) = \frac{U/r - (EN_3^r T_f + n_3 T_p)}{U/r} \tag{17}$$

where n_3 is the number of PM actions, $n_3 = \max\{j : j(L/r + T_p) \le U/r\}$; EN_3^r is the conditional expected number of failures, $EN_3^r = \sum_{j=0}^{n_3-1} \int_{j(L/r)}^{(j+1)(L/r)} \lambda_j(t\,|\,r)\,dt + \int_{n_3(L/r)}^{U/r-n_3 T_p} \lambda_{n_3}(t\,|\,r)\,dt$.

Finally, removing the conditioning on R, the expected warranty servicing cost $EC(\Omega)$ and operational availability $EA(\Omega)$ in Case a, can be obtained as, respectively:

$$EC(\Omega) = \int_{r_{\min}}^{r_2} EC_1(\Omega)\,g(r)\,dr + \int_{r_2}^{r_1} EC_2(\Omega)\,g(r)\,dr + \int_{r_1}^{r_{\max}} EC_3(\Omega)\,g(r)\,dr \tag{18}$$

$$EA(\Omega) = \int_{r_{\min}}^{r_2} EA_1(\Omega)\,g(r)\,dr + \int_{r_2}^{r_1} EA_2(\Omega)\,g(r)\,dr + \int_{r_1}^{r_{\max}} EA_3(\Omega)\,g(r)\,dr \tag{19}$$

Similar to Case a, we also need to consider the following three subcases in Case b. They are: (1) $r \le r_1$; (2) $r_1 < r \le r_2$; (3) $r > r_2$. Following the similar procedures, the warranty cost and availability models for Case b can be obtained. Limited the length of this paper, detailed derivation process of Case b is omitted.

Under the availability-based warranty policy, the manufacturer optimizes PM schedules during the warranty period, so as to meet a negotiated availability level A_0 of the equipment with the lowest cost, that is, the optimal PM decision space Ω to minimize $EC(\Omega)$ with an availability constraint. Therefore, the availability-based warranty optimization model is given by:

$$\begin{cases} \min \quad EC(\Omega) \\ s.t. \quad EA(\Omega) \ge A_0 \\ K, L > 0,\ 0 \le m \le M \end{cases} \tag{20}$$

3. Case Study

Considering a repairable automobile component, the manufacturer provides an availability-based warranty service for each product with warranted availability level 0.96, and warranty coverage of 3 years/ 30, 000 km. In addition, the manufacture implements 2D PM activities free of charge so as to minimize warranty cost with the availability requirement. It is assumed that the usage rate R follows uniform distribution with $r_{\min}$ = 0.2 and $r_{\max}$ = 1.8 (10^4 km/year). The values of θ_0, θ_1, θ_2 and θ_3 are set to be 0.1, 0.2, 0.7 and 0.7, respectively. The corresponding maintenance time are as follows: T_f = 0.02 years, T_p = 0.004 years. Besides, set m = 0, 1, 2, 3, 4, 5, and the relevant PM cost is $C_p(0)$ = \$0, $C_p(1)$ = \$10, $C_p(2)$ = \$30, $C_p(3)$ = \$60, $C_p(4)$ = \$100 and $C_p(5)$ = \$160 (Cheng, et al. 2015; Wang and Su, 2016).

The grid search in specific regions is performed to optimize PM decisions in this work. For the sake of efficient maintenance management, the search steps of K and L are one month and 10^3 km respectively (Wang and Su, 2016). Under 2D PM strategy, the optimal decisions Ω^* = (K^*, L^*, m^*), the expected warranty cost EC and expected operational availability EA with varying minimal repair cost C_f are summarized in Table 1. For example, when C_f = \$150, PM actions should be scheduled every 12 months or

10, 000 km and the corresponding expected warranty cost and operational availability are EC = \$548.12 and EA = 0.97457 > A_0. As shown in Table 1, appropriate 2D PM activities carried out by manufacturers can effectively satisfy products' availability requirements during the warranty period. In addition, it is found that K^* and L^* gradually decrease as C_f increases, while m^* tends to increase as C_f increases. It can be interpreted by the fact that the manufacturer tends to improve the level of PM actions or reduce PM intervals, so as to reduce the occurrence of failures and compensate the increase of warranty cost.

Table 1 Optimal PM decisions under 2D PM strategy.

C_f	K^*	L^*	m^*	EC	EA
50	18	15	2	226.96	0.96783
100	12	10	3	405.41	0.97457
150	12	10	3	548.12	0.97457
200	9	8	3	690.35	0.97529
250	9	8	3	817.94	0.97529
300	9	8	3	945.53	0.97529
350	9	8	4	1065.10	0.97810
400	9	8	4	1174.40	0.97810
450	9	8	4	1283.70	0.97810
500	9	6	4	1391.12	0.97809

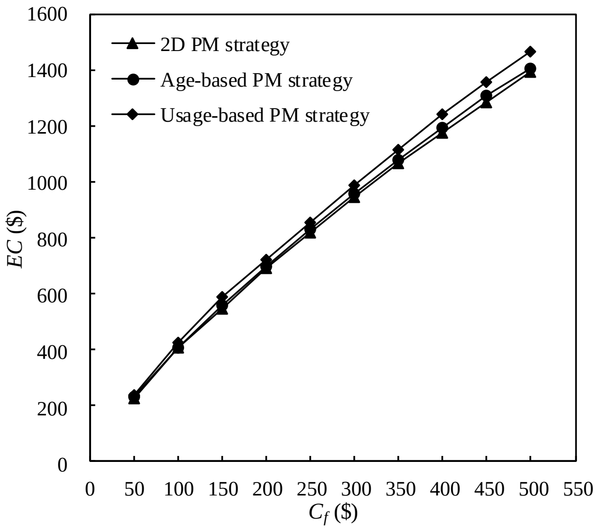

Figure 3. Expected warranty cost under PM strategies.

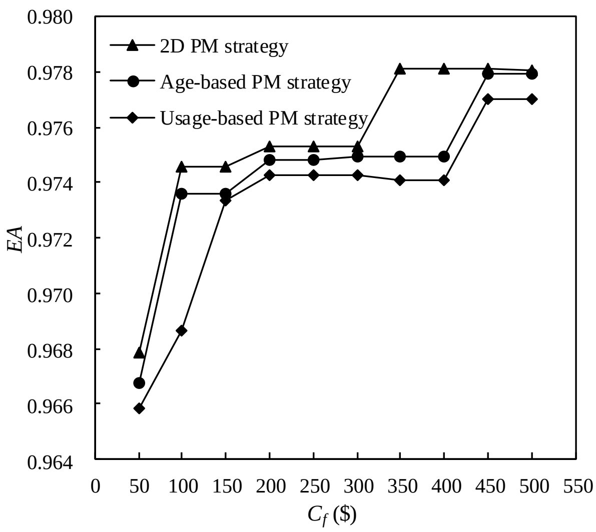

Figure 4. Expected operational availability under PM strategies.

In order to further illustrate the superiority of 2D PM strategy, the results for age-based and usage-based PM strategies can be used as the references. From Figure 3, the expected warranty cost *EC* under three PM strategies is compared. It is obvious that in terms of warranty cost, 2D PM strategy is superior or at least identical to 1D PM strategy. What's more, as can be seen from Figure 4, the expected operational availability *EA* for 2D PM strategy is higher in comparison to the other PM strategies. Hence, whether in terms of availability or warranty cost, 2D PM strategy is superior to the age-based and usage-based strategies under the availability-based warranty policy.

4. Conclusion

In this paper, the availability-based warranty policy is investigated in the framework of 2D warranty. From the manufacturer's perspective, rational 2D PM schedules are carried out with a view to satisfying the availability requirement of product and minimizing warranty cost within the warranty period. Numerical example shows that under the availability-based warranty policy, 2D PM strategy is superior to the age-based and usage-based strategies. In future studies, the PM expenditure is shared partially by customers, instead of being paid by manufacturers. The usage rate can also be assumed to be other distributions, e.g., normal distribution or Weibull distribution.

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