Applications of the gamma process in storage models: Energy Harvesting Systems

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Abstract The gamma process is a stochastic process extensively used to model the degradation of a system over the time. In this paper, gamma process is used in a different perspective: to model the harvested energy for an Energy Harvesting System (EHS). Energy harvesting refers to harnessing energy from the environment or other energy sources and converting it to electrical energy. Specifically, in this work, gamma process is used to model the harvested energy for a Radio Frequency Energy Harvesting System. A management policy of this EHS is described.

1. Introduction

In general terms, an energy harvesting system is a system that captures small amounts of readily available energy in the environment and converts it into usable electrical energy. Nowadays, EHS's are emerging as a promising technology for charging batteries used to supply ultra low power micro controllers and sensors (Pantelopoulos and Pantelopoulos, 2010). Applied to sensor nodes, energy from external sources can be harvested to power the nodes and in turn, increase their lifetime and capability (Sudevalayam and Kulkarni, 2011).

In an EHS, the incoming energy is managed using two points of view.

- Harvest-store-use (HSU) where harvested energy can be accumulated for future use. It requires an energy storage device or battery along with an appropriate management of the stored energy.
- Harvest-use (HU) where energy can not be stored and it is used immediately.

Focusing on the HSU architecture and depending on the capacity of the storage device, finite and infinite capacity of the battery can be considered.

- Infinite capacity implies that the incoming energy can always be saved in the battery. It can be, for example, devices with a very large energy storage capacity compared to the rate of the harvested energy flow (Ozel and Ulukus 2012).
- HSU with finite capacity of the storage battery implies that the harvested energy can be stored up to a limit. Beyond this limit, incoming energy can not be stored and would be lost (Tutuncuoglu and Aylin, 2012).

In this paper, an EHS system with a HSU architecture is analyzed. The EHS system contains

(a) An antenna that gathers energy from the environment.

(b) A battery with finite capacity to store the harvested energy.

(c) A device powered by the battery. The device received energy from the battery at certain times called *feeding times*.

The harvested energy can be treated as a stochastic process due to the random nature of the energy source. Common stochastic processes in the literature to describe the harvested energy include Poisson processes (Yang *et. al* 2016) and compound Poisson processes (Khuzani and Mitran, 2014). However, in this paper, the process used to model the harvested energy by the EHS is the gamma process. Gamma process is a versatile stochastic process used in many engineering fields to model, for example, the system degradation over the time, the water storage by dams and the aggregate claims process in classical collective risk theory (van Noortwijk, 2009). From a theoretical point of view, a gamma process can be thought as a compound Poisson process with Poisson rate tends to infinity and increment sizes tend to zero in proportion. The definition of gamma process is first recalled.

DEFINITION 1. *We say that $\{X(t), t \geq 0\}$ is a homogeneous gamma process with parameters α and β if it fulfills the following properties.*

- $P(X(0) = 0) = 1$
- *Increments $X(t+s) - X(t)$ are independent for all t and s.*
- *Increments $X(t+s) - X(s)$ follow a gamma distribution with parameters αt and β.*

The technical justification for the use of a gamma process to model the harvested energy for a Radio Frequency Energy Harvesting System is described in detail in Castro *et. al* (2018).

2. System Description

We assume a harvesting system which contains an antenna and a battery to store the harvested energy. The device is powered by the battery at certain times called *feeding times*. Let $T_1, T_2, \ldots,$ the successive feeding times.

The system functioning is described as follows.

(a) Let $E_H(t)$ be the harvested energy by the system at time t. We assume that $\{E_H(t), t \geq 0\}$ follows an homogeneous gamma process with parameters α and β. For $s < t$, we get that the probability distribution function of the harvested energy in the interval $(s, s+t)$,, $E_H(s, s+t)$, is given by

$$f_{E_H(s,s+t)}(x) = \frac{\beta^{\alpha t}}{\Gamma(\alpha t)} x^{\alpha t - 1} \exp\left(-\beta x\right), \quad x > 0, \tag{1}$$

where $\Gamma(\cdot)$ denotes the gamma function given by

$$\Gamma(\alpha t) = \int_0^\infty x^{\alpha t - 1} e^{-x} dx. \tag{2}$$

(b) Let $E_B(t)$ be the stored energy in the battery at time t with $E_B(0) = E_0$ where E_0 denotes the initial stored energy. There is a *safety threshold* in the battery that corresponds to the minimum energy reserve of the battery that can not be consumed. It means that the level of stored energy must always exceed a threshold level E_M for security reasons with $E_0 > E_M$.

(c) The capacity of the battery is finite and equals to E_K hence the stored energy in the battery at time t is bounded

$$E_M \leq E_B(t) \leq E_K, \quad \forall t. \tag{3}$$

It means that, when the level of energy in the battery reaches the threshold E_K, the incoming energy is overflowed.

(d) The device can be in two modes: ON and OFF depending on the feeding process.

- In mode ON, the device is operational and powered by the battery. It consumes E_C energy units in each feeding time.
- In mode OFF, the device is turned off and the feeding process of the device stops.

(e) Let $E_B(T_n^-)$ be the stored energy in the battery right before the feeding time T_n for $n = 1, 2, \ldots$.

- If $E_B(T_n^-) - E_C \geq E_M$, the device is properly powered and a *feasible feeding task* is performed at time T_n.
- If $E_B(T_n^-) - E_C \leq E_M$, the feeding of the device starts but, when the system detects that the stored energy is going to drop below E_M, the feeding process stops. In this case, the device is not properly powered and it is called an *unfeasible feeding task*. Right after an unfeasible feeding task, the energy stored in the battery is equal to the threshold E_M and the device enters in OFF mode and the system keeps on harvesting energy. When the stored energy reaches the level E_0, the device enters in ON mode and the feeding process starts again.

(f) Feeding times are periodic, that is, $T_i - T_{i-1} = T$ and $T > 0$ for $i = 1, 2, \ldots$.

Let $\left\{E_H^{(i)}(t), t \geq 0\right\}$ replicas of the process $\{E_H(t), t \geq 0\}$ for $i = 1, 2, \ldots$. Starting from 0, the system evolves as follows

$$E_B(t) = E_H^{(1)}(t) + E_B(0), \quad 0 \leq t < T.$$

The harvested energy in $[0, T]$ is noted by $E_H^{(1)}(T)$ with density given by (1). Right before T, the energy stored in the battery is equal to

$$E_B(T^-) = \min\left(E_H^{(1)}(T) + E_B(0), E_K\right).$$

At time T,

- If $E_B(T^-) - E_C \geq E_M$, the feeding task is performed properly and the device remains in mode ON. The stored energy in the battery at time T^+ is equal to

$$E_B(T^+) = E_B(T^-) - E_C.$$

- If $E_B(T^-) - E_C < E_M$, the feeding task is not performed properly (*unfeasible*) and the device enters in mode ON. The stored energy in the battery at time T^+ is equal to

$$E_B(T^+) = E_M.$$

Hence, the stored energy in the battery at time T^+ is equal to

$$E_B(T^+) = \max\left(E_B(T^-) - E_C, E_M\right).$$

In a general setting, let $E_B(iT^-)$ be the stored energy in the battery right before a feeding time iT.

- If $E_B(iT^-) - E_C \geq E_M$, the device is properly powered and it is called a *feasible feeding task* and

$$E_B(iT^+) = E_B(iT^-) - E_C.$$

- If $E_B(iT^-) - E_C < E_M$, the feeding of the device starts but, when the system detects that the stored energy is going to drop below E_M, the feeding process is stopped and

$$E_B(iT^+) = E_M.$$

Hence

$$E_B(iT^+) = \max\left(E_B(iT^-) - E_C, E_M\right).$$

Notice that $E_M = 0$ implies that, as long as there is energy in the battery, the device remains in mode ON.

Let

$$V_i = \left\{E_B(iT^-) - E_c > E_M\right\}, \tag{4}$$

for $i = 1, 2, \ldots$ that corresponds to a feasible feeding task at time iT.

Considering an finite horizon time T^*, the probability that the device is in mode ON from 0 to T^* is equal to

$$\mathbb{P}\left(\cap_{i=1}^{n^*} V_i\right), \quad \text{with} \quad n^* = \lfloor \frac{T^*}{T} \rfloor,$$

where V_i are given by (4) Whenever (3) is fulfilled in $[0, T^*]$, the total energy consumed by the device due to the feeding tasks at time T^*, $E_D(T^*)$, is equal to

$$E_D(T^*) = \lfloor T^*/T \rfloor E_C.$$

Figure 1 shows the evolution of the energy stored in the battery. Blue arrows corresponds to feasible feeding tasks. Red arrows correspond to unfeasible feeding task, that is, the energy stored in the battery was not enough to feed the device under the safety constraint given by (3).

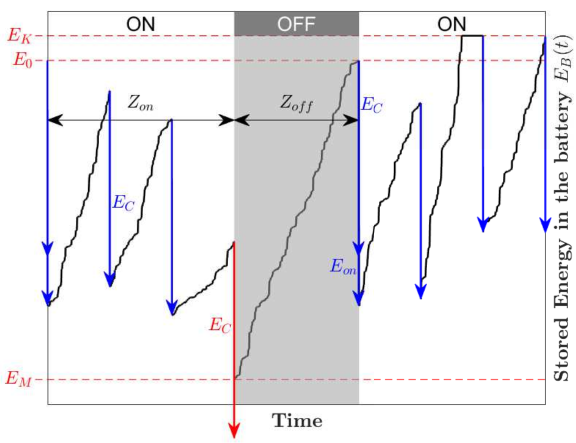

Figure1. Evolution of the energy stored in the battery due to the operation of the EHS

3. Management policy of the EHS

Renewal techniques are used in this work to describe management policies of this system. We recall that a renewal process is an arrival process in which the inter-arrival intervals are independent and identically distributed random variables. Applying the renewal process to this system, a renewal cycle takes place every time that the period OFF ends.

After the switching on at time $t=0$, the energy stored in the battery is E_0. The device remains in mode ON until the first unfeasible feeding time. After this unfeasible feeding time, the device enters in model OFF and the system keeps harvesting energy until the energy stored in the battery exceeds the level E_0, time in which the system again enters in ON mode (see Figure 1).

Let $n_{uf}T$ be the first unfeasible feeding time, that is,

$$n_{uf} = \inf\left\{i \geq 0, E_B(iT^-) - E_c < E_M\right\}. \tag{5}$$

Starting from $t=0$, at time $n_{uf}T$, the energy stored in the battery is equal to

$$\mathbb{E}\left[n_{uf}T^+\right] = E_M.$$

After this unfeasible feeding task, the device remains in OFF mode until the energy stored in the battery reaches the level E_0. Then Z_{off} follows the same distribution as $\sigma_{E_0-E_M}$ where $\sigma_{E_0-E_M}$ denotes the first hitting time to the level E_0-E_M of an homogeneous gamma process with parameters α and β. That is,

$$\sigma_{E_0-E_M} = inf\left\{t \geq 0, X(t) \geq E_0 - E_M\right\},$$

where $X(t)$ denotes an homogeneous gamma process with parameters α and β. The distribution function of Z_{off} is given by

$$\begin{aligned}
F_{\sigma_{E_0-E_M}}(t) &= P[\sigma_{E_0-E_M} \leq t] \\
&= P[X(t) \geq E_0 - E_M] \\
&= \int_{E_0-E_M}^{\infty} f_{E_H(0,t)}(x)dx \\
&= \frac{\Gamma(\alpha t, (E_0-E_M)\beta)}{\Gamma(\alpha t)},
\end{aligned}$$

where $\Gamma(\alpha t)$ is given by (2) and $\Gamma(\alpha t, (E_0-E_M)\beta)$ denotes the incomplete gamma function given by

$$\Gamma(\alpha t, \beta(E_0-E_M)) = \int_{\frac{\beta(E_0-E_M)}{\alpha}}^{\infty} z^{\alpha t-1} e^{-z} dz. \tag{6}$$

Due to the jump discontinuities of the sample paths of a gamma process, the energy stored in the battery in the period OFF fulfils

$$E_B(Z_{off}) \geq E_0 - E_M,$$

hence the overshoot

$$E_B(Z_{on} + Z_{off}) - E_0,$$

is positive almost surely. To avoid complicated mathematical technicality, in this paper this overshoot is disregarded what it implies that the energy stored in the battery right after the end of the period OFF is equal to E_0, that is,

$$E_B(Z_{on} + Z_{off}) = E_0.$$

Let $R_1, R_2, \ldots,$ be the time between successive renewals of the system. Random variables R_i are independent and identivally distributed with $R_i \equiv_d R$ where R is given by

$$R = Z_{on} + Z_{off} = n_{uf}T + Z_{off} \tag{7}$$

where $n_{uf}T$ is given by (5).

The optimal management policy corresponds to the value of T that maximises the expected number of feasible feeding tasks per unit time in a renewal cycle. As objective function, the ratio

$$T_f^{-1} = \frac{\mathbb{E}\left[n_{uf}\right] - 1}{\mathbb{E}\left[R\right]},$$

where R and n_{uf} are given by (7) and (5) respectively is chosen. The problem of finding the optimal management policy is formulated as

$$\max_{T>0} \left(\frac{\mathbb{E}\left[n_{uf}\right] - 1}{\mathbb{E}\left[R\right]} \right).$$

4. Conclusions and final remarks

In this paper, the management policy for an EHS system is analyzed. The main novelty of this paper is the use of the gamma process to model the harvested energy in a system. Although complex stochastic processes have been used to describe the harvested energy, as far as we are concerned, it is the first time in which the gamma process is used in the context of harvesting energy. Due to the properties of the gamma process, the model is analytically tractable and useful to provide insights for solving some theoretical management policies. Although the technical justification of the use of gamma process has been described by Castro *et. al* (2018), the model has not been justified with empirical measurements.

For an EHS system with infinite capacity of storage (that is, $E_K = \infty$), the problem exposed in this paper can be seen as a system with degradation modeled using a gamma process and subject to periodic imperfect repairs. These imperfect repairs decreases the degradation by a deterministic and constant amount whenever the degradation of the system exceeds a fixed threshold.

Two important aspects considered in this paper is that the battery does not suffer degradation (that is, battery can save in a proper way the energy that receives) and also that the energy demand is periodic (that is, feeding times are periodic). However, future works can consider extensions of this work where the inefficiency of the storage can be considered and also the demand of the energy by the device can be continuous.

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