Practical uses of the Weibull distribution for decision making

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Abstract. Weibull analysis is often seen as a complicated technique employed by reliability engineering enthusiasts and statistical analysis aficionados, with un-relatable features such as "beta parameters" and "characteristic life". The array of information that is made available from a quick web search doesn't do much to convince the casual browser of anything different. What does exist is a dearth of information or guidance on just how useful the Weibull distribution is, therefore no compelling case is made for the casually interested party to look further into the use of this as part of an analysis technique. Analysis using the Weibull distribution include determining an appropriate improvement activity or asset management tactic, assessing test data for product life or supporting investment decisions. This paper gives an overview of the Weibull distribution in plain language, explaining the terminology in relatable terms to the general population. Also covered is the way in which each input variable can be calculated or estimated. Applications for Weibull analysis are explored, with proposals of action based upon the results. The different ways in which the data can be presented is also shown, with key points to consider when proposing asset management activities.

1. A brief introduction to the Weibull distribution

The Weibull distribution is one of the most commonly used statistical models for determining failure patterns. The distribution is useful in terms of describing the behaviour of an item under study i.e. the probability of survival to a certain life, or inversely the likelihood of failure in future.

The Weibull distribution is a continuous distribution and is defined with three parameters. It is acknowledged that these parameters can have different notations depending upon the information source viewed but nonetheless serve the same function. The parameters described in terms of their use in reliability engineering analysis are as follows:

1. shape parameter (β): gives a rough indication of the failure behaviour that can be expected (see Section **Erreur ! Source du renvoi introuvable.**).
2. scale parameter (η): or more descriptively the characteristic life, which is broadly related to the mean time between failures (MTBF) but more accurately it is the time at which 63.2% of the items under study have failed (see Section *2.2*).
3. location parameter (μ): used when some guaranteed life is expected.

The parameters are all expressed as a function of a variable *t*, which typically represents time; although other units may be used, for example number of cycles or loading to failure.

Continuous random variables can be defined using a Probability Density Function (PDF) and a Cumulative Distribution Function (CDF). The PDF can be expressed using all three parameters. However, if the location parameter is taken as zero, this means that the time, *t*, is the start of the failure prediction and that failures cannot occur before this time. This reduces the PDF and CDF to a two parameter definition given in Equation (1) and Equation (2), respectively:

$$f(t) = (\beta/\eta).(t/\eta)^{\beta-1}\ e^{-(t/\eta)^{\beta}} \tag{1}$$

$$F(t) = 1 - e^{-(t/\eta)^{\beta}} \tag{2}$$

2. Weibull distribution parameters

The Weibull distribution is used to understand the lifetime characteristics of an item for use in reliability engineering studies, be it an individual component or a complex machine, and hence determining asset management activities. The spread and shape of the distribution gives a useful prediction of the reliability, or failure rate, of an item at a given point in its life and how the susceptibility to failure changes during the lifecycle.

Linear regression is used establish the Weibull parameters for the item under study, using data collected either through testing or from observations; e.g. from a computerised maintenance management system (CMMS). The two-parameter CDF of a Weibull distribution (Equation (2)) can be rearranged into the format of an equation for a straight line:

$$ln\left[ln\left\{\frac{1}{1-F(t)}\right\}\right] = \beta lnt - \beta ln\eta. \tag{3}$$

In plotting $ln\left[ln\left\{\frac{1}{1-F(t)}\right\}\right]$ against lnt a straight line graph is produced with gradient, β, and intercept, -$\beta ln\eta$. For example, using a sample, n, of tests of an item to failure, the data can be given a rank, r, in order from shortest to longest time to failure. Then, F(t) is estimated using the median rank method, given by:

$$F(t) = (r - 0.3)/(n + 0.4). \tag{4}$$

A plot, such as that given in Figure 1, can be derived and a straight line fit to the data to establish the gradient and intercept. The shape and scale parameters of the Weibull distribution can then be determined.

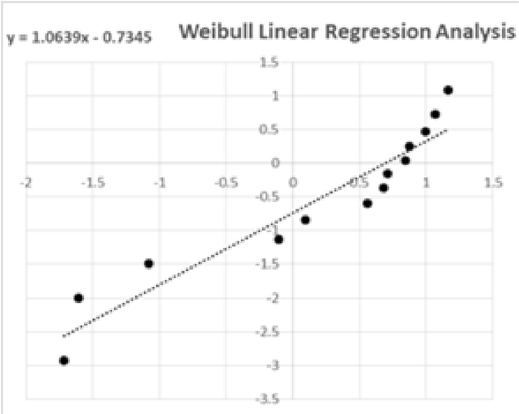

Figure 1. Weibull Linear Regression plot

2.1 The shape parameter as an indicator of failure behaviour

The shape parameter, β, represents the behaviour of the item under study and indicates if the failure rate is increasing, constant or decreasing. This is useful to categorise items into early life (or infant mortality), failure and wear out (or end of life) failure patterns, where:

$\beta < 1$ represents early life failure; $\beta = 1$ represents random failure; and, $\beta > 1$ represents wear out behaviour.

An example of the PDF and CDF plots obtained for differing shape parameters, given a consistent scale parameter, is shown in Figure 2.

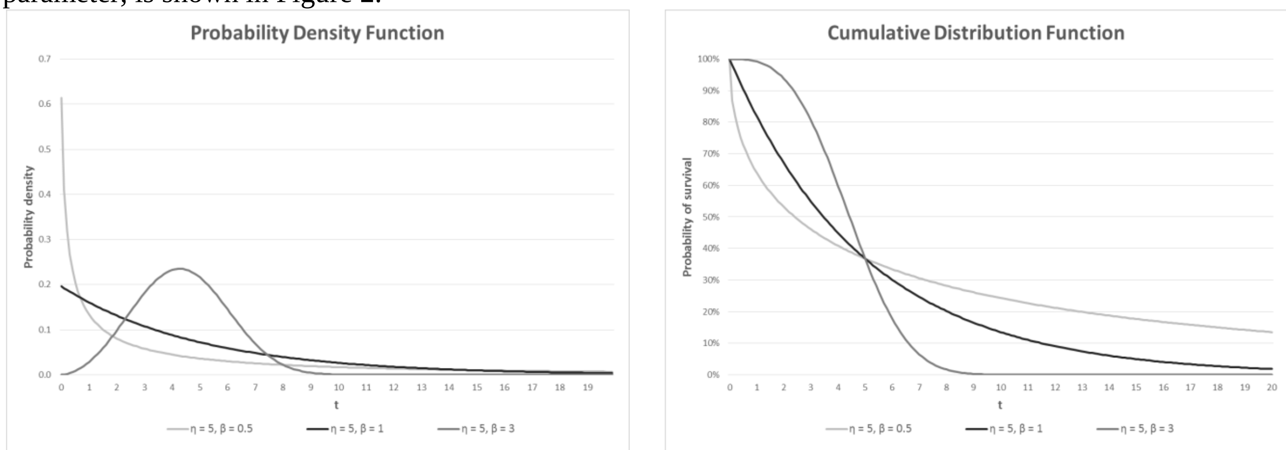

Figure 2. PDF and CDF for β = 0.5, 1 and 3

The linear regression plots for varying β values can be likened to the failure patterns proposed by the Reliability Centred Maintenance (RCM) methodology (Moubray), as shown in Figure 3.

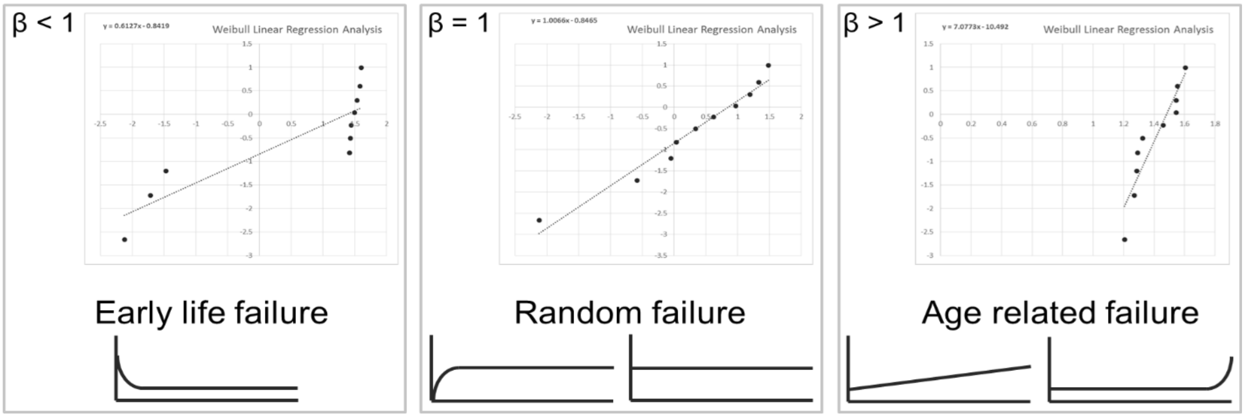

Figure 3 – Weibull distribution β parameters related to RCM failure patterns

Another failure pattern which may be observed is known as the 'bathtub curve', whereby three distinct slopes are evident in the linear regression plot, as shown in Figure 4.

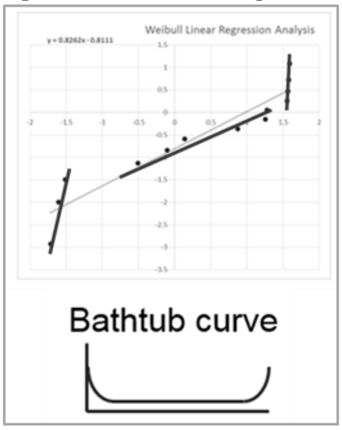

Figure 4 – Linear regression plot showing the bathtub curve failure pattern

The bathtub failure pattern presents a unique challenge in analysis as the failure behaviour would likely be represented with $\beta \approx 1$. If only the value of the shape parameter is considered, this may lead the analyst to believe that random failure is apparent when in truth the failure pattern may actually be pseudo-random. Therefore, it is important to review the linear regression plot to help understand the type of failure behaviour.

For example, consider the failure information which may be available related to a pump. Just having the times to failure tells us that the overall performance of the pump offers the opportunity to be improved. However, if data on the exact failure modes is reviewed this can help with identifying specific performance improvement activities. Taking into consideration each section on the bathtub curve, it was discovered that the failures with the lowest time to failure were due to misalignment, mechanical looseness in the mounting and a faulty mechanical seal. All of these infer problems with build quality which is a typical early life failure root cause. Improvements which may be considered include enhanced training and quality control measures. The failures on the random section of the linear regression plot were found to be poor lubrication practices and start up / shut down practices. These can be addressed through improving operational discipline. The end of life failures were discovered to be due to impeller wear, which might be acceptable from a design life point of view but could offer an opportunity for a design upgrade to extend the pump life.

2.2 The scale parameter as an indicator of characteristic life

The scale parameter, η, is used in reliability analysis to determine the intersection whereby 63.2% of failures have occurred. This allows an assessment of performance between different items given a similar β value. The significance of 63.2% on the Weibull distribution is this is the point at which the characteristic life is truly equal to MTBF when $\beta = 1$ (Warwick.ac.uk).

3 Available data to derive input parameters

As with any analysis the quality and availability of data is key. In reliability analysis, as much relevant information should be gathered regarding the failure instance, such as the reason for failure (the failure mechanism), the configuration / style / model / material of the item, the date / time of failure, ambient conditions, location etc. Input data which is validated either from testing or from other data gathering methods, such as product warranty or a well-managed CMMS, is considered to provide the most robust assessments.

3.1 Maintenance Management Systems

The way in which assets are identified and structured within any maintenance management system is probably the most critical to the ability to analyse in-service items. Within a CMMS, it must be ensured that asset tags are consistent and that fields are used to identify equipment by type (i.e. pump, compressor), duty, location, configuration etc. All failures should be logged within the system, as should any other maintenance activities that may "reset the clock", such as replacement of components subject to wear or fatigue. This will allow the analyst to extract the data and carry out at least the most basic of failure assessments.

The majority of modern CMMSs allow the entry of fault and cause codes when a fault is logged against an equipment. Consistent use of this functionality will allow the analyst to draw deeper conclusions.

There are a number of published standards which offer guidance on how to structure systems and data to allow the capture and interrogation for maintenance purposes. One such standard is ISO 14224, developed for the petrochemical industry, but which offers a good structure that might be applied to most situations.

3.2 Highly accelerated life testing (HALT)

HALT is a widely used technique for demonstrating an items resistance to predicted environmental stresses such as loading, vibration, temperature, voltage and moisture. The application of the technique is dependent upon the item type and the performance standard required.

A company developing items and using HALT techniques presents the ideal opportunity for the collection of data and analysis using the Weibull distribution. Engineers who are involved in developing and deploying HALT are well placed to get the maximum benefit from the use of a Weibull based analysis to present either detailed information to designers on how to modify the items, or to end users who can incorporate the knowledge into their own Reliability, Availability, Maintenance (RAM) simulations (see Section 4.4 for further details on RAM) .

3.3 Alternative Data Sources

Where data from testing or say a CMMS is not available, does not exist or cannot be fully trusted, alternative data sources are required. The experience of experts may be used and by asking the right questions, this may not only lead to an average life being suggested (which can be approximated to η) but also a failure pattern. An example conversation may go:

"So how often do these pumps fail?"

"I would say about every two years give or take a few months"

" Every time a pump is installed, does it always make it to two years?"

"Generally yes, but over the fifteen years' experience I have had here a handful have failed in the first few months"

The failure behaviour suggested by this response could lead the analyst to use a β of between 0.5 and 0.7 (some evidence of early life failures) and a η of somewhere between 1.75 and 2.25 years (for an average time to failure of around two years). The usefulness of the estimation would depend upon the need for the data to incorporate into the analysis. For example, if the purpose of the analysis were to determine whether effort should be expended upon improving maintenance practices (reducing early hour failures) or to replace with a more robust item (improving characteristic life) then "what if" analyses can be carried out. Such analyses help determine the most appropriate course of action by comparing the cost of the improvements vs the expected return, either in improved uptime, reduced maintenance costs or a combination of the two.

Next in the hierarchy, vendor data may also be available. The validity of this data must be assessed as it may not be representative of the operational environment of the item. Vendors may provide data based upon warranty performance, or empirically derived data from similar components. The ability to have a discussion with a vendor as to the performance of their items, backed with data and analysis, is a very powerful way to ensure that operating needs are met.

Finally, published data sources are available such as the Offshore and Onshore Reliability Data (OREDA) handbook for the oil and gas industry and the FIDES (Latin: Trust) guide for electronic systems and components. It is unlikely that these data sources will be truly representative of the unique operating context of the item to be assessed and there may also only be limited population that the data was drawn from. Therefore, thought must be given as to why the data is being used and the manner in which it is being applied. Results from studies using published data should be used with caution, as there is increased uncertainty in this as an input data source.

4. Using the analysis to determine appropriate actions

If quality data is collected, then the analyst will be able to draw better conclusions and propose appropriate actions from the assessment. A common technique deployed to investigate and propose corrective actions is Root Cause Analysis (RCA). RCA using the Weibull distribution is an extremely useful tool, as it can be used to validate any proposed failure mechanisms as part of the investigation. Understanding the exact reason for failure allows control and mitigation measures to be implemented that target the true cause therefore preventing recurrence of the problem.

4.1 Quality assurance and control

Early life failures are typically due to quality assurance and control issues. Technical and performance standards may include maintenance or operational tasks which were not completed effectively which led to a component that has failed unexpectedly or material that does not conform to specification.

Where there has been a failing in task execution, consideration may be given to reviewing procedures and processes. Robust procedures that are followed are a cornerstone for reliability whereby operational and maintenance interventions are inevitable.

Should a component not conform to standard, then the RCA will likely head into the supply chain. This includes a review of the steps an organisation takes to ensure the quality of incoming goods such as inspections, vendor audits and factory acceptance tests. Whereby components may fail unexpectedly during early life, but then conform to a random failure pattern, Highly Accelerated Stress Screening (HASS) may be an applicable technique. This testing technique is designed to subject items to higher than normal operational stresses (but which are still within the components design capability) in order to weed out any defective items prior to supply to the customer. Attention should also be paid to component shelf life, as the reason for failure may actually be deterioration during storage as opposed to true early life failure.

Material failures may head even further into the supply chain, looking into the raw materials and their processing methods. In principle, each step of the supply chain should be assuring the as-supplied material, but the organisation who actually suffers the effects of a failing may be several steps removed. For highly critical components, such as those used within the nuclear and aerospace industry, such far reaching traceability of components may be absolutely necessary for critical items.

4.2 Replacement / remediation

Whereby a definite end of life behaviour is exhibited, then an ideal opportunity is presented to carry out planned replacement or remediation maintenance activities. The time (or life) at which items should be replaced can be determined from the Weibull distribution CDF. In order to determine the intervention time, an appropriate failure probability limit would be assigned. For example, should there be a desire to accept no more than 0.1% probability (or once every thousand years) that a failure would occur then the intervention activity would be undertaken at a time before this limit is reached.

The timeframe for intervention may be read straight from the CDF plot at the desired survival probability level, 99.9% in this case. An example is shown in Figure 5, which would result in a replacement task within two years. Consideration must be given to how steeply the probability of failure increases over time. Whereby a failure may lead to severe safety concerns then the probability of failure using the Weibull distribution analysis is used in risk assessment.

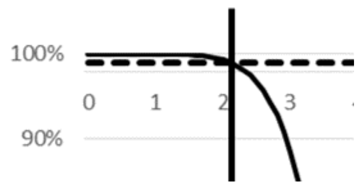

Figure 5. - 99.9% reliability

4.3 Condition monitoring

Pure random failure behaviour lends itself to condition monitoring, as there is no benefit to improving initial installation practices or assigning a retirement / decommissioning age. However, where limited windows for maintenance are available, tackling items that exhibit random failure on such an ad hoc basis may not be desirable. Understanding the failure characteristics allows residual risk to be expressed. The analyst may recommend the replacement of such components if the risk is deemed unacceptable. Very few items in truth conform to pure random failure, many more fall into the infant mortality pattern. The best decision may be to leave well alone; acknowledge the risk and plan for a failure should it occur.

4.4 Design - Reliability, Availability & Maintainability (RAM) Simulation

RAM simulation is used during the design of complex processes and systems in order to determine the overall performance over time. Its greatest use is as a comparator of competing designs, allowing decisions to be made on an optimum configuration.

A typical RAM simulation starts with the development of a base case, which is the base system design represented using a reliability block diagram to illustrate the reliability dependencies of the system. Each block in the diagram represents an item, which may be a component, a piece of equipment, a subsystem or even (during concept stage for mega-projects) entire systems or plants. The anticipated performance of each item in terms of a failure rate and a repair time can be expressed in terms of Weibull distribution parameters. Sensitivity cases can be developed, changing system configurations and/or the base data to allow an optimum design to be determined. The β and η parameters can be adjusted and the simulation run to determine if there will be any detrimental effect upon lifecycle performance. The validity of the output of RAM simulation is heavily dependent upon the data used. By virtue of the fact that the item under study, or process/system in design, is early in the lifecycle then it is highly likely that failure data will not currently exist to demonstrate behaviour. Generally, published data sources are used unless a company has its own data set.

Summary

The practical deployment of the Weibull analysis technique has been presented, along with advice that anyone working within an engineering environment can use to improve performance. Gaining an understanding of failure behaviour leads to robust engineering solutions in a much faster timeframe than by, say, experimenting and hoping for the best. The author encourages anyone involved in reliability (and indeed any field of) engineering to become familiar with the technique and try it – you will undoubtedly learn at least something.

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