Joint maintenance scheduling and routing optimization for geographically dispersed production systems

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Abstract We develop a joint optimization between maintenance planning based on dynamic grouping approach and maintenance team routing for a geographically dispersed production system (GDPS). First, the interactions between the maintenance planning and routing scheduling are analyzed and formulated. A joint implementation of Local Search Genetic Algorithm (LSGA) and a new maintenance routing algorithm based on Branch and Bound (BAB) is then proposed to solve the joint optimization problem. The uses and advantages of the proposed joint optimization approach are illustrated through a numerical example of a GDPS consisting of fifteen components located in five different sites.

1. Introduction

In the literature, a large number of maintenance models has been proposed for multi-component systems, see for instance (Nicolai and Dekker.,2008; Vu et al.,2014; Keizer et al. 2017). However, majority of them investigate for centralized production systems (CPS) containing components which are located in close proximity to each other. In the framework of GDPS system, whose production is done over a number of production sites which are geographically located far apart from each other, the existing maintenance models cannot be directly adapted. In fact, planning a maintenance strategy for GDPS is a more complex work than CPS due to the dispersed location of production sites which need to schedule optimal maintenance routes. In addition, a GDPS system some specific constraints which is not exist in CPS such as disruption, possibility of accident, weather or road condition, etc (Hameed and Vatn.,2012) should be considered in maintenance models. Recently, some maintenance strategies have been developed for GDPS systems, see Diaz-Ramirez et al (2014); Camci (2015). However, these works mainly focus on the problem of maintenance routing rather than maintenance planning and optimization ones. Besides, the number of PM activities in a short-term planning period is always assumed to be known in advance, and then introduced as a constraint for the maintenance routing model.

The aim of paper is to develop a dynamic grouping maintenance strategy for a GDPS system which integrate both maintenance scheduling and routing into joint maintenance model simultaneously. The economic dependence is here considered in the proposed model. Moreover, to solve optimization problem for maintenance model, the joint implementation of a Local Search Genetic Algorithm (LSGA) and a new route optimization algorithm based on Branch and Bound (BAB) is consider. This work ensures that obtained result is global optimal solution.

The rest of this paper is organized as follows. Section 2 is devoted to describe mathematical model for a GDPS system. Section 3 focuses on performing joint optimization for GDPS integrating both maintenance scheduling and routing. A numerical example investigated to validate the performance of the proposed approach is shown in section 4. Finally, the conclusions deduced from this work are presented in the last section.

2. Problem descriptions

We consider a GDPS containing n components located at different production sites dispersed over a large area. The components are subjected to random failures which is modeled by Weibull distribution. To prevent the system from an intensive regime of failures, preventive maintenance (PM) is carried out at predetermined times. A component is considered to be in “as good as new” state after the PM. Whereas, corrective maintenance (CM) is immediately carried out after the component failures in order to restore the failed components into their operational state as soon as possible. After a minimal repair action, component is in “as bad as old” state. It should be noted that in this paper, we consider that to perform a CM action, local resources (repair team, repair tool and spare part) at maintenance sites are enough. Otherwise, to do a PM action, the external maintenance resources from the maintenance center are needed. For this reason, when PM is performed jointly on a group of components situated in different sites, the maintenance resources have to be transported from the maintenance center, then passed all the maintained sites and finally returned to the departure center.

Assume that a group of several components, denote G^k, are preventively maintained together. These components are located in m_s different sites. The itinerary of maintenance team/spare part to the PM of all components of the group should contains all m_s sites. The itinerary has to be optimized and could be represented by decision variable X_{jv}. $X_{jv} = 1$ if site j is the v^{th} site visited by the maintenance team, otherwise, X_{jv} is equal to 0. The whole maintenance itinerary can be represented by the following matrix $X_{(n_s \times m_s)}$ which n_s is the number of all sites and m_s is the number of sites in group G^k:

$$X = \begin{pmatrix} X_{11} & \dots & X_{1m_s} \\ \vdots & \ddots & \vdots \\ X_{n_s1} & \dots & X_{n_sm_s} \end{pmatrix} \text{ with } \begin{array}{l} \sum\limits_{j=1}^{n_s} X_{jv} = 1, \forall v = [1, 2 \dots m_s] \\ \sum\limits_{v=1}^{m_s} X_{jv} \leq 1, \forall j = [1, 2 \dots n_s] \end{array}$$

There are a lot of possible itineraries to visit all n_s sites of group G^k. The total maintenance cost of the group depends on the itinerary that we selected. In the following paragraph, we will present how to calculate the maintenance cost of G^k as a function of a itinerary X.

Maintenance costs. The maintenance cost of group G^k contains the following parts: cost of spare part ($C_{G^k}^{sp}$), downtime cost (${G^k}^{dt}$), labor cost ($C_{G^k}^{lb}$), site preparation cost ($S_{G^k}^0$), and travel cost ($S_{G^k}^{tr}$).

$$\begin{aligned} C_{G^k}^p &= C_{G^k}^{sp} + C_{G^k}^{dt} + C_{G^k}^{lb} + S_{G^k}^0 + S_{G^k}^{tr} \\ &= \sum_{i \in G^k} C_i^{sp} + R_{j_{G^k}}^{dt} . \sum_{j \in G^k, i \in j} w_i + R_{G^k}^{lb} . w_{G^k} + \sum_{j=1}^{n_s} \sum_{v=1}^{m_s} X_{jv} . S_j \\ &+ R^{tr} . \left(L_{0v_1} + L_{v_{end}0} + \sum_{j=1}^{n_s} \sum_{v=2}^{m_s} \sum_{m=1, m \neq j} X_{jv} . X_{m(v-1)} . L_{m,j} \right) \end{aligned} \quad (1)$$

where, $R_{j_{G^k}}^{dt}$ and w_i are the downtime cost rate and the maintenance duration of component i located at site j; $R_{G^k}^{lb}$ and w_{G^k} are the labor cost rate and the total maintenance duration of group G^k; R^{tr} is travel cost rate; L_{0v_1} is the distance between the maintenance center and the first site of the itinerary; L_{0v_1} is the distance between the maintenance center and the last site of the itinerary; L_{mj} is the distance between site m and site j; S_j is the site preparation cost of site j. When several components of the same site are maintained together, only one S_j has to be paid.

Grouping economic profit. To evaluate the effectiveness of grouping maintenance strategy, grouping economic profit is normally used. The grouping economic profit is defined as the difference between the total maintenance costs that have to be paid when the components are grouped and when they are separately maintained. The grouping economic profit of group G^k, denoted EPG, can be expressed as follows

$$\begin{aligned} EPG(G^k, t_{G^k}, L_{G^k}) &= \sum_{i \in G^k} C_i^p - C_{G^k}^p - H_{G^k} \\ &= (\sum_{i \in G^k} S_{ij} - S_{G^k}^0) + (\sum_{i \in G^k} S_{ij} - S_{G^k}^{tr}) - (C_{G^k}^{lb} - \sum_{i \in G^k} R_i^{lb} \cdot \omega_i) - H(G^k, t_{G^k}) \\ &= \Delta S_{site}(G^k) + \Delta S_{tr}(G^k, L_{G^k}) - \Delta C_{lb}(G^k) - H(G^k, t_{G^k}) \end{aligned} \quad (2)$$

where,

- $\Delta S_{site}(G^k)$ is the site-preparation cost saving when several components of the same site are preventively replaced.$\Delta S_{site}(G^k) = \sum\limits_{j \in G^k, i \in j} S_{ij} - \sum\limits_{j=1}^{n_s} \sum\limits_{v=1}^{m_s} X_{jv} . S_j$
- $\Delta S_{tr}(G^k, L_{G^k}) = \Delta S_{tr}(\overline{G^k}) - \Delta S_{tr}(G^k)$ is the travel cost saving. $\Delta S_{tr}(\overline{G^k})$ is the travel cost when the components are maintained separately. It means that the maintenance team travels from the maintenance center, then performs the PM on only one component at a time, and returns to the maintenance center. $\Delta S_{tr}(G^k)$ is the total travel distance when the maintenance team performs the maintenance actions on all the components of

the group before returns to the maintenance center.

$$\Delta S_{tr}(G^k, L_{G^k}) = R^{tr} \cdot \left[\sum_{j \in G^k} \sum_{i \in j} \left(L_{0j} + L_{j0} \right) - \left(L_{0v_1} + L_{v_{end}0} + \sum_{j=1}^{n_s} \sum_{v=2}^{m_s} \sum_{m=1, m \neq j} X_{jv}.X_{m(v-1)}.L_{mj} \right) \right]$$

- Labor penalty cost is the difference between the labor cost paid for the maintenance of group G^k and that of its components. We consider that the labor cost rate of components and a group of components may be different. $\Delta C_{lb}(G^k) = R^{lb}_{G^k} \omega_{G^k} - \sum_{i \in G^k} R^{lb}_i \cdot \omega_i$
- $H(G^k, t_{G^k})$ is the penalty cost occurred due to the change of the maintenance dates of the components in the group. Note that to be grouped in the same group, the maintenance dates of individual components in the group have to be modified. The detail calculations of this penalty cost can be found in Do et al. (2014).

In a short-term planning horizon, there are many PM activities and a grouping solution of this horizon therefore may contains several groups. A grouping solution or grouping structure can be defined as a collection of mutually exclusive groups $SG = \{G^1, G^2, ., G^e\}$ with $G^h \cap G^k = \emptyset$ and $G^1 \cup G^2 \cup \ldots \cup G^e$ covers all maintenance activities in the planning horizon. According to this definition, a grouping structure is represented as

$$Y = \begin{pmatrix} y_{11} & \cdots & y_{1b} \\ \vdots & \ddots & \vdots \\ y_{a1} & \cdots & y_{ab} \end{pmatrix} \text{ with } \quad y_{ab} = \begin{cases} k, & group\ number \\ 0, & otherwise \end{cases}$$

where, a and b are index of component and site respectively. $y_{ab} = k$ means that the component a located in site b is in k^{th} group.

The total economic profit of a grouping structure is equal to the sum of the economic profit of its groups.

$$EPS(Y, X) = \sum_{k=1}^{ng_{\max}} EPG\left(G^k, t_{G^k}, L_{G^k}\right) \tag{3}$$

where, ng_{max} is the total number of group in grouping structure Y.

3. Joint optimization of maintenance scheduling and routing

The aim of this section is to present the proposed joint optimization between maintenance grouping and routing (see Fig. 1 for more details). The optimization process can be divided into two following phases: *(a)* optimization at level of group of components to maximize the group economic profit (EPG); *(b)* optimization at grouping structure level where all groups of components of a grouping solution are considered to maximize the economic profit the considered short-term horizon (EPS).

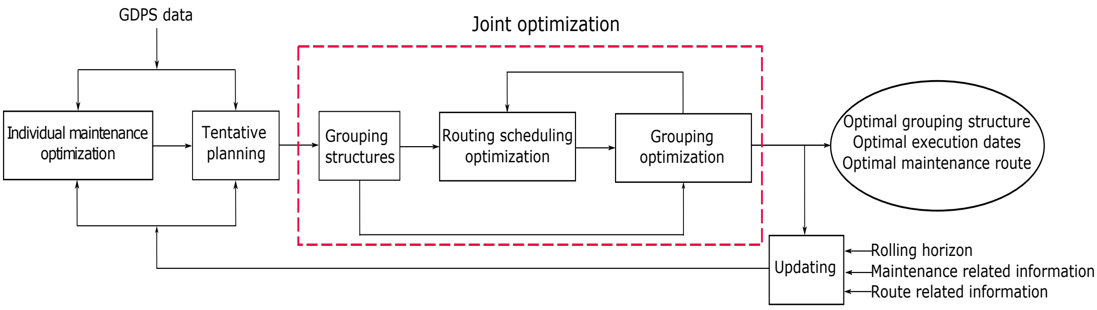

Figure 1. Joint optimization for dynamic grouping maintenance

3.1. *Optimization at the group level*

In the fist phase, we assume that the grouping structure is known. With each group of a grouping structure, we find the optimal itinerary for the maintenance team to do the maintenance of all components of the group at the lowest penalty and travel costs. To this end, a new joint optimization approach based on the Branch and Bound is proposed. Branch and bound method is a critical enumeration of the search space. It enumerates, but constantly tries to rule out parts of the search space that cannot contain the best solution, by using lower estimated bounds . In more details, consider the iteration q ($q = 1, ..., m_s$) being the q^{th} site of the itinerary. The lower bound at this iteration, denoted by LB_q, is defined as the lowest maintenance cost of the considered group (corresponding to the maximum of group economic profit) that we can obtained if a site j is decided to be the q^{th} visited one. Given that the optimal itinerary has to be optimal for the maintenance grouping and routing at the same time, the lower bound is designed as $LB_q = LBF_q + LBE_q$, where LBF_q denotes the total of travel cost and penalty cost of the fist part of the itinerary starting from the maintenance center and finishing at the q^{th} maintained site. This part has been already planned, LBF_q is then can be calculated exactly. Otherwise, LBE_q is the total of travel and penalty cost of the remaining part of the itinerary starting from the q^{th} maintained site and finishing at the maintenance center. Given that in the second part, the itinerary has not been planned yet, exact calculations of LBF_q are impossible. In our work, this second part is estimated by considering the shortest time needed to visit all the remaining sites of the second part.

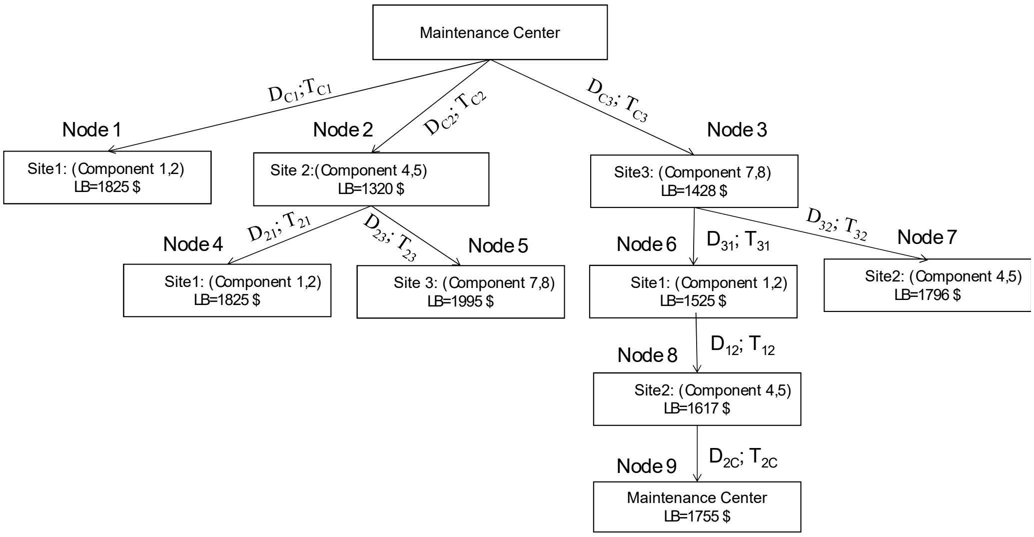

Figure 2. Search tree representation of the developed joint optimization approach

At each iteration q, a node (a maintenance site) with the lowest LB_q value is chosen for expansion. The expanded and generated nodes are called the parent and the child nodes respectively. After expansion, lower bounds of all child nodes are recalculated. The space of solutions is updated to cover all unexpanded nodes of the previous iterations and the new child nodes. The process is repeated until all the maintained sites are planned. To facilitate the understanding, let us consider an example in Figure 2. The BAB process is applied to a group containing 8 components. The components $\{1,2\}$, $\{4,5\}$, $\{7,8\}$ are located in the three different sites 1, 2, 3 respectively. The BAB process stars at the maintenance center (node 0, level 0). Since at this level there is only one maintenance center note, the expansion is directly applied to this node. Nodes 1, 2 and 3 are three child ones. The process jumps to level 1 ($q = 1$). The lower bound is evaluated for all nodes of this level. Node 2 is then selected as the expansion node of level 1, since its lower bound value is the smallest one among all the nodes of the level. The expansion is then done on node 2. Nodes 4 and 5 are generated as children nodes. The process go to level 2. The lower bound is evaluated for nodes 4 and 5. Node 3 is then selected for expansion. The process then returns to the level 1, and so on. The process is stopped at node 9, and the optimal maintenance itinerary is to start at the maintenance center, then travel across sites 3, 1, 2 respectively, and come back to the maintenance center.

3.2. *Optimization at the grouping solution level*

In this second phase, the joint optimization between maintenance grouping and routing is considered at grouping solution level. To solve the joint optimization problem, a joint implementation of a Local Search Genetic Algorithm (LSGA) and the modified BAB approach (developed in the previous subsection) is proposed. For more details, LSGA generates first a random population of grouping structures. To evaluate the performance (total economic profit) of a grouping structure, the modified BAB is applied to find the optimal itineraries of all groups in a grouping structure. Promising grouping solutions will be selected based on their total economic profit. At each iteration, local search, elitism, crossover and mutation operators are applied to the promising solutions. The best solution is found after certain iterations (Ombuki and Ventresca, 2004).

4. Numerical examples

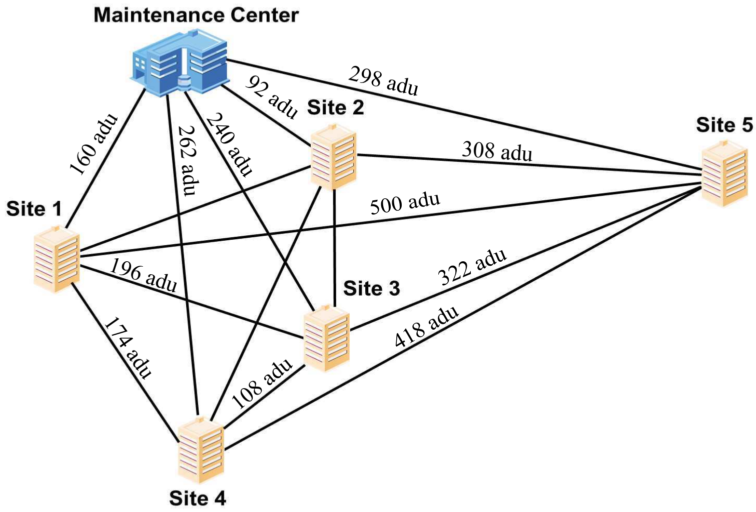

Figure 3. GDPS containing 3 production sites

The aim of this section is to demonstrate the performance of the proposed joint optimization between maintenance grouping and maintenance routing, named JOMSR. To do this, the performance of JOMSR will be compared to the separate optimization approach named SOMSR. Most steps of SOMSR are the same as that of JOMSR, except that the maintenance grouping and maintenance routing are separately considered. The penalty costs of the grouping maintenance are not considered during the finding of the optimal itineraries. The optimal itineraries are determined based on the total travel distance only. Both JOMSR and SOMSR are applied to find the best grouping solution within the planning horizon $PH = [0, t_{51} + \omega_5] = [0, 8460.7]$, and optimal repair team itineraries of a GDPS containing 15 components located in 5 different sites (Figure 3). The given data and the tentative maintenance dates of 15 components are reported in Table 1, and the labor cost rates R^{lb} are fixed at 100, 200, 300 for the required skill levels of repair team 1, 2, 3 respectively. It should be noticed that, in this study, all parameters are given in arbitrary units, e.g., arbitrary time unit (atu), arbitrary distance unit (adu) and arbitrary cost unit (acu).

The results provided by JOMSR are shown in Tables 2 and 3. The joint optimization helps to reduce up to 59.85% travel distance of the maintenance team as well as the travel cost. The reduction of travel distance is very important since it is not only meaningful from the economic point of view, but also the sustainable one (reduce energy consumption, travel-related risks, environmental negative impacts). Given the above performance, the grouping maintenance also leads to slight increases of the labor and the CM costs which is around 7.44%.

Similarly, the results provided by the use of SOMSR are reported in Tables 4 and 5. By comparing the obtained results of SOMSR and JOMSR, we can conclude that both JOMSR and SOMSR provide the same grouping structure, however, the optimal itinerary and the best departure time of the maintenance team to maintain the group G^1 are not the same. Given a higher performance in reducing the travel distance as well as the travel cost of SOMSR when compared to the JOMSR, the routing solution provided by SOMSR is not optimal. The reason is that SOMSR consider only the travel distance during the finding of the optimal route. The obtained itinerary is then only optimal for maintenance routing but not for the grouping maintenance. The travel cost of the itinerary is minimal, but the penalty costs related to the maintenance date are too large. Overall, it leads to a lower performance of SOMSR when compared to that of JOMSR. In our case study, JOMSR helps to save up to 4.79% of the total maintenance cost when compare to SOMSR. The joint optimization is then important to ensure the global optimality of the maintenance grouping and routing.

Table 1. Data and individual optimization at component level

Unit	λ_i	β_i	C_i^{sp}	C_i^c	w_i	t_i^e	sl_i	S_i^{tr}	x_i^*	ϕ_i^*	t_i^1
1	2499	2.85	1757	561	30	1398	1	4800	6861.5	4.1452	5526.5
2	3257	2.77	2546	669	48	2729	2	4800	10592	4.5875	7955.5
3	2423	3.72	3651	628	63	781	3	4800	5852.3	10.613	5071.3
4	2815	2.87	1874	665	27	2418	1	2760	6846.6	3.5731	4428.6
5	3505	2.74	2676	722	45	2649	2	2760	10972	4.1107	8415.7
6	2327	3.69	3397	619	66	1142	3	2760	5728.5	11.076	4613.5
7	2645	2.82	1704	517	33	3281	1	7200	8303.6	4.4215	5022.6
8	3057	2.76	2341	547	51	3799	2	7200	11512	5.0947	7818.1
9	2624	3.54	3313	611	72	1035	3	7200	7246.7	10.881	6244.7
10	2446	2.88	1656	445	30	2498	1	7860	7778.5	4.6119	5346.5
11	2963	2.74	2449	628	45	2999	2	7860	10564	5.3044	7661.1
12	2327	3.73	3332	690	66	1155	3	7860	5730.6	12.951	4575.6
13	2698	2.84	1659	689	36	1546	1	8940	7331.3	4.5637	5908.3
14	3359	3.16	2432	609	51	3598	2	8940	9227.8	5.0827	5629.8
15	2483	3.65	3532	314	72	1819	3	8940	7564.3	8.8367	5796.3

Table 2. Grouping solution provided by JOMSR

Optimal grouping structure	ΔS_{tr}	ΔS_{Site}	ΔC_{lb}	H	EPG	EPS
$G_1 = \{3^1, 6^1, 9^1, 12^1, 13^1, 14^1, 15^1\}$	31530	380	12300	3938.5	15671.5	40462.8
$G_2 = \{1^2, 4^2, 7^2, 10^2\}$	12570	0	0	257.9	12312.1	
$G_3 = \{2^3, 5^3, 8^3, 11^3\}$	12570	0	0	90.8	12479.2	

Table 3. Maintenance itineraries and departure times provided by JOMSR

Groups	Departure times	Maintenance routes	Travel distances
$G_1 = \{3^1, 6^1, 9^1, 12^1, 13^1, 14^1, 15^1\}$	4992.1	$0 \to 2 \to 1 \to 4 \to 3 \to 5 \to 0$	1194
$G_2 = \{1^2, 4^2, 7^2, 10^2\}$	5073.2	$0 \to 2 \to 3 \to 4 \to 1 \to 0$	670
$G_3 = \{2^3, 5^3, 8^3, 11^3\}$	7852.4	$0 \to 1 \to 4 \to 3 \to 2 \to 0$	670

5. Conclusions

In this work, a joint optimization between the maintenance grouping and routing is modeled and formulated for geographically dispersed production systems with taking into account the interactions between components at both component and site levels. To solve the joint optimization problem, the SOMSR optimization process is proposed based on the LSGA and the BAB algorithms. The numerical examples shown that the grouping maintenance is an interesting approach for the maintenance management of GDPS systems, and the joint optimization between the maintenance scheduling and routing is better than the separate optimization. In the

Table 4. Grouping solution provided by SOMSR

Optimal grouping structure	ΔS_{tr}	ΔS_{Site}	ΔC_{lb}	H	EPG	EPS
$G_1 = \{3^1, 6^1, 9^1, 12^1, 13^1, 14^1, 15^1\}$	31980	380	12300	5468.8	14591.2	39382.5
$G_2 = \{1^2, 4^2, 7^2, 10^2\}$	12570	0	0	257.9	12312.1	
$G_3 = \{2^3, 5^3, 8^3, 11^3\}$	12570	0	0	90.8	12479.2	

Table 5. Maintenance itineraries and departure times provided by SOMSR

Groups	Departure times	Maintenance routes	Travel distances
$G_1 = \{3^1, 6^1, 9^1, 12^1, 13^1, 14^1, 15^1\}$	4914.8	$0 \to 2 \to 5 \to 3 \to 4 \to 1 \to 0$	1164
$G_2 = \{1^2, 4^2, 7^2, 10^2\}$	5073.2	$0 \to 2 \to 3 \to 4 \to 1 \to 0$	670
$G_3 = \{2^3, 5^3, 8^3, 11^3\}$	7852.4	$0 \to 1 \to 4 \to 3 \to 2 \to 0$	670

future work, other kinds of component interactions (such as stochastic and/or functional dependencies) and production constraints will be investigated and integrated in the proposed joint optimization model.

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