Reliability Demonstration Tests Considering Performance Degradation with Measurement Error

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Abstract Reliability demonstration tests (RDTs) have been widely used in engineering design to verify if a product has met a certain reliability requirement. Such tests are usually conducted and analyzed based on binomial theory for the number of failures or analysis of failure times. Unlike these traditional methods, a degradation-based RDT method is proposed in this paper. Appropriate implementation of this method will speed up reliability demonstration, especially for highly reliable products. However, a big challenge is the measurement error that cannot be avoided in degradation data collection. To incorporate the impact of measurement errors in degradation-based RDT, a random effects stochastic process model explaining both the evolution of product degradation and measurement error is proposed. Under this model, a statistical inference method based an expectation-maximization algorithm is developed to estimate the model parameters. Moreover, the optimal design of degradation-based RDT is developed to minimize the total testing cost considering both the producer's and consumer's risks. A numerical example is presented to illustrate the use of the proposed RDT method in practice.

1. Introduction

Reliability Demonstration Testing (RDT) has been widely used to verify if a product has met a certain reliability requirement. In the literature, two groups of RDT methods have been developed for non-repairable systems based on either the number of failures (Leemis (2006)) or failure times (Lawless (1978); McKane et al. (2005)). Guo and Liao (2012) provide detailed discussions on the relationship between the two groups of methods and illustrate when they will lead to similar RDT designs. In practice, the method based on the number of failures is the most commonly used. This method is called "binomial test" as it uses the binomial equation for calculating the design variable based on the given quantities such as the confidence level, the maximum number of failures allowed, and lower bound on the demonstrated reliability. The drawback of this method is that the test must be conducted up to the target lifetime. Unlike the binomial test method, a parametric alternative is to perform reliability demonstration by fitting a failure time distribution based on observed failure times. The limitation of this method is that it would be difficult, if not impossible, to estimate the model parameters if no failures are observed in the test. For repairable systems, Guo et al. (2011) develop an RDT design method applicable for both Homogeneous Poisson Process and Non-homogeneous Poisson Process cases. The method can effectively determine the required sample size and testing time to demonstrate the required Mean-Time-Between-Failures. Apparently, these traditional methods are not efficient for highly reliable products with high reliability requirements.

Degradation data analysis has been extensively studied in reliability engineering (Meeker and Escobar (1998); Liao and Elsayed (2006); Ye and Chen (2014)). Compared to failure time data analysis, this type of approach can speed up reliability modeling without waiting for actual failures to occur. Moreover, it can provide valuable insights into the underlying physics of failure. Although degradation data analysis has shown many advantages in reliability estimation, degradation-based RDT has not been reported in the literature. This paper focuses on the design and analysis of degradation-based RDT, which will provide a new direction for improving the efficiency of RDT. To the best of our knowledge, this is the first study on the design and analysis of degradation-based RDT.

When performing degradation analysis, a big challenge is how to handle measurement errors. Without considering the impact of measurement errors, the developments of a degradation model for the actual degradation process and of the optimal RDT plan will be misled by the contaminated degradation data. To incorporate the impact of measurement errors in degradation-based RDT, a stochastic process model explaining both the evolution of actual product degradation and measurement errors is proposed in this paper. In particular, a random effects Inverse Gaussian process is used to model the actual degradation processes for different units while the measurement error is described by a normal probability distribution.

In planning RDT, the producer's and consumer's risks are usually considered. In the context of RDT, the producer's risk is the probability of rejecting a product when its reliability is higher than the target value. Similarly, the consumer's risk is the probability of accepting a product when its reliability is lower than the target value. In this paper, the optimal design of RDT plan is developed to minimize the total testing cost considering both the producer's and consumer's risks. To assist in solving the optimization problem, a method for checking the problem feasibility is developed considering the producer's and consumer's risks.

The remainder of this paper is organized as follows. Section 2 provides the random effects degradation model considering measurement errors as well as the corresponding statistical inference method. The optimal design of degradation-based RDT is addressed in Section 3, where both the producer's and consumer's risks are considered. In Section 4, a numerical example is provided to illustrate the use of the proposed degradation-based RDT method in practice. Finally, Section 5 draws conclusions.

2. Degradation-based Reliability Modeling

2.1. *Random effects degradation model*

Consider a product whose performance degrades over time. We assume that the performance evolution of each unit of the product can be described by a monotone degradation process $Y(t)$, and the degradation increments $\Delta Y_j = Y(t_j) - Y(t_{j-1})$ in time interval $[t_{j-1}, t_j]$ follow an Inverse-Gaussian distribution (Ye and Chen (2014)) with probability density function (pdf):

$$f(\Delta y_j|\mu,\lambda,\Delta h_j) = \sqrt{\frac{\lambda \Delta h_j^2}{2\pi \Delta y_j^3}} \exp\left(\frac{-\lambda(\Delta y_j - \mu \Delta h_j)^2}{2\mu^2 \Delta y_j}\right), \tag{1}$$

where (μ, λ) are the mean and diffusion parameters, respectively, $\Delta h_j = h(t_j) - h(t_{j-1})$, and $h(t)$ is a monotonically increasing time transformation.

We assume that the RDT is conducted by testing n units of the product and each unit i has unit-specific parameters (μ_i, λ_i) drawn from a joint Truncated Normal-Gamma distribution (i.e., random effects) with pdf:

$$g_r(\lambda, \mu^{-1}) = \frac{\beta^\alpha}{\Gamma(\alpha)} \lambda^{\alpha-1} \exp(-\beta\lambda) \left(\frac{\lambda\tau}{2\pi}\right)^{1/2} \exp\left(-\frac{\lambda\tau(\mu^{-1}-\nu)^2}{2}\right) \frac{1}{1-\Upsilon_{2\alpha}(-\nu\sqrt{\tau\alpha/\beta})}, \tag{2}$$

where $\Upsilon_{2\alpha}$ is the cumulative distribution function (cdf) of a Student-t distribution with 2α degrees of freedom. Under the random effects model, $f(\Delta y|\Delta h_j)$ can be derived as:

$$\begin{aligned} f(\Delta y_j|\Delta h_j) &= \int_0^\infty \int_0^\infty f(\Delta y_j|\mu,\lambda,\Delta h_j) g_r(\lambda,\mu^{-1}) d\mu d\lambda \\ &= \left[1 - \Upsilon_{2\alpha+1}\left(-(\Delta h_j + \tau\nu)\sqrt{\frac{2\alpha+1}{2\gamma(\Delta y_j+\tau)}}\right)\right] \frac{1}{1-\Upsilon_{2\alpha}(-\nu\sqrt{\tau\alpha/\beta})} \times \\ &\frac{\Delta h_j}{\sqrt{2\pi\Delta y_j^3}} \frac{\beta^\alpha}{\Gamma(\alpha)} \sqrt{\frac{\tau}{\Delta y_j+\tau}} \frac{\Gamma(\alpha+1/2)}{\gamma^{\alpha+1/2}}, \end{aligned} \tag{3}$$

where

$$\gamma = \frac{\Delta h_j^2}{2\Delta y_j} + \beta + \frac{\tau\nu^2}{2} - \frac{(\Delta h_j + \tau\nu)^2}{2(\Delta y_j + \tau)}.$$

Let Z_{ij} be the jth degradation measurement of unit i, which is composed of the true degradation signal Y_{ij} and an error term ε_{ij} that follows the Normal distribution $N(0, \sigma^2)$ with mean 0 and known variance determined from the specifications of measurement equipment. As a result, the observed degradation increment of unit i in time interval $[t_{j-1}, t_j]$ can be expressed as: $\Delta z_{ij} = \Delta y_{ij} + \Delta\varepsilon_{ij}$, where $\Delta\varepsilon_{ij} = \varepsilon_{ij} - \varepsilon_{ij-1}$. Under this model, the reliability of a new unit given a fixed failure threshold y^T and target lifetime T is:

$$R(T, y^T) = \int_0^{y^T} \int_{-\infty}^{z} f(z-\varepsilon|h(T))\phi(\varepsilon/\sigma)/\Phi(z/\sigma)d\varepsilon dz, \tag{4}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are the pdf and cdf of the standard normal distribution, respectively. To evaluate the reliability value using this expression, Monte-Carlo (MC) integration will be performed for the inner integral, and a numerical routine will be implemented for the outer integral.

2.2. *Model parameter estimation*

Let $\Theta = \{\alpha, \beta, \nu, \tau\}$ be the parameter vector of the degradation process. To facilitate parameter estimation, an expectation-maximization (EM) algorithm is developed. First, consider the likelihood of the observed degradation increment Δz with complete data $f_z(\Delta z) = f(\Delta z - \Delta\varepsilon|\Delta\varepsilon, \lambda, \mu)g_r(\lambda, \mu^{-1})\phi(\Delta\varepsilon/(\sqrt{2}\sigma))$. After testing n units with m measurements each and observing the matrix $Z = \{Z_{ij}, \forall i, j\}$ of degradation signals, the log-likelihood function for the complete information can be expressed as:

$$\begin{aligned}\ln\mathscr{L}(\Theta|Z) \propto & \sum_{i=1}^{n}\sum_{j=1}^{m}\left[\frac{\ln(\lambda_i\Delta h_{ij})}{2} - \frac{\lambda_i(\Delta y_{ij} - \mu_i\Delta h_{ij})^2}{2\mu_i^2\Delta y_{ij}}\right] \\ & + m\sum_{i=1}^{n}\left[\alpha\ln(\beta) + (\alpha - 1/2)\ln(\lambda_i) - \lambda_i\beta + \frac{\ln(\tau)}{2} - \ln(\Gamma(\alpha)) - \frac{\tau\lambda_i\mu_i^{-2}}{2} + \nu\tau\lambda_i\mu_i^{-1} - \frac{\tau\nu^2\lambda_i}{2}\right] \\ & - mn\ln\left[1 - \Upsilon_{2\alpha}(-\nu\sqrt{\tau\alpha/\beta})\right] - \sum_{i=1}^{n}\frac{\Delta\varepsilon_{i1}^2}{2\sigma^2} - \sum_{i=1}^{n}\sum_{j=2}^{m}\frac{\Delta\varepsilon_{ij}^2}{4\sigma^2},\end{aligned} \tag{5}$$

For the E-step, $\hat{\Delta y_{ij}} = E[\Delta y_{ij}]$ with $E[\Delta y_{ij}] = \Delta z_{ij} - E[\Delta\varepsilon_{ij}]$ and $E[\Delta\varepsilon_{ij}]$ is the expected value of a Truncated Normal distribution with an upper bound at $-z_{ij}$. Since the variance of measurement errors σ^2 is known, this estimate can be used for all iterations of the EM algorithm. In addition, using Bayesian results (Banerjee and Bhattacharyya 1979), we can calculate $E[\lambda_i|Z_i]$ and $E[\ln(\lambda_i)|Z_i]$ for each unit i with degradation signals $Z_i = \{Z_{i,j}, \forall j\}$ as:

$$E[\lambda_i|Z_i] = \left(\alpha + \frac{m}{2}\right)\left[\beta + \frac{\tau\nu^2}{2} + \sum_{j=1}^{m}\frac{\Delta h_{ij}^2}{2\hat{\Delta y_{ij}}} - \frac{(\tau\nu + \sum_{j=1}^{m}\Delta h_{ij})^2}{2(\tau + \sum_{j=1}^{m}\hat{\Delta y_{ij}})}\right]^{-1} = \frac{\alpha'}{\beta_i'}, \tag{6}$$

$$E[\ln(\lambda_i)|Z_i] = \psi(\alpha') - \ln(\beta'), \tag{7}$$

where $\psi(\cdot) = \Gamma'(\cdot)/\Gamma(\cdot)$ is the digamma function, in which $\Gamma(\cdot)$ and $\Gamma'(\cdot)$ are the gamma function and its first derivative, respectively.

Note that $(\mu_i^{-1}|Z_i)$ follows a Truncated Student-t (Tt) distribution with lower bound 0, mean ν, standard deviation $\sigma = \sqrt{\beta/\tau\alpha}$, and 2α degrees of freedom. Moreover, the expected value and variance of a Tt distribution with lower bound of $-\nu/\sigma$, mean 0, standard deviation 1, and d degrees of freedom are (Kim 2008):

$$E[Tt|d, \Theta] = d^{d/2}\frac{\Gamma((d-1)/2)}{\Gamma(d/2)\Gamma(1/2)}\frac{(d + \nu^2\tau\alpha/\beta)^{-(d-1)/2}}{2(1 - \Upsilon_d(-\nu\sqrt{\tau\alpha/\beta}))} \tag{8}$$

$$\mathrm{Var}(Tt|d, \Theta) = \frac{d}{d-2} - E[Tt|d, \Theta]\left(\nu\sqrt{\tau\alpha/\beta} + E[Tt|d, \Theta]\right). \tag{9}$$

By defining $\tau' = \tau + \sum_{j=1}^{m}\Delta y_{ij}$, $\nu' = (\tau\nu + \sum_{j=1}^{m}\Delta h_{ij})/\tau'$ and $\Theta' = \{\alpha', \beta', \nu', \tau'\}$, and using the transformation $E[\mu^{-1}|Z_i] = \nu + \sigma E[Tt]$, the expected values of $\mu_i^{-1}, \lambda_i\mu_i^{-1}$, and $\lambda_i\mu_i^{-2}$ can be derived as:

$$E[\mu_i^{-1}|Z_i] = \nu' + \sqrt{\frac{\beta'}{\tau'\alpha'}}E[Tt|2\alpha', \Theta']$$

$$E[\lambda_i\mu_i^{-1}|Z_i] = \frac{\alpha'\nu'}{\beta'} + \frac{\alpha'}{\beta'}\sqrt{\frac{\beta'}{\tau'(\alpha'+1)}}E[Tt|2\alpha'+2, \Theta']$$

$$E[\lambda_i\mu_i^{-2}|Z_i] = \frac{\alpha'}{\beta'}\left[\nu'^2 + \frac{\beta'}{\tau'\alpha'} + \nu'\sqrt{\frac{\beta'}{\tau'(\alpha'+1)}}E[Tt|2\alpha'+2, \Theta']\right]$$

Note that not all the terms in Equation 5 are independent of Θ. For example, for the M-step we only need to numerically maximize:

$$\ln \mathscr{L}(\Theta|Z) \propto (\alpha - 1/2)\sum_{i=1}^{n} E[\ln(\lambda_i)|Z_i] - \left(\beta + \frac{\tau\nu^2}{2}\right)\sum_{i=1}^{n} E[\lambda_i|Z_i] - \frac{\tau}{2}\sum_{i=1}^{n} E[\lambda_i\mu_i^{-2}|Z_i] + \nu\tau\sum_{i=1}^{n} E[\lambda_i\mu_i^{-1}|Z_i] \\ + n\left[\alpha\ln(\beta) - \ln(\Gamma(\alpha)) + \frac{\ln(\tau)}{2} - \ln\left[1 - \Upsilon_{2\alpha}(-\nu\sqrt{\tau\alpha/\beta})\right]\right] \tag{10}$$

Alternatively if $P\{\mu^{-1} < 0\} \approx 0$, it is possible to approximate the maximum likelihood estimates by solving the following system of equations:

$$\hat{\beta} = \frac{\alpha}{\sum_{i=1}^{n} E[\lambda_i|Z_i]/n} \qquad \ln(\hat{\alpha}) - \ln(\sum_{i=1}^{n} E[\lambda_i|Z_i]/n) + \sum_{i=1}^{n} E[\ln(\lambda_i)|Z_i]/n - \psi(\hat{\alpha}) = 0$$

$$\hat{\nu} = \frac{\sum_{i=1}^{n} E[\lambda_i\mu_i^{-1}|Z_i]}{\sum_{i=1}^{n} E[\lambda_i|Z_i]} \qquad \hat{\tau} = n\left[\sum_{i=1}^{n} E[\lambda_i\mu_i^{-2}|Z_i] - 2\hat{\nu}\sum_{i=1}^{n} E[\lambda_i\mu_i^{-1}|Z_i] + \hat{\nu}^2\sum_{i=1}^{n} E[\lambda_i|Z_i]\right]^{-1}$$

It is worth pointing out that the corresponding statistical inference methods developed in this paper can be used for other stochastic degradation process models, such as the Wiener process and gamma process.

3. Optimal Design of Degradation-based RDT

Consider a RDT experiment where n units will be tested for a fixed amount of time τ, and m degradation measurements will be taken on each unit periodically at equal time intervals such that the time between measurements is τ/m. Given the cost of a unit c_u that includes testing and unit manufacturing cost, and the measurement cost c_m, the decision maker wants to minimize the total cost of testing subject to performance guarantees in terms of the producer's risk r_p and the consumer's risk r_c by determining the optimal values of n and m. The optimal RDT design problem can be formulated as:

$$\begin{aligned} &\underset{n,m}{\text{minimize}} && c_u n + c_m nm && \\ &\text{subject to} && P\{R(\hat{\Theta}) \le R_0 | R(\Theta_p) > R_0, \Omega\} \le r_p && \text{Producer's risk} \\ & && P\{R(\hat{\Theta}) \ge R_0 | R(\Theta_c) < R_0, \Omega\} \le r_c && \text{Consumer's risk} \\ & && Lb \le (n,m) \le Ub && \text{Lower and upper bounds for}(n,m) \end{aligned}$$

where $\Omega = \{n, m\}$ represents the experimental design for a bounded design space, and the competing parameter sets Θ_p and Θ_c are specified by the producer and consumer, respectively.

We only discuss how to determine the feasibility of the producer's risk with techniques that are also applicable for the consumer risk's. The exact probability involves nm dimensional integral of the form:

$$P\{R(\hat{\Theta}) \le R_0 | R(\Theta_1) > R_0, \Omega\} = \int_0^\infty \cdots \int_0^\infty I(R(\hat{\Theta}|Z) \le R_0) f_Z(Z|\Theta_1, \Omega) dz_{11} \cdots dz_{nm}. \tag{11}$$

where $I(R(\hat{\Theta}|Z) \le R_0)$ is an indicator function of the test being rejected for the null hypothesis: $R \ge R_0$ and the alternative hypothesis: $R < R_0$. Note that an EM algorithm for parameter estimation has to be used to compute $\hat{\Theta}$ for each design candidate. Moreover, since the sample size n tends to be small when minimizing the total cost of testing, it is not appropriate to use large-sample approximation for calculating the variance-covariance matrix of $\hat{\Theta}$, which only leaves a bootstrap procedure as a reasonable way for performing the hypothesis test. All these complications make the numerical integration of this probability computationally prohibitive.

Homem-de-Mello and Bayraksan (2015) solve a chance constrained stochastic optimization problem by constructing constraints of the form $P \ge \alpha$, where $\hat{P} = \sum_{k=1}^{N} I(\cdot)/N$ is estimated as the average of an indicator function and α is the minimum required probability. By constructing a test with $\rho < \alpha$ such that if it passes there is a $(1-\beta)$ confidence that the constraint is satisfied. The quantity ρ is estimated as $\sup \rho \in (0,1)$ such that $B(N\hat{P}, N, \rho) \le \beta$ for $B(\cdot)$ being the cdf of the binomial distribution with the probability of success ρ and $N\hat{P}$ successes out of N trials. Using this method, we need to transform the producer's risk constraint to $1 - P\{R(\hat{\Theta}) \le R_0 | R(\Theta_p) > R_0, E\} \ge 1 - r_p$, then specify β and N to do a Monte Carlo simulation of N tests for a given Θ_p.

4. Numerical Example

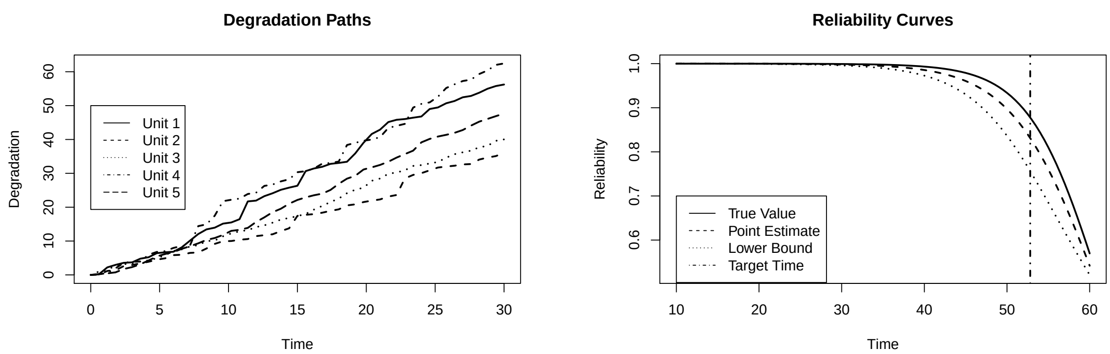
This section provides a numerical example to illustrate the parameter estimation for the proposed random effects degradation model and the design of optimal degradation-based RDT plan.

4.1. *Problem setting*

Assume a manufacturer wants to sell a new product to a consumer. The minimum reliability requirement for this product is $R_0 = 0.8779$ after approximately $T = 53$ days of operation. The degradation parameters that achieve this target are $\Theta = \{4, 2, 1, 50\}$. In the degradation test, a time transformation $h(t) = t^{1.12}$ is considered (so the target life time after the transformation is $h(T) = 85$ days), and measurement errors have mean 0 and variance $\sigma^2 = 0.001$. A test unit is considered failed if its degradation process crosses a failure threshold of 100 (i.e., soft failure). The RDT is conducted for a fixed 30 days.

4.2. *Parameter estimation*

To illustrate the proposed reliability estimation method for the random effects degradation model, we consider an RDT plan that takes 50 measurements on each of 50 units with model parameters drawn from Θ. Figure 1a shows the simulated degradation paths of 5 (out of 50) units. We implement the EM algorithm presented in Section 2.2 and use the function "auglag" from the package "nloptr" in R to solve the M-Step routine with initial solution of $\Theta_0 = \{1, 1, 1, 1\}$. The hypothesis test on the product's reliability is conducted using a bootstrap procedure with 200 nonparametric bootstrap samples. A one-sided 95% confidence interval on the reliability is constructed using the pivot inversion procedure, and the acceptance is declared if the attained reliability lower bound is greater than R_0. The parameter estimates from the simulated degradation data are $\hat{\Theta} = \{4.369, 2.416, 0.9932, 35.56\}$, the point reliability estimate is 0.8312, and the bootstrap lower bound of reliability is 0.7639. Figure 1b shows the true reliability of the product, the point estimate and the lower bound. The result is unfavorable to the producer because of the under-estimation of the product's reliability.



(a) Simulated degradation paths (5 out of 50 units). (b) True and estimated reliability functions.

Figure1. Simulated degradations paths and reliability functions.

4.3. *Optimal RDT plan*

Assume that the cost per unit c_u is 100, the cost of each measurement c_m is 2, and both the number of measurements and units to be tested have a lower bound of 20 and an upper bound of 100, respectively. According to the producer, the product degrades with parameters $\Theta_p = \{4, 2, 1.1, 50\}$, so that its reliability at the target 53 days is 0.9693. On the other hand, the consumer proposes the parameters $\Theta_c = \{4, 2, 0.9, 50\}$ with reliability of 0.6623. Without loss of generality, we consider that both the consumer's and producer's risks are 5%. The feasibility of each candidate test plan is evaluated using the method outlined in Section 3 with a 95% confidence on the feasibility ($\beta = 0.05$) and 100 independent simulations of the test plan. Table 1 presents the feasibility and costs of candidate test plans. ρ_c and ρ_p are the values obtained for the consumer and producer respectively, and a test plan is feasible only if both values are greater than 0.95. The optimal test plan requires the use of 50 units and taking 50 measurements on each test unit. The total cost is 10000.

Table 1: Costs and feasibility of candidate testing plans.

m	n	ρ_c	ρ_p	Cost	m	n	ρ_c	ρ_p	Cost	m	n	ρ_c	ρ_p	Cost
20	20	1	0.8803	2800	48	48	1	0.9452	9408	60	60	1	0.9801	13200
40	40	1	0.9133	7200	49	49	1	0.9452	9702	100	100	1	0.9735	30000
45	45	1	0.9463	8550	50	50	1	0.9769	10000	–	–	–	–	–

5. Conclusions

In this paper, a new methodology is proposed for the use of degradation data in modeling and planning RDT. The EM algorithm developed in this study can effectively estimate the parameters of the proposed random effects degradation model considering measurement errors. The result shows that the proposed degradation-based RDT can significantly speed up reliability demonstration. In addition, the proposed simulation methodology enables the feasibility assessment of an RDT plan involving both the consumer's and producer's risks.

It is worth pointing out that the numerical results suggest that the most important restriction in planning the RDT is the producer's risk. Sometimes, the consumer's risk may be dropped from the formulation without affecting the feasible region. In practice, the methodology proposed in this paper can be adapted to a Bayesian model in order to explore if the bias in the reliability estimation can be reduced.

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