Insight into cost of defective state using delay-time concept

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Abstract. Delay-time concept has been used in a large number of applications for modelling important aspects of reality, such as the dynamics of failures, which in practice occurs in two stages for the most devices. Concisely, a defect arises in a component or system followed by a subsequent failure after an interval called delay time. The effort to repair a defect is related to the time from when it can be firstly observed until its discovery by an inspection. However, a small number of contributions deal with the cost related to the time in which the system undergoes in the defective state. In addition, a repair action on a system that undergoes in defective state for a short period of time can have different consequences than the same action after a long period of time. Two examples are: a pipeline leakage defect and the repair of a defect in a tooth. In both cases, the consequences vary depending on the time in defective state. Hence, this paper proposes a way to take the defect cost into consideration according to the time in which the system undergoes in this state.

1. Introduction

Modelling a maintenance policy is a way to understand the behavior of a system, equipment or component and to try to establish a clarification or a solution for an aspect that is crucial for the system. Nevertheless, the maintenance models had a late development when compared to models related to operations. Scarf (1997) suggests that the reasons for a late development on maintenance modelling are related to the fact that maintenance actions are carried out on plant, not on product, and also because it is difficult to associate the spending on maintenance and its respective results in production performance.

Currently, maintenance is no longer considered a necessary evil and represents an important function for competitiveness (Pintelon and Parodi-Herz, 2008). In fact, the way that maintenance is organized and structured can highly influence the performance and competitiveness of an organization. Pinjala, Pintelon and Vereecke (2006) indicate that maintenance has now a more important role to achieve cost-effective operations and superior product quality. Due to changes in maintenance perspective along the years and its greater importance in the current industrial scenario, traditional maintenance policies have been continuously studied and applied in order to accomplish better results in terms of cost, availability and other aspects.

Maintenance models have been adjusted to better fit some particular situations. A positive consequence is the creation of new models that have been extensively applied to similar maintenance conditions. As a result, interesting modifications in maintenance models have been suggested to take into consideration specific points that were not addressed on previous models.

This paper proposes to analyse the cost of a defect while a component undergoes in defective state. The model analysed is: delay time model for single component (Christer, 1976, 1982; Christer and Waller, 1984; Wang, 2008, 2012). The basis for the modification is supported by the fact that a penalty cost should be associated to a defect when it implies in a reduction in the performance of a system or the sojourn time of the defect increases the difficulty of the maintenance action. Besides, the modification proposal is in line with practice in many situations and it is not difficult to be applied to real cases, which are important aspects in a maintenance model (Scarf, 1997). In fact, models with a higher complexity and number of parameters are more difficult to be applied to real cases.

The outline of the paper is as follows: the model is reviewed in section 2, the adjustments are suggested in section 3, a model analysis is presented in section 4, possible applications for revised model are presented in section 5, some consideration about application on multi-component system is presented in section 6 and conclusions and future work are discussed in section 7.

2. Reviewing the model

In this section, the delay-time model for single component is reviewed as it was first proposed and the theory related to it is briefly explained. A single component system may be interpreted as a component or a component and a socket that together execute an operational function (Ascher and Feingold, 1984; Scarf and Cavalcante, 2012).

The delay time model was first introduced by Christer (1976) and it considers that the failure is regarded as a two-stage process. A defect arises in the component at some time *x*, followed by a subsequent failure after an interval *h* (Baker and Christer, 1994), as shown in Figure 1.

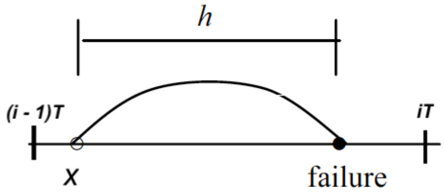

Figure 1. Delay time for a defect (Adapted from Wang, 2008: p. 347).

Clearly, the failure is preceded by a defect that undergoes during the interval *h*. This interval represents an opportunity to check the system and correct the defect before it turns into a failure. In this sense, the interaction between equipment performance and inspections can be evaluated using the delay-time concept (Wang, 2012). In fact, it is possible to determine an inspection interval that minimizes the total expected cost over an inspection cycle, $C(T)$, which includes: inspection cost (C_i), average cost to repair a defect (C_p) and failure cost (C_f).

First, we assume that the following assumptions are valid (Lee, 1999):

1) An inspection takes place every T time units, cost C_i units and requires d_i time units, where $d_i << T$.
2) Inspections are perfect and the defect present will be identified.
3) Defect identified at an inspection will be repaired within the inspection period at an average cost of C_p.
4) Failure is repaired as soon as it arises, incurring on average *df* downtime and C_f cost, where $C_f > C_p$ and $d_f << T$.
5) The initial time *x* of a defect is uniformly distributed over ($0, T$), and is independent of *h*, with defect arises at a rate of *k* per unit time.
6) The probability density function (*pdf*) of delay time *h*, $f(h)$, is known.

Considering Figure 1, $C(T)$ can be determined considering two possibilities: failure occurs on the interval ($x + h \in [(i-1)T, iT]$) or failure does not occur on the interval ($x \in [(i-1)T, iT]$ and $h > iT - x$). A possible way to model these possibilities, according with the notation used in Figure 1 and considering the assumptions above, is presented in Equation 1:

$$C(T) = \sum_{i=1}^{\infty}[iC_i + C_p] \int_{(i-1)T}^{iT} f(x) R_h(iT - x) dx + \sum_{i=1}^{\infty}[(i-1)C_i + C_f] \int_{(i-1)T}^{iT} f(x) F_h(iT - x) dx \qquad (1)$$

Similarly, the lifetime of a component, $L(T)$, can be modelled as suggested in Equation 2.

$$L(T) = \sum_{i=1}^{\infty} iT \int_{(i-1)T}^{iT} f(x) R_h(iT - x) dx + \sum_{i=1}^{\infty} \int_{(i-1)T}^{iT} f(x) \left[\int_{0}^{(iT-x)} (x+h) f(h) dh \right] dx \qquad (2)$$

The first part of Equations 1 and 2 represents the situation when the failure does not occur on the interval and the second part represents the case in which the failure occurs on the interval. The sum from 1 to infinite number of inspections considers all necessary inspections due to the many defects and failures that can arrive in the failure process of a single component system along time.

The sequence of inspections should be determined in a way that the total expected cost over an inspection cycle is minimized. In many practical situations, the failure cost is much higher than the inspection cost and the cost to repair a defect item, because it is influenced by some inconvenient brought to the system, such as the downtime period (d_f) from failure to repair action. Consequently, the inspection intervals tend to be determined to avoid failures and their negative influences on system.

This initial model does not consider that the time in defective state can have a penalty cost due to the insertion of possible limitations in performance and productivity. The inclusion of this possibility is commented in the next section.

3. Insight into cost of defective state

In this section, the insight into the cost of defective state is provided. This cost can be modelled as a part of the cost to repair a defect and as a part of the failure cost. As the failure cost is sufficiently high in most cases, the cost of defective state is modelled in this paper only as a part of the cost to repair a defect.

As shown in section 2, the delay time modelling for a single component considers a two-stage process and the failure can occur on the inspection interval ($x + h \in [(i-1)T, iT]$) or out of the inspection interval ($x \in [(i-1)T, iT]$ and $h > iT - x$). For both cases, the cost associated with the time in defective state is not considered. If the defect is found, the repair cost is the same (Cp) no matter the sojourn time of the defect. However, there are situations in which the consequences of a period in this state should be measured in order to better represent the reality, for instance, a leakage defect. In this case, the consequences vary depending on the time in defective state and the more time in this state the more waste of material and penalty cost.

Aven and Castro (2009) propose a discounted cost based on the number of failures of the system and the time spent in the defective state (delay time). In our paper, however, the defect cost per unit time is suggested to be considered in delay time model as an increment in the cost to repair a defect at an inspection.

In this sense, when the failure does not occur on the interval, the defect cost per unit of time can be considered making the cost to repair a defect at an inspection varies as a function of the time that the system undergoes in defective state ($C_p(y) = C_p + C_c(y)$, with $y = iT - x$). Hence, the first part of Equation 1 can be reformulated as follows in Equation 3. Possible formats for $C_c(y)$ can be linear, quadratic, exponential or other according to the specific problem. The next section considers two of them for a model analysis.

$$C(T)_1 = \sum_{i=1}^{\infty} iC_i \int_{(i-1)T}^{iT} f(x)[\mathrm{C}_p + C_c(y)]R_h(iT-x)dx$$

(3)

When failure occurs on the interval, the defect cost per unit time can be considered making the failure cost varies as function of the time in which the system undergoes in defective state as well. In this paper, we didn't consider this variation. However, we present a way that it would be considered. The failure cost could be modelled as follows: $C_f(h)$ is the sum of the fixed penalty cost relative to the general consequences of the failure, C_{ff}, and the time-related penalty cost in defective state, h, which interferes in variable failure cost, $C_c(h)$. So, $C_f(h) = C_{ff} + C_c(h)$ and the second part of Equation 2 can be reformulated as follows in Equation 4.

$$C(T)_2 = \sum_{i=1}^{\infty} [(i-1)C_i] \int_{(i-1)T}^{iT} f(x) \int_{0}^{iT-x} f(h)C_f(h)dhdx \qquad (4)$$

This modification proposal in delay time model has two important characteristics: (1) it can describe an important and recurrent practice situation, which can be associated with a significant number of real cases; (2) it does not increase the complexity of the model, so that is applicable to reality.

4. Model analysis

This section evaluates the delay time model for a hypothetical problem. It consists in assessing the optimal interval between inspections and the respective optimal cost rate per unit of time for previous and revised models. This analysis considers a linear function ($C_c(y) = cy$) and a quadratic function ($C_c(y) = cy^2$) for representing the cost along the time in defective state.

We consider that *f(x)* follows a Weibull distribution with shape parameter equals to 3 and scale parameter equals to 1, $f(h) = 2e^{-2h}$, C_i = \$0.5/inspection, C_p = \$1/repair of a defect, C_f = \$5/failure and *c* as defined in table 2 (\$/unit time in defective state).

Table 2: Comparing the two models.

	No cost in defective state	Linear cost in defective state $C_c(y) = cy$				Quadratic cost in defective state $C_c(y) = cy^2$			
c	not applicable	1	2	3	4	1	2	3	4
Insp. period (*T*)	1.191	1.186	1.181	1.177	1.172	1.183	1.176	1.170	1.164
C(T)	3,588	3.732	3.877	4.021	4.165	3.665	3.742	3.819	3.896
L(T)	1.178	1.178	1.177	1.177	1.176	1.178	1.177	1.176	1.176
E(T) = C(T)/L(T)	3.046	3.169	3.293	3.417	3.541	3.113	3.180	3.247	3.314

Table 2 shows that the cost in the defective state directly affects the optimal interval between inspections and the minimal *E(T)*. When the parameter *c* rises, the interval between inspections is lightly reduced and the E(T) is significantly higher, as expected. The increase in the expected cost is illustrated in Figure 2 for linear case and in Figure 3 for quadratic case.

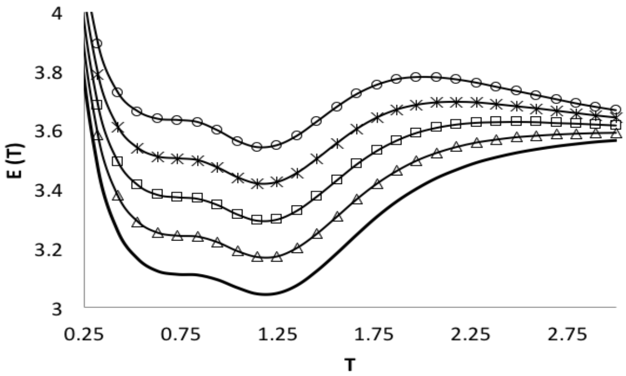

Figure 2. Expected cost per unit time for the defective cost as a linear function and different values of c: c = 0 (¯); c = 1 (−Δ−); c = 2 (−• −); c = 3 (−∗−); c = 4 (−o−).

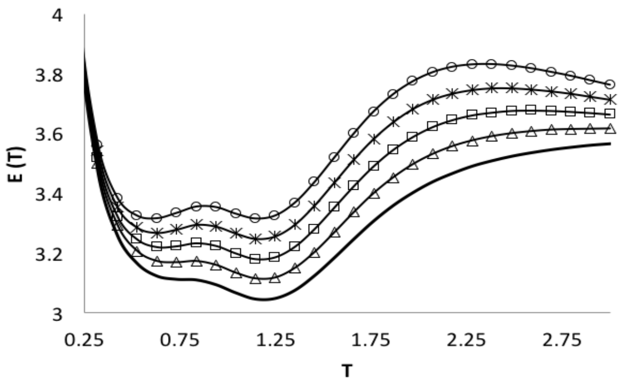

Figure 3. Expected cost per unit time for the defective cost as a quadratic function and different values of c: c = 0 (¯); c = 1 (−Δ−); c = 2 (−• −); c = 3 (−∗−); c = 4 (−o−).

Figures 2 and 3 represent the expected cost behavior along the time. As observed, the higher the cost in defective state, the higher is the increase in expected cost. Although the period between inspections does not suffer a significant change for this particular example, the variation in the expected cost may affect the way that maintenance is planned. It gives a more representative view from reality and can offer a more reliable cost prediction for the multiples possible situations related to the maintenance of the system.

Besides, it is important to emphasize that in real cases the defective cost function should be carefully modelled to fit the characteristics of the specific problem and the parameters used in the model should be precisely defined. Considering these aspects, the delay time modelling can be better applied. The next section suggests possible real applications in which the revised model can be applied.

5. Possible applications for revised model

This section presents two practical situations in which the revised model may be applied: leakage defect in pipelines and dental defects. In both cases, the period in defective state considerably affects the features of the system and entails negative consequences.

In the former case, a defect or failure causes high losses of product, energy, capital and can even cause human and environmental damages. Consequently, the inspection on pipelines over regular periods of time is critical (Sorabh, Raghav and Sengar, 2017). The proposal modification presented in Section 3 may be an alternative to model the inspection interval to this specific context.

In the latter case, a dental defect is influenced by the time that the tooth undergoes in defective state, so the longer the period of time in this state, the higher the negative consequences associated with it, such as, decrease in dental health and increase in repair costs. In the context of defective dental restorations, for instance, the repair brings advantages as: less loss and more preservation of tooth structure, reduction of pain, reduction of treatment time, reduced costs to the patient, and increased longevity of the restoration (Hickel, Brüshaver and Ilie, 2013). In this sense, the inclusion of variable defect cost over time in delay time modelling may serve to model, for example, the increase in cost associated with a dental defect over time.

Finally, other practical situations may be characterized and modelled according to the proposals described in this paper. They may be applied in situations in which the time in defective state brings negative impacts that may be accounted in terms of cost, for instance, low performance, environmental consequences, changes in the operating characteristics of the equipment, other penalties, so on.

The next section presents some considerations for possible future work on multi-components systems in order to meet the requirements of particular cases with the same characteristics of a penalty cost associated with the cost in defective state.

6. Extending context and application for multi-component systems

As in the single-component system, there are practical situations that require a multi-component approach. One interesting example is the hammers of a shredding machine. The hammers are components that tend to wear out considerably before fail because of its performance features (Zhou et al, 2016). This wear can decrease efficiency on machine performance and, therefore, a significant portion of the maintenance cost is function of the defective state of each hammer in the system.

A model that would provide a way to represent this situation is presented by Anbar (1976) and suggests that the similar components works in parallel in a system with identical exponential failure distribution. The system works if and only if k out of n units are working. The failures can be discovered only by inspections and when an inspection occurs all failure units are replaced by new ones.

This model would provide a good representation for this particular situation if two important modifications were done. The first one is to insert the possibility of a defect to occur and the second one is to provide a way to account for the cost in defective state. An analysis on delay time modelling for multi-component system may assist in this situation.

7. Conclusion and future work

This paper contributes to the development of maintenance modelling by taking into consideration the cost related to the period in defective state. The proposal modification in delay time model for single component system allows some practical situations to be better modelled. In these cases, the sojourn time in defective state negatively influences the performance of a system. Some examples of real context were presented and other possible related cases can be identified based on their characteristics. Besides, there is not a significantly increase in model complexity, so that the proposal is not difficult to be applied to real cases. In future work, the revised model may be applied to the practical situations illustrated in Section 6 and the findings compared with the ones obtained by delay time model. Also, a multi-component system analysis is expected to be developed, as suggested in Section 6.

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