An unpunctual preventive maintenance policy for repairable items sold with a two-dimensional warranty

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Abstract The optimization of preventive maintenance (PM) strategies for repairable items sold with warranty contracts has received much attention in the literature. However, the existing research implicitly assumes that maintenance actions within the warranty period are punctual. In practice, it is not uncommon that the actual maintenance instants deviate from the scheduled instants. In this paper, an unpunctual imperfect PM strategy, which allows customers to advance or postpone the scheduled PM actions in a tolerable range, is proposed for repairable items sold with a two-dimensional warranty. The objective of this work is to determine the optimal unpunctual PM strategies under the given warranty period so as to minimize the manufacturers total expected warranty servicing cost. It is shown that the unpunctual PM strategy contains its punctual counterpart as a special case and tends to result in slightly higher warranty servicing cost.

1. Introduction

Nowadays, almost all products are sold with warranty contracts due to competition, industrial obligations, and/or customer requirements. However, most manufacturers regard warranty as an overhead item, and they are always under pressure to keep warranty servicing costs staying within limits. Incorporating appropriate preventive maintenance (PM) programs into the warranty policy is one feasible way of reducing warranty servicing cost (Wang and Xie 2018). From the cost-benefit perspective, the optimal trade-off between the reduction of warranty cost and the additional cost incurred by the PM program becomes necessary in order to obtain an optimal PM strategy for the manufacturer.

This paper is interested in investigating optimal imperfect PM strategies for repairable items sold with a two-dimensional (2-D) warranty. A 2-D warranty simultaneously employs two variables, usually age and usage, to characterize the warranty policies. In the 2-D warranty context, there are some references dealing with the optimization of PM strategies; see Wang et al. (2015), Wang and Su (2016), Su and Wang (2016), and Huang et al. (2017), for recent ones. Specifically, Wang and Su (2016) proposed a 2-D PM strategy for items sold with a 2-D warranty, under which the items are preventively maintained every K units of age or every L units of usage, whichever occurs first.

Nevertheless, one of the main drawbacks of the studies above is that they implicitly assume that the PM actions within the warranty period are punctual. In practice, however, the unpunctuality of PM activities that results from the separate nature of PM specification and implementation, is quite common in a B2C (business-to-customer) context (He et al. 2017a). For instance, in an automobile warranty and maintenance setting, it is not uncommon that some automobile owners do not follow the prescribed PM schedule strictly. In the literature, the optimal planning of unpunctual perfect PM (replacement) policies has aroused the interests from a few researchers. He et al. (2017a) studied the optimal age replacement policy with and without minimal repair in anticipation of maintenance unpunctuality. Besides, Zhao et al. (2017) discussed a random age replacement policy in which the unit is replaced before failure at a random point of time.

To our knowledge, there is currently little research dealing with unpunctual imperfect PM strategy in the context of warranty. This paper proposes an unpunctual imperfect PM strategy for repairable items sold with a 2-D warranty, which allows customers to advance or postpone the scheduled PM activities in tolerable ranges. Under this unpunctual PM strategy, the jth PM action should be performed at age $jK \pm \Delta K$, or at usage $jL \pm \Delta L$, whichever occurs first (Note that, K and L are scheduled age- and usage-based PM intervals, respectively). The remainder of the article is organized as follows. Section 2 develops the optimization model for determining the optimal PM schedules under the unpunctual PM strategy. Section 3 conducts a case study to demonstrate the proposed unpunctual PM strategy. Finally, section 4 concludes this paper.

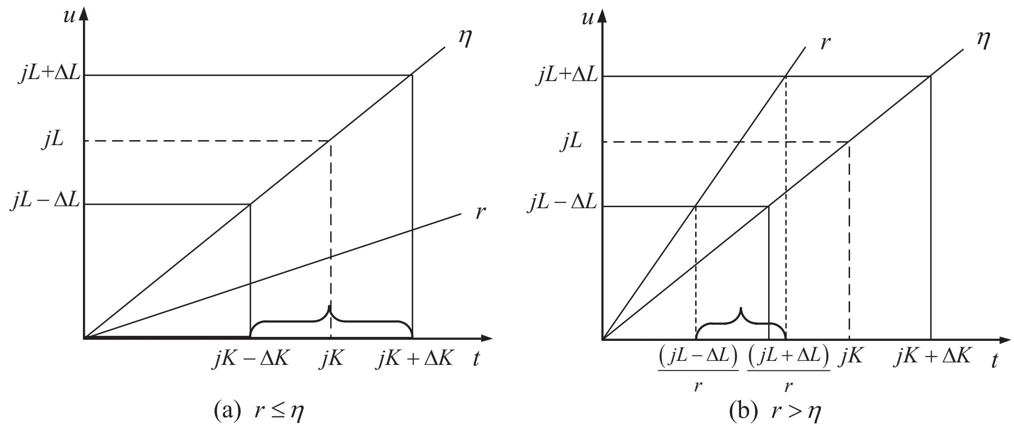

Figure1. Illustration of the tolerable range of the jth unpunctual PM activity.

2. Model formulation

2.1. *Two-dimensional warranty and failure modeling*

Consider that a repairable item is sold with a 2-D warranty policy, which is characterized by a rectangular region $\Omega_R(W,U) = \{(t,u) : t \in (0,W) \text{ and } u \in (0,U)\}$. Within the warranty region, all item failures are rectified by the manufacturer at no cost to the customer. This free repair warranty expires when either the item age exceeds its limit W or the total usage reaches its limit U, whichever occurs first.

In this study, we look at a repairable item as a black-box system, and use the marginal approach to model its failures in terms of age and cumulative usage (Wang and Xie 2018). For an elapsed time t, the cumulative operating time and total usage of the item can be denoted by $T(t)$ and $U(t)$, respectively. For simplicity, we assume that the item's cumulative operating time equals its age, i.e., $T(t) = t$. An essential assumption adopted here is that different customers have different usage rates but they will not alter their own usage rates during the warranty period (Ye et al. 2013). As a result, the usage rate R can be modelled as a random variable with distribution function $G(r) = P(R \le r),\ 0 \le r < \infty$. Conditional on $R = r$, the total usage $U(t)$ of an item at age t can be given by $U(t) = rt$.

Item failures can thus be modeled by a point process with a conditional intensity function $\lambda(t|r)$ that is dependent on both age t and usage $U(t)$. Given that all failures are minimally repaired with negligible repair time and no PM is performed, failures over time occur according to a non-homogeneous Poisson process (NHPP) with conditional intensity function $\lambda(t|r)$. In this study, following Murthy and Wilson (1991), $\lambda(t|r)$ is modeled as a simple polynomial function:

$$\lambda(t|r) = \theta_0 + \theta_1 r + \theta_2 T(t) + \theta_3 U(t) = \theta_0 + \theta_1 r + (\theta_2 + \theta_3 r)t \tag{1}$$

where θ_0, θ_1, θ_2, and θ_3 are positive constants.

2.2. *Punctual 2-D PM strategy*

In this study, what causes the actual PM instants to deviate from the scheduled instants is the potential unpunctuality of customers returning their items for PM activities in a B2C context. The proposed unpunctual PM strategy allows the earliness or tardiness of customers performing each (originally periodical) PM activity, and the tolerable ranges are $\pm\Delta K$ and $\pm\Delta L$ in the two dimensions of age and usage, respectively. In other words, under the unpunctual PM strategy, the jth PM activity should be performed at age $jK \pm \Delta K$ or at usage $jL \pm \Delta L$, whichever occurs first. Here, we assume that $K = W/(n+1)$ and $L = U/(n+1)$, which is common in practice. For the sake of simplicity, we further assume that $\Delta L/\Delta K = L/K = \eta$. It should be noted that when $\Delta K = \Delta L = 0$, the unpunctual PM strategy reduces to its punctual counterpart.

Figure 1 illustrates the age-based tolerable range for the jth unpunctual PM activity. As can be seen, for a customer with usage rate $r \le \eta$, the jth PM activity should be performed within the time interval $[jK - \Delta K, jK + \Delta K]$; while for a customer with usage rate $r > \eta$, the jth PM activity should be carried out within the time interval $[(jL - \Delta L)/r, (jL + \Delta L)/r]$.

Under the unpunctual PM strategy, an important issue is how to characterize the maintenance unpunctuality, as different customers may have different unpunctual behaviors. Since the customers' unpunctual behaviors are random, this study models the jth age-based PM instant τ_j for all customers as a uniform random variable defined in a finite support. This assumption is acceptable because 1) it has been adopted in He et al. (2017) and Zhao et al. (2017); and 2) the uniform distribution is often employed for analytical modeling of an unobservable parameter (Kurata and Nam 2013). Nevertheless, an appropriate distribution can be selected and estimated if real datasets are available.

2.3. *The optimization model*

In this study, the effect of imperfect PM activities on the item reliability is described by the age reduction model in Kim et al. (2004). The amount of age reduction depends on the PM effort level (m) applied. Larger value of m corresponds to greater PM effort and more reliability improvement. Such PM effort is assumed to be imperfect and constrained so that $0 \leq m \leq M$, with $m = 0$ implying no PM action and M indicating the upper limit of the PM effort.

In order to analyze the expected warranty expenses under the unpunctual PM strategy, two cases should be considered, i.e., $r \leq \eta$ and $r > \eta$.

Case 1: $r \leq \eta$. In this case, for a given customer with usage rate $r \leq \eta$, the 2-D warranty period would terminate at time W, with the corresponding usage being rW. Also, the jth age-based PM instant τ_j is a random variable with conditional probability density function (pdf) $z(\tau_j|r \leq \eta) = 1/(2\Delta K)$, $\tau_j \in [jK - \Delta K, jK + \Delta K]$.

Conditioning on the jth PM instant τ_j, the virtual age after the jth PM activity can be obtained as (Kim et al. 2004)

$$v_j = v_{j-1} + \delta(m)(\tau_j - \tau_{j-1}) = \delta(m)\tau_j, j = 1, 2, \ldots, n, \tag{2}$$

with $\tau_0 = 0$ and $\tau_{n+1} = W$.

Note that, $\delta(m)$ is a decreasing function of m with $\delta(0) = 1$ and $\delta(M) = 0$. If $m = M$, then the damage during the $(j-1)$th and the jth PM activities is fully compensated, i.e., $v_j = v_{j-1}$; if $m = 0$, then no PM action is taken, i.e., $v_j = \tau_j$; while in a more general case $0 < m < M$, the item is partially restored, i.e., the PM activity is imperfect. Here, we assume that $\delta(m) = (1+m)e^{-m}$, as in Kim et al. (2004).

We assume that any failure within the warranty period will be rectified through minimal repair with negligible duration, under which the failure rate after repair is nearly the same as that immediately before failure. Hence, the failure process between two successive PM activities is also an NHPP. Then, the conditional expected number of item failures within the warranty period

$$E[N^{\mathscr{U}}(n,m)|r \leq \eta; \tau_1, \ldots, \tau_n] = \int_0^{\tau_1} \lambda(t|r)\mathrm{d}t + \sum_{j=1}^{n-1} \int_{\delta(m)\tau_j}^{\delta(m)\tau_j + \tau_{j+1} - \tau_j} \lambda(t|r)\mathrm{d}t + \int_{\delta(m)\tau_n}^{\delta(m)\tau_n + W - \tau_n} \lambda(t|r)\mathrm{d}t \tag{3}$$

Removing the conditioning on τ_j, the conditional number of failures within the warranty period becomes

$$E[N^{\mathscr{U}}(n,m)|r \leq \eta] = \int_{nK-\Delta K}^{nK+\Delta K} \cdots \int_{K-\Delta K}^{K+\Delta K} E[N^{\mathscr{U}}(n,m)|r \leq \eta; \tau_1, \ldots, \tau_n] \prod_{j=1}^{n} z(\tau_j|r \leq \eta)\mathrm{d}\tau_1 \ldots \mathrm{d}\tau_n \tag{4}$$

where $z(\tau_j|r \leq \eta) = 1/(2\Delta K)$ and $E[N(n,m)|r \leq \eta; \tau_1, \ldots, \tau_n]$ is given by (3).

Case 2: $r > \eta$. For a given customer with usage rate $r > \eta$, the 2-D warranty period would terminate at time U/r, when the corresponding usage reaches its limit U. Moreover, the jth age-based PM instant τ_j is a random variable with conditional pdf $z(\tau_j|r > \eta) = r/(2\Delta L)$, $\tau_j \in [(jL - \Delta L)/r, (jL + \Delta L)/r]$.

Conditioning on the jth PM instant τ_j, the virtual age after the jth PM activity is given by $v_j = \delta(m)\tau_j$, $j = 1, 2, \ldots, n$, with $\tau_0 = 0$ and $\tau_{n+1} = U/r$. Then, the conditional number of item failures during the warranty period is

$$E[N^{\mathscr{U}}(n,m)|r > \eta; \tau_1, \ldots, \tau_n] = \int_0^{\tau_1} \lambda(t|r)\mathrm{d}t + \sum_{j=1}^{n-1} \int_{\delta(m)\tau_j}^{\delta(m)\tau_j + \tau_{j+1} - \tau_j} \lambda(t|r)\mathrm{d}t + \int_{\delta(m)\tau_n}^{\delta(m)\tau_n + \frac{U}{r} - \tau_n} \lambda(t|r)\mathrm{d}t \tag{5}$$

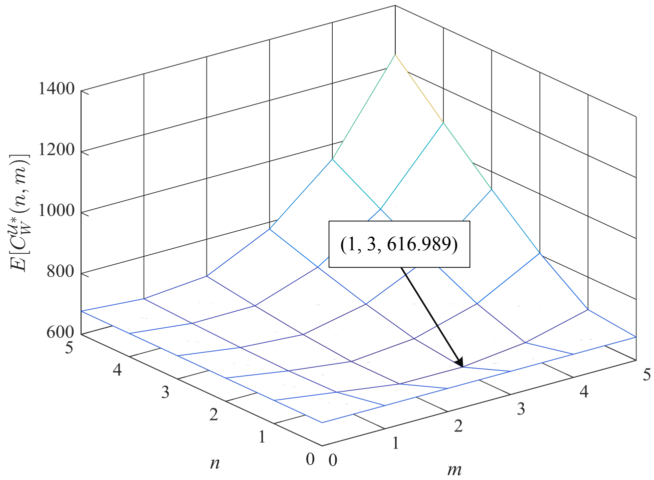

Figure2. Illustration of the existence of the optimal PM strategy.

Similarly, removing the conditioning on τ_j, the conditional number of item failures during the warranty period becomes

$$E[N^{\mathscr{U}}(n,m)|r>\eta] = \int_{\frac{nL-\Delta L}{r}}^{\frac{nL+\Delta L}{r}} \cdots \int_{\frac{L-\Delta L}{r}}^{\frac{L+\Delta L}{r}} E[N^{\mathscr{U}}(n,m)|r>\eta;\tau_1,\ldots,\tau_n] \prod_{j=1}^{n} z(\tau_j|r>\eta)\mathrm{d}\tau_1\ldots\mathrm{d}\tau_n \tag{6}$$

where $z(\tau_j|r>\eta) = r/(2\Delta L)$ and $E[N(n,m)|r>\eta;\tau_1,\ldots,\tau_n]$ is given by (5).

Finally, removing the conditioning on R, the total expected warranty servicing cost under the unpunctual PM strategy becomes

$$E[C_W^{\mathscr{U}}(n,m)] = c_f \int_0^{\eta} E[N^{\mathscr{U}}(n,m)|r\le\eta]g(r)\mathrm{d}r + c_f \int_{\eta}^{\infty} E[N^{\mathscr{U}}(n,m)|r>\eta]g(r)\mathrm{d}r + nC_p(m) \tag{7}$$

where c_f is the expected cost of a minimal repair, and $C_p(m)$ is the expected PM cost with level m. Generally, $C_p(m)$ is an increasing function of m, $0 \le m \le M$.

When the conditional failure rate $\lambda(t|r)$ is given by (1), then (6) can be expressed as:

$$\begin{aligned} E\left[C_W^{\mathscr{U}}(n,m)\right] = c_f &\left\{ \begin{array}{l} \left[(1+n\delta(m))\dfrac{W^2}{2(n+1)} + \dfrac{1}{3}(1-\delta(m))n(\Delta K)^2\right] \times \\ \left[\theta_2 G(\eta) + \theta_3 \displaystyle\int_0^{\eta} rg(r)\mathrm{d}r\right] + \theta_0 W G(\eta) + \theta_1 W \displaystyle\int_0^{\eta} rg(r)\mathrm{d}r \end{array} \right\} \\ + c_f &\left\{ \begin{array}{l} \left[(1+n\delta(m))\dfrac{U^2}{2(n+1)} + \dfrac{1}{3}(1-\delta(m))n(\Delta L)^2\right] \times \\ \left[\theta_3 \displaystyle\int_{\eta}^{\infty} \dfrac{1}{r} g(r)\mathrm{d}r + \theta_2 \displaystyle\int_{\eta}^{\infty} \dfrac{1}{r^2} g(r)\mathrm{d}r\right] + \theta_1 U \bar{G}(\eta) + \theta_0 U \displaystyle\int_{\eta}^{\infty} \dfrac{1}{r} g(r)\mathrm{d}r \end{array} \right\} + nC_p(m) \end{aligned} \tag{8}$$

From (8), it is easily known that when $\Delta K = 0$, $E[C_W^{\mathscr{U}}(n,m)] = E[C_W^{\mathscr{P}}(n,m)]$, where $E[C_W^{\mathscr{P}}(n,m)]$ represents the expected warranty cost under the punctual PM strategy.

Under both PM strategies, the optimization problem is to identify the optimal PM parameters, i.e., $n^{\omega*}$ and $m^{\omega*}$, over the warranty period to minimize $E[C_W^{\omega}(n,m)]$, where $\omega = \{\mathscr{U},\mathscr{P}\}$. Mathematically, it can be expressed as follows:

$$min\; E[C_W^{\omega}(n,m)] s.t. 0 \le m^{\omega} \le M, n^{\omega} \in Z^+ \cup \{0\}, \omega = \{\mathscr{U},\mathscr{P}\} \tag{9}$$

3. Case study

In this section, a case study is given to illustrate the implementation of the proposed unpunctual PM strategy. Consider a Chinese truck manufacturer whose trucks are sold with a 2-D warranty of 3 years and 400000 km (He et al. 2017b). Based on warranty data analysis, the usage rates r (in 100000 km/year) of 2016-mode

Table 1. Sensitivity analysis

c_f	Punctual PM			Unpunctual PM											
				ΔK = 1 weeks				ΔK = 2 weeks				ΔK = 4 weeks			
	$n^{\mathscr{P}*}$	$m^{\mathscr{P}*}$	$E[C^{\mathscr{P}*}]$	$n^{\mathscr{U}*}$	$m^{\mathscr{U}*}$	$E[C^{\mathscr{U}*}]$	Ψ	$n^{\mathscr{U}*}$	$m^{\mathscr{U}*}$	$E[C^{\mathscr{U}*}]$	Ψ	$n^{\mathscr{U}*}$	$m^{\mathscr{U}*}$	$E[C^{\mathscr{U}*}]$	Ψ
50	1	2	339.373	1	2	339.375	0.01‰	1	2	339.383	0.03‰	1	2	339.414	0.12‰
100	1	3	646.277	1	3	646.284	0.01‰	1	3	646.305	0.04‰	1	3	646.387	0.17‰
150	2	3	936.574	2	3	936.595	0.02‰	2	3	936.657	0.09‰	2	3	936.904	0.35‰
200	2	3	1208.765	2	3	1208.793	0.02‰	2	3	1208.875	0.09‰	2	3	1209.206	0.36‰
250	2	3	1480.957	2	3	1480.991	0.02‰	2	3	1481.094	0.09‰	2	3	1481.507	0.37‰
300	3	3	1750.306	3	3	1750.368	0.04‰	3	3	1750.554	0.14‰	3	3	1751.298	0.57‰
350	3	3	2012.024	3	3	2012.096	0.04‰	3	3	2012.313	0.14‰	3	3	2013.181	0.57‰
400	3	3	2273.741	3	3	2273.824	0.04‰	3	3	2274.072	0.15‰	3	3	2275.064	0.58‰
450	3	3	2535.459	3	3	2535.552	0.04‰	3	3	2535.831	0.15‰	3	3	2536.947	0.59‰
500	3	4	2790.654	3	4	2790.771	0.04‰	3	4	2791.122	0.17‰	3	4	2792.528	0.67‰

trucks follow a Weibull distribution with scale parameter $\alpha = 1.5259$ and shape parameter $\beta = 2.1062$. The minimum and maximum usage rates are 0.0365 and 5.65, respectively, which are directly obtained through the warranty claims data. Moreover, the parameters in the failure rate function (1) are estimated as $\theta_0 = 0.9558$, $\theta_1 = 0.4603$, $\theta_2 = 0.3427$ and $\theta_3 = 0.6085$. Also, the average repair cost of each failure is approximately \$95. Furthermore, we assume that the PM cost is $C_p(0) = \$0, C_p(1) = \$10, C_p(2) = \$30, C_p(3) = \$60, C_p(4) = \$100$ and $C_p(5) = \$160$, as in Kim et al. (2004). We further assume that the manufacturer pre-specifies $\Delta K = 2$ weeks (i.e., $1/26$ year) for the unpunctual PM strategy. This means that customers are entitled to have a 4-week tolerable range of maintenance unpunctuality.

Here, simple numerical search approach is adopted to determine the optimal PM parameters, since the number of PM actions, n, and the corresponding PM level, m, are integers. Based on the parameters above, the optimal number and level of PM actions under the unpunctual strategy are solved as $n^{\mathscr{U}*} = 1$ and $m^{\mathscr{U}*} = 3$, respectively, and the corresponding expected warranty cost is $E[C_W^{\mathscr{U}*}] = \$616.989$; while under its punctual counterpart carrying out $n^{\mathscr{P}*} = 1$ PM actions with level $m^{\mathscr{P}*} = 3$ is also the optimal strategy, and the resulting warranty cost is expected to be $E[C_W^{\mathscr{P}*}] = \$616.963$, which is slightly lower than \$616.989. This shows that the unpunctual PM strategy will lead to lightly higher warranty cost than the punctual strategy in this case. To obtain more insights, we perform a sensitivity analysis with respect to c_f and ΔK, as shown in Table 1. Note that, $\Psi = (E[C_W^{\mathscr{U}*}] - E[C_W^{\mathscr{P}*}])/E[C_W^{\mathscr{P}*}]$ represents the percentage growth of warranty cost of the unpunctual PM strategy in comparison to its punctual counterpart.

As can be seen, for both PM strategies, as the minimal repair cost c_f increases, the optimal number of PM actions, the optimal PM degree, and the total expected warranty costs increase, as to be expected. Moreover, under the unpunctual PM strategy, the total expected warranty cost increases as ΔK increases. This is reasonable since more serious unpunctuality behaviors will result in more warranty servicing cost. Furthermore, it is interesting to obverse that the warranty cost growth of the unpunctual PM strategy is very small (less than 1‰) in all cases. This implies that the unpunctual PM strategy may be of practical interest to the manufacturers given its potential of boosting future sales by increasing customer satisfaction. Actually, this unpunctual PM strategy has been adopted by a Chinese automaker (SAIC-GM-Wuling Automobile Company) in their warranty and maintenance practice.

4. Conclusions

This paper investigates an unpunctual PM strategy for repairable items sold with a 2-D warranty, under which the jth PM activity should be performed at age $jK \pm \Delta K$ or at usage $jL \pm \Delta L$, whichever occurs first. The unique characteristic of the proposed unpunctual PM strategy is that it allows customers to advance or postpone the scheduled PM activities in tolerable ranges, which has great potential to increase the customer satisfaction. Case study and sensitivity analysis show that applying the unpunctual PM strategy leads to slightly higher warranty cost than the punctual strategy. Nevertheless, the increase of warranty cost due to the introduction of unpunctual PM strategy is not significant in most cases. In view of its possibility of boosting future sales by increasing customer satisfaction, it is therefore of potential interest for manufacturers to adopt this PM strategy.

In this study, the optimal PM parameters and the corresponding warranty cost under the unpunctual PM strategy is almost the same as those under the punctual strategy. This may be due to our assumption of uniform distribution of unpunctual PM instants τ_i, $j = 1, 2, \ldots, n$. Investigating the impact of other distributions on the optimal PM parameters and the corresponding warranty cost will be our future research topic.

References

He K, Maillart LM, Prokopyev OA (2017a) Optimal planning of unpunctual preventive maintenance. *IISE Transactions*, 49, 127–143.

He S, Zhang Z, Zhang G, He Z (2017b) Two-dimensional base warranty design based on a new demand function considering heterogeneous usage rate. *International Journal of Production Research*, 55, 7058–7072.

Huang Y-S, Huang Ch-D, Ho J-W (2017) A customized two-dimensional extended warranty with preventive maintenance. *European Journal of Operational Research*, 257, 971–978.

Kim CS, Djamaludin I, Murthy DNP (2004) Warranty and discrete preventive maintenance. *Reliability Engineering & System Safety*, 84, 301–309.

Kurata H, Nam S-H (2013) After-sales service competition in a supply chain: Does uncertainty affect the conflict between profit maximization and customer satisfaction?. *International journal of production economics*, 144, 268–280.

Murthy DNP, Wilson RJ (1991) Modelling two-dimensional warranties. *Proceedings of the Fifth International Symposium on Applied Stochastic Models and Data Analysis*, pp.481–492.

Su C, Wang X (2016) A two-stage preventive maintenance optimization model incorporating two-dimensional extended warranty. *Reliability Engineering & System Safety*, 155, 169–178.

Wang X, Su C (2016) A two-dimensional preventive maintenance strategy for items sold with warranty. *International Journal of Production Research*, 54, 5901–5915.

Wang X, Xie W (2018) Two-dimensional warranty: A literature review. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, DOI: 1748006X17742776.

Wang Y, Liu Z, Liu Y (2015) Optimal preventive maintenance strategy for repairable items under two-dimensional warranty. *Reliability Engineering & System Safety*, 142, 326–333.

Ye Z-S, Murthy DNP, Xie M, Tang L-C (2013) Optimal burn-in for repairable products sold with a two-dimensional warranty. *IIE transactions*, 45, 164–176.

Zhao X, Al-Khalifa KN, Hamouda AM, Nakagawa T (2017) Age replacement models: A summary with new perspectives and methods. *Reliability Engineering & System Safety*, 161, 95–105.