Dynamic imperfect condition-based maintenance for systems subject to nonlinear degradation paths

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Abstract. This paper presents a maintenance optimization framework for systems suffering from nonlinear continuous degradation. The inspection interval is dynamically determined by the historical system conditions. We model the degradation path as a nonlinear Wiener process with time-varying drift parameter. Techniques to predict remaining useful life (RUL) is utilized to optimize maintenance policy by minimizing the expected cost rate. The effect of imperfect maintenance is assumed to be random in the sense that the maintenance action reduces the systems degradation level by a random proportion described by a beta distribution. Two thresholds on the degradation are determined for the preventive imperfect maintenance and perfect replacement, respectively. We evaluate expected cost rate using Monte Carlo simulation. A dataset from the real-world example is used to provide the pilot parameters as input for the optimization maintenance policies. Afterward, numerical examples are presented to illustrate the proposed method.

1. Introduction

With the rapid development of technologies to monitor and measure system deterioration, condition-based maintenance (CBM) is playing a more important role to maintain systems that suffer measurable degradations (Lu and Meeker, 1993). To facilitate condition-based maintenance, maintenance operations are scheduled based on the online measurements of quality characteristics, such as degradation level, instead of following a fixed schedule or age policy. For modern manufacturing systems, the planning of maintenance based on online status is of great importance to ensure production efficiency and products' quality (Zied et al., 2011; Beheshti Fakher et al., 2017).

The advance in sensing and inspecting technology significantly facilitates the lifetime prediction via system quality characteristics. Degradation models have drawn significant attention from both researchers and engineers to analyze highly reliable degrading systems. Two common types of degradation models are general path models and stochastic process models (Ye and Xie, 2015; Zhai and Ye, 2017). Under the degradation models, CBMs are the policies to schedule maintenance actions based on a system's actual degradation level which is measured under a certain inspection policy. Stochastic degradation models have been widely used in CBM optimization because of their clear physical explanations and tractable mathematical properties. For an overview, one can refer to Alaswad and Xiang (2017).

Although there exist numerous studies that address the maintenance optimization problems from various aspects, most of them have assumed linear degradation models. However, in real applications, it is found that many degradation paths have nonlinear behaviors. For example, as described in Si et al. (2012), nonlinear degradation characteristics widely exist in electronic devices, such as LED lamps and carbon-film resistors. Further, a very common assumption adopted in literature on maintenance optimization is that inspection is scheduled periodically (Liu et al., 2017; Chen et al., 2015). Nonlinearity in degradation paths makes the periodic inspections inappropriate since the mean degradation increments between two inspections can be drastically different, which brings more risks of system failure. Thus, non-periodic inspections are preferred in this scenario. Do et al. (2015) proposed a proactive maintenance policy with the aid of remaining useful life (RUL), and the inspection interval was dynamically determined based on the system status.

Another concern in maintaining systems is the choice between repair and replacement. More often than not, repairs are imperfect (Pham and Wang, 1996). On the one hand, from the perspective of degradation, an imperfect repair can generally reduce the degradation level though cannot make the system as-good-as-new. On the other hand, replacements are usually conducted with new systems thus reset the system degradation to the initial status that is the same with new systems. A tradeoff is that replacement is better in the extent of improvement but it is generally more expensive than imperfect repairs. Decision makers need to determine whether to repair or replace a system upon a maintenance action to save more costs.

In this paper, we present a RUL-based dynamic maintenance policy for systems subject to nonlinear degradation paths. Two maintenance thresholds are determined for repairs and replacements. Optimal maintenance policy is obtained by Monte Carlo simulation. The remainder of the paper is organized as follows. Section 2 describes the nonlinear Wiener process for reliability modeling. The optimization of maintenance policy is presented in Section 3 in detail. Section 4 presents a numerical example to illustrate the methods. Finally, Section 5 concludes the paper.

2. Nonlinear Wiener process for degradation modeling

Wiener process is widely used to model non-monotone degradation paths (Li et al., 2017; Ye et al., 2013). A well-adopted expression of drifted Wiener process $\{X(t), t > 0\}$ is given as follows:

$$X(t) = \beta\Lambda(t) + \sigma B\big(\Lambda(t)\big), \tag{1}$$

where $\beta > 0$ and $\sigma > 0$ are the drift and diffusion parameters, respectively, and $B(\cdot)$ is the standard Brownian motion. Further, $\Lambda(\cdot)$ is a parametric transformation function that describes the nonlinearity of the degradation path. For example, $\Lambda(t) = t$ implies that the expected degradation path is linear. Note that $\Lambda(\cdot)$ can be in various expressions, such as $\Lambda(t) = t^b, \Lambda(t) = \exp(t)$ and $\Lambda(t) = \log at$. Under the current degradation model, the failure time of a system is typically determined by the first passage time (FPT) of $Y(t)$ to a specific failure threshold L. The system failure time, denoted by T, follows an inverse Gaussian (IG) distribution of which the density function is given by

$$F_T(t; \beta, \sigma, b, L) = \Phi\left\{\sqrt{\frac{L^2}{\sigma^2\Lambda(t)}}\left[\frac{\beta\Lambda(t)}{L} - 1\right]\right\} + \exp\left(\frac{2\beta L}{\sigma^2}\right)\Phi\left\{-\sqrt{\frac{L^2}{\sigma^2\Lambda(t)}}\left[\frac{\beta\Lambda(t)}{L} + 1\right]\right\}. \tag{2}$$

3. A two-threshold maintenance policy with dynamic inspection intervals

3.1 *Assumptions*

Consider a single-component system of which the degradation can be well modeled a parametric nonlinear Wiener process as in Eq. (1). The following assumptions are adopted in this work:

- The failure is the system is assumed to be hidden. In other words, system failures can only be revealed by inspections.
- The initial system degradation is zero. The assumption also applies to systems with non-zero but identical initial degradation levels. Further, when a system is replaced, the new system also has a degradation level of zero.
- Replacement and repairs can only be carried out upon inspections, and their durations can be neglected compared to the system life.
- The degradation rate of the system is not influenced by maintenance at any time.
- The system is discretely inspected at time epochs $T_1, T_2, \ldots$, and the inspection intervals are dynamically determined. For convenience, we let $\Delta T_i = T_i - T_{i-1}$, where $i = 1,2, \ldots$ and $T_0 = 0$.

3.2 *Imperfect repair with random improvement factor*

We propose a random improvement factor to describe the effect of repair actions. Let X_i be the observed degradation level at the time T_i, i.e., the ith inspection. If a repair is conducted on the system at T_i, the system degradation level after the repair, denoted by X_i^+. We have

$$X_i^+ = \alpha_i X_i, \tag{3}$$

where $0 < \alpha_i < 1$ is the improvement factor of the imperfect repair. To characterize the randomness in the effect of improvement, we assume that $\alpha_i, i = 1,2, \ldots$ follow i.i.d. beta distributions with parameters u and v (Zhang et al., 2015), and the density function is given by

$$f_{\alpha_i}(z; u, v) = \frac{\Gamma(u+v)}{\Gamma(u)\Gamma(v)} z^{u-1}(1-z)^{v-1}, \quad 0 \le z \le 1. \tag{4}$$

When $\mu > v$, the distribution is left skewed and the mean is greater than 0.5. the Note that the random improvement factor only applies to repairs, and replacement will reset the degradation level to zero. Figure 1 shows various shapes of the density curves of beta distribution under different parameters.

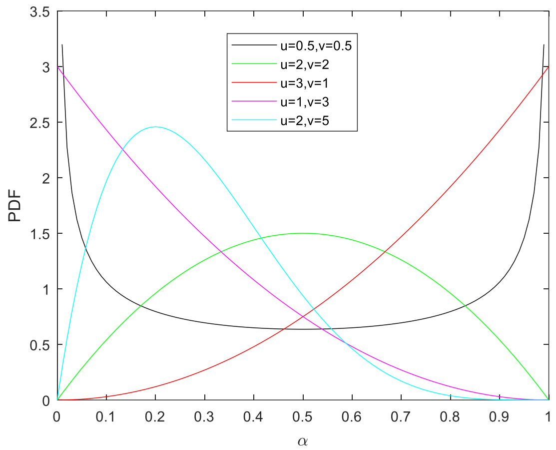

Figure 1. Various shapes of density functions for beta distributions

3.3 *Maintenance policy under dynamic RUL-based inspection intervals*

As part of the maintenance policy, inspection schedules are of great importance to reduce the risks of system failures and maintenance cost. We adopt the RUL-based inspection policy proposed by Do et al. (2015). Specifically,

$$\Delta T_i = \Delta T_i(Q, X_{i-1}) = \{\Delta t: \Pr(T - T_{i-1} \leq \Delta T | X_i) = Q\}, \tag{5}$$

where

$$\begin{aligned} &\Pr(T - T_{i-1} \leq \Delta T | X_i) = F_T'(\Delta T; \beta, \sigma, b, L - X_{i-1}) \\ &= \Phi\left\{\sqrt{\frac{L^2}{\sigma^2[\Lambda(T_{i-1}+\Delta T) - \Lambda(T_{i-1})]}}\left[\frac{\beta[\Lambda(T_{i-1}+\Delta T) - \Lambda(T_{i-1})]}{L} - 1\right]\right\} \\ &+ \exp\left(\frac{2\beta L}{\sigma^2}\right)\Phi\left\{-\sqrt{\frac{L^2}{\sigma^2[\Lambda(T_{i-1}+\Delta T) - \Lambda(T_{i-1})]}}\left[\frac{\beta[\Lambda(T_{i-1}+\Delta T) - \Lambda(T_{i-1})]}{L} + 1\right]\right\}. \end{aligned} \tag{6}$$

$Q \in (0,1)$ is the probability that the system fails between T_{i-1} and T, thus, generally we assign a relatively small value. We define the following types of cost that can be incurred to the system: inspection cost C_I; preventive replacement cost C_{PR}; repair cost C_R; corrective replacement cost per unit of time C_{CR}. The two-threshold maintenance policy has three decision variables to be determined by the planners, Q, M_1 and M_2, where M_1 and M_2 are the thresholds for repair and replacement. Upon each inspection, if the degradation level is smaller than M_1, the system will continue to operate until the next inspection; if the degradation level is larger than M_1 while smaller than M_2 , the system is subject to imperfect repair; otherwise, the system is replaced immediately.

The cumulative maintenance cost induced to the system by time, denoted by $C(t; Q, M_1, M_2)$, can be expressed by

$$C(t; Q, M_1, M_2) = C_I N_I(t) + C_R N_R(t) + C_{PR} N_{PR}(t) + C_{CR} N_{PR}(t), \tag{7}$$

where $N_I(t)$, $N_R(t)$, $N_{PR}(t)$ and $N_{CR}(t)$ are the number of inspections, imperfect repairs, preventive replacements and corrective replacement. The asymptotic cost rate can be obtained by

$$CR(Q, M_1, M_2) = \frac{E[C(H; Q, M_1, M_2)]}{E(H)}, \tag{8}$$

where H is the length of the renewal cycle, i.e., the time at which the system is replaced for the first time. We employ Monte Carlo simulation to evaluate the $CR(Q, M_1, M_2)$ for various combinations of Q, M_1 and M_2, denoted by maintenance policy $\mathcal{M}$. Note that the distribution function in Eq. (6) reflects that $T - T_{i-1} \leq \Delta T | X_i$ also follows inverse Gaussian distribution.

4. Numerical example

In this section, we use a numerical example similar to that in Ye et al. (2013). In the example, the wear of magnetic heads of HDDs is modeled as a degradation characteristic by Wiener process. The degradation path is deemed to be nonlinear. The following preliminary parameters are given for the maintenance optimization:

$$L = 10, \beta = 0.133, \sigma = 0.0627, \Lambda(t) = t^{1.518}, \alpha_i \sim \text{Beta}(3,2). \tag{9}$$

Additionally, the cost vector is given by $C = (C_I, C_R, C_{PR}, C_{CR})' = (10,50,70,100)'$. For various combinations of Q, M_1 and M_2, we calculate the asymptotic cost rate to obtain the optimal maintenance policy. Table 1 shows part of the results.

Table 1. Asymptotic cost rate of various $\mathcal{M}$

$\mathcal{M}$			$CR(\mathcal{M})$
Q	M_1	M_2	
0.02	3	5	2.2675
0.02	3	6	2.2688
0.02	3	7	2.2682
0.02	4	6	2.2674
0.02	4	7	2.2680
0.02	4	8	2.2957
0.02	5	7	2.2681
0.02	5	8	2.2964
0.02	5	9	2.7371
0.05	3	5	2.2538
0.05	3	6	2.2532
0.05	3	7	2.2547
0.05	4	6	2.2543
0.05	**4**	**7**	**2.2505**
0.05	4	8	2.2590
0.05	5	7	2.2513
0.05	5	8	2.2631
0.05	5	9	2.5777
0.10	3	5	2.2562
0.10	3	6	2.2588
0.10	3	7	2.2584
0.10	4	6	2.2589
0.10	4	7	2.2543
0.10	4	8	2.2642
0.10	5	7	2.2564
0.10	5	8	2.2617
0.10	5	9	2.4799

From the results, we can observe that the optimal maintenance policy is close to $\mathcal{M} = (Q, M_1, M_2)' = (0.05, 4, 7)'$. To obtain a more accurate solution, a numerical search around $\mathcal{M} = (0.05, 4, 7)'$ is conducted, and $\mathcal{M}^* = (0.073, 4.3, 7.4)'$is the global optimal solution for the maintenance policy. It is noted that further sensitivity analyses are needed to investigate the robustness of the optimal maintenance policy.

5. Conclusions

In this paper, we present a condition-based maintenance policy with two maintenance thresholds and dynamic inspection policy. Preliminary analytical results are given to facilitate the Monte Carlo simulation to search for the optimal maintenance policy. A simplistic numerical example from previous literatures is revisited to illustrate the proposed method.

The main challenge in optimizing the maintenance policy lies in the computational burden induced by Monte Carlo simulation. Thus, it is of great interest to derive the analytical form of the asymptotic cost rate for the problem.

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