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Invited Talk - The Emerging Discipline of Theoretical History

Dr Marc Widdowson (Amarna Ltd)

In the natural sciences, we are used to the idea that we can plot data on a graph and get a definite pattern like a straight line, and that the way to study this is through mathematics. It is generally thought that social science is too fuzzy and context-dependent for this kind of thing, but that assumption may be wrong. Growing numbers of researchers are finding mathematical patterns in historical and archaeological data, suggesting that societies are governed by deep underlying principles and creating the possibility of a discipline of theoretical history that can help interpret the forces and dynamics shaping our world today. This talk will present some of the surprising regularities in how societies behave, and will show how they are being understood through compact, analytical mathematical models. It will then discuss how these insights from history and archaeology shed light on contemporary questions affecting security such as: the influence of increasing global connectedness on development and growth; thresholds of conflict and instability; and the social adjustments caused by a disruptive technology like artificial intelligence. It is hoped that more people with mathematical skills will take an interest in this area, leveraging the increased sophistication of social theory, the new conceptual tools for studying complex systems, and the massive expansion of behavioural datasets from online activity, to produce a revolution in our ability to study and reason about collective human behaviour, the need for which is increasingly obvious.

Tomographic Non-Destructive Testing of Manufactured Components using Bimodal Convolutional Deep Belief Network for Sinogram Enhancement

Emilien Valat, Thomas Blumensath (Institute of Sound and Vibration Research, University of Southampton)

When manufacturing high value components, non-destructive imaging techniques have to be used to guarantee component integrity and manufacturing standards.

X-ray computed tomography (XCT) is a powerful non-destructive imaging technique that uses x-rays to generate 3D images of the internal structure of an object. However, when imaging objects with complex external geometries or objects manufactured from materials that significantly absorb x-rays, then current techniques do often not provide good images as not all of the required measurements can be taken. This then means that the computational inverse problem that computes the 3D image from the x-ray data becomes significantly ill posed.

However, in many cases, prior knowledge about a manufactured object is available. In this project, we want to harness information from Computer Assisted Design (CAD) data to enhance CT acquisitions by generating missing measurements. By doing so, one can hope to get either an image with a better resolution from a normally sampled acquisition or to match the existing resolution standards from an under-sampled acquisition.

Inspired by Lee et al (2009), we have developed a convolutional variant of the bimodal Deep-Belief Network (DBN) proposed by Ngiam et al (2011) that can be used to estimate missing information. Our architecture allows unsupervised feature extraction from a CT scan and its associated CAD data. In addition, it allows for joint learning of the extracted high-level features using an auto-encoder like supervised fine-tuning step. The finetuning focuses on interpolating evenly spaced missing acquisitions and extrapolating several consecutive acquisitions at random locations in the sinogram. The training is performed using transfer learning on data of increasing complexity.

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We compare the performance achieved by our bimodal convolutional DBN to other ML method (Li et al, 2019; Tovey et al, 2019). In addition, we study the quality of the reconstructions in cases where the CAD data is not used.

Our approach to sinogram inpainting is novel by several ways. Firstly, the use of prior knowledge about the object's CAD features is novel. Learning the joint distribution of an object and its CAD allows enhancement of both modalities and with fully connected neural networks, one can generate the CAD drawing given the scan data and vice-versa. Secondly, the network we developed is novel. To our knowledge, no approach uses the generative power of the energy-based models for sinogram inpainting. In addition, the use of convolutional layers in a bimodal DBN setting has not previously been explored.

Object Classification in Metal Detection using Spectral MPT Data

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The location and identification of hidden conducting security threats in metal detection is an important, yet challenging, inverse problem. Applications in security and defence include early detection of terrorist threats, finding landmines and locating unexploded ordnance. Current approaches to metal detection use simple thresholding and are incapable of determining an object's size, shape and material properties from the measurements of the perturbed magnetic field. Instead, in our approach, an asymptotic expansion of the perturbed magnetic field separates shape and material parameter dependence from inclusion position for small objects. The former is characterised by a small number of parameters through a magnetic polarizability tensor (MPT) [1,2].

This approach provides a model reduction of the metal detection inverse problem by reducing it to, separately, identifying an object's location, which can be done using a MUSIC algorithm, and then identifying information about the shape and material properties of the hidden object according to the measured MPT coefficients. In this context, exploiting the spectral behaviour of the MPTs [3], by taking measurements over a range of frequencies, is of particular interest.

For object identification from the measured MPT coefficients, we are interested in using a statistical based classification as it can provide end users with a level of confidence with the prediction. To inform the classifier, we will describe in the talk how a dictionary containing the spectral behaviour of the invariants of the MPTs for different objects can be computed using finite elements accelerated by a reduced order model [4]. We will also describe how knowledge of the scaling of MPT coefficients under parameter changes can be used to further accelerate this off-line computation.

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Detection of Fleeting Radio Signals in a Congested Spectrum

Dr. David J. Sadler (Roke Manor Research Ltd)

A compressive sensing based technique to decompose radio spectra into background signals of little interest and foreground, target signals of new emissions is presented. The algorithm is executed on-line so that data is processed immediately after it is generated in real-time, i.e. there is no significant latency whilst a large block of data is collected. The approach directly operates on wideband spectral data and is completely blind in the sense that no a priori knowledge of the target signals' time of appearance, centre frequency, bandwidth or modulation type is required.

The mathematics of the five fundamental stages of the algorithm are presented, and the mechanisms behind the operation of the algorithm are discussed. The proposed algorithm is inspired by signal processing methods for video streams used to separate foreground image layers, such as moving people and objects, from static background image layers. Training provides an initial basis for the subspace which contains the background signals. Orthogonal projection is used to extract a subspace which contains the foreground signals. Estimating the spectrum of the foreground signals then becomes a problem of sparse recovery in the presence of noise. A sparse solution is found by constrained $\ell 1$-norm minimization via convex optimization. Estimation of the background spectrum directly follows. Finally, a subspace update procedure is applied to permit the tracking of a time-varying background subspace so that the foreground (target) signals are detectable even in dynamic electromagnetic environments.

A number of experiments were completed using both simulated data and off-air data recorded within the high frequency band. The impact of various effects were explored: the density of both background and foreground signals, target signal-to-noise ratio, target signal duration, the proximity of a target signal to background emissions, and time-varying background signals. In all cases spectral decomposition has proven to be feasible. The demonstrated advantages of the proposed method are as follows:

• Robustness to the number and strength of background signals present.

• Operation when the foreground signals are not particularly sparse.

• Noise reduction for the foreground signals due to additive noise being assigned to the background.

• Ability to discern foreground signals which are close in frequency to background signals, even when they are partially overlapping the sidebands of a background signal.

• Operation for genuinely short duration signals.

• Ability to adapt to changing background signals whilst still separating the spectrum into foreground and background.

Multi-channel self-reset analog-to-digital converters for high dynamic range sensing.

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In many defense related applications (e.g., synthetic aperture radar imaging), saturation (or clipping) effects of analog-to-digital converters (ADC) pose great challenges when sensing a high-dynamic range complicated signal and/or when unexpected strong targets appear in a scene. They may lead to false targets identification and require sophisticated, highly sensitive automatic gain controllers (AGCs) or manual gain controllers. Over the past few years, a novel anti-saturation sampling framework called unlimited sampling has been developed using self-reset analog-to-digital converters (SR-ADCs) for high dynamic range digitization. Unlike conventional ADCs, an SR-ADC takes the modulo measurements of the signal. However, most of existing works focus on single-channel SR-ADC, which requires high sampling rates and the reconstruction could be very unstable at low signal to noise ratio.

In this work, we consider the problem of recovering a sparse signal from multi-channel modulo folded measurements, where each channel uses a different modulo size. Our work is based on recent developments of robust remaindering problem in number theory and sparse recovery in signal processing. Specifically, in the noiseless case, we show that perfect reconstruction can be achieved using only twice the number of samples in conventional compressive sensing measurements. Besides, by exploiting Chinese remainder theorem for real-numbers, we derive the optimal modulo sizes for a 2-channel SR-ADC system. For the noisy case, we first develop a low-complexity, robust lattice-based optimization for stable reconstruction for each sample. Then, we exploit extended l_1 optimization to recover the sparse signal and remove impulsive reconstruction errors. For noisy measurements, we develop a lattice-based optimization for stable reconstruction. As a quick demo, we present simulation results of multi-channel SR-ADC systems to sample sparse sinusoidal mixtures, which is a fundamental problem in many

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application areas including source localization and direction-of-arrival estimation. We show that compared with single-channel SR-ADC system, the proposed multi-channel systems can offer better performance at much lower sampling rates with much reduced number of samples. Simulation results are presented to demonstrate the effectiveness of the proposed algorithms. To our best knowledge, this is the first work that exploits multi-channel SR-ADCs for sensing of sparse signals.

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The Unit Re-balancing Problem

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We describe the problem of optimally re-balancing several military units distributed over a large geographic area of locally independent domains (such as islands). Each unit consists of three components: the number of people, their armor, and their equipment. A real number between 0 and 1 (representing 0% to 100%) describes the current status of each component. For each of the three components, a nonlinear function is introduced that converts the numerical status into an assessment, which is a real number, say, from 0 to 10, where zero is the weakest and ten is the strongest. It allows comparing the components between different units as well as with other components of the same unit. Based on this, we define the strength and the cost of each unit in the following way: The lowest assessment determines the strength, and the highest assessment determines the cost of a unit.

Over time, it turned out that some units became slightly unbalanced so that they are too costly and too inefficient at the same time. Now that military leaders identified this issue, they desire to move components between different units by transferring people and shipping material. The desired goal is to have units that are equally well equipped at the lowest possible cost. On a secondary level, the cost for the re-balancing should also be as low as possible. We describe a mixed-integer nonlinear programming formulation for this problem. This model formulation describes the potential movement of components between units as a multi-commodity flow at minimum cost. It is also possible to shut down a unit completely and re-distribute all its components to the others. Additional constraints identify the lowest and the highest assessment, where the nonlinear functions enter the model formulation. These functions are typically table-based piecewise-linear functions, and thus can be re-formulated by introducing additional binary variables and constraints to fit into the framework of mixed-integer linear programming. After such a transformation to a mixed-integer linear program, numerical standard solvers (such as Cplex and Gurobi) can find proven optimal solutions. This approach works well for up to 100 units. We present numerical solutions for a set of test instances and a bi-criteria objective function, demonstrating the trade-off between cost and efficiency.

Predicting the descent into extremism and terrorism

Richard Lane, H.M. State-Davey, W.J. Holmes, C.J. Taylor, A.J. Wragge (QinetiQ)

Increasingly, malicious activities are carried out online rather than in the physical world. Current intelligence processes dictate that individuals who pose a threat to security are manually selected based on actions or statements they make, before being monitored in more detail. The pool of such people is increasingly diverse, and there are a range of ways they promote propaganda through the internet, using blogs, social media, and videos. Their online and real-world behaviour can be analysed to determine their threat level, so that appropriate action can be taken. However, large data volumes mean that automated processes are required to assist analysts' understanding of risk and inform the actions of influence teams that endeavour to prevent harmful behaviour.

This paper proposes that analysts will use a language-independent system to track statements made by people over time and determine whether they are likely to be, or to become, involved in extremism or terrorism (Figure 1), as well as providing other forms of analysis. The proposed system comprises: online statement collation, a text-to-vector generator, machine learning (ML), a tracker, and a visualisation system. Text data from social media, forum posts, speech transcripts from video, or other sources is fed into the vector generator. The ML module uses manually labelled data and the vectors to learn a model of terrorist, extremist, and normal behaviour. A series of new unlabelled statements can then be processed and tracked. The ML module uses the tracker output to estimate whether the individual is currently or likely to be involved in extremism or terrorism.

The concept has been tested using quotes made by terrorists, extremists, campaigners, and politicians, obtained from wikiquote.org. A set of features were extracted for each quote using the state-of-the-art Universal Sentence Encoder, which produces 512-dimensional vectors. The Universal Sentence Encoder is an extension of word vectors, which encode relationships like "pilot is to airplane as driver is to car", for arbitrary length pieces of text. A 2D projection of labelled vector data using linear discriminant analysis (LDA) is shown in Figure 2. The data were used to train and test a support vector machine classifier using 10-fold cross-validation. The system was able to correctly detect intentions and attitudes associated with extremism 81% of the time and terrorism 97% of the time, on a dataset of 1578 quotes.

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Tracking techniques were used to perform a temporal analysis of the data. Each quote is considered to be a noisy measurement of a person's state of mind. Each person's "mind state" was tracked over time to obtain more accurate estimates of the current state and predicted trajectory than is possible with individual quotes. Evaluation of the tracking algorithms showed that they were able to detect both trends over time and sharp changes in attitude that could be attributed to major events. This analysis has been applied on an individual, group, and population level. Further group analysis showed that additional characteristics of the population can be determined, such as polarisation over particular issues and large scale shifts in attitude.
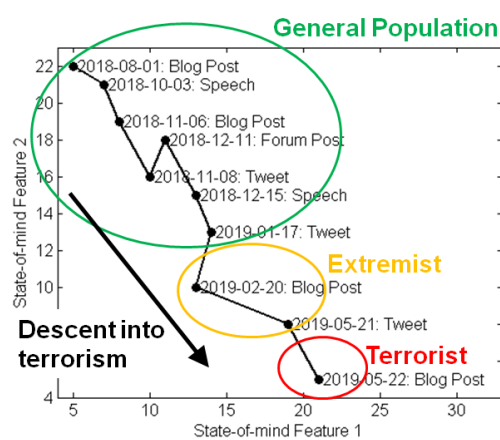

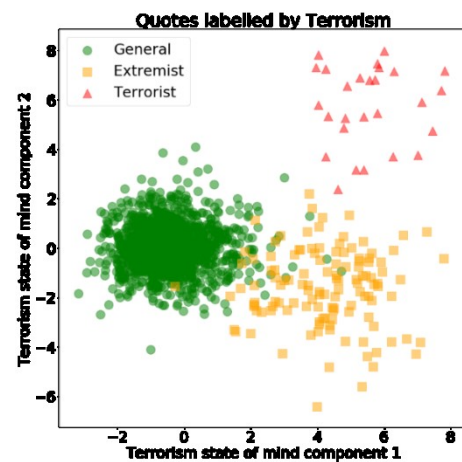Figure 1: System concept for an individual

Figure 2: LDA projection of labelled quotes for all people

SIER and Seers: Mathematics and Decision Making

Stephen Coulson (Changing Character of War Centre, Pembroke College, University of Oxford.)

The arrival of the Covid-19 pandemic to the UK in March 2020 led to what was probably an unprecedented demand, in peacetime, for modelling and forecasting capability across Whitehall and the NHS. As well as mathematicians and scientists within the Civil and Crown Service, many others from academia and industry supported the cross-government effort to tackle the virus. Some were called up as military reservists; others volunteered their time and expertise through the UK Research Council Covid portal[1]; or by joining informal, research groups to investigate different aspects of the epidemic. This paper considers the experience of some of these researchers, in the period between March and June 2020, to explore the relationship between mathematics and decision making.

The mathematics of decision making encloses a rather wide field that has been explored elsewhere. Here, rather than develop mathematical models of decision making process, we explore the interplay between mathematics and decision making: how mathematics informs decision makers, and how decision makers direct their mathematical requirements.

We first consider the application of compartmental models of infectious diseases to the Covid pandemic. Starting with the most generalised form of the Susceptible-Exposed-Infectious-Recovered (SEIR) model[2]. We consider the stability of solutions and their sensitivity to initial conditions, comparing results with previous researchers[3].

A simplified SEIR model, which only considers Susceptible-Infectious-Susceptible (SIS) populations is developed. This SIS model retains some of the non-linear features of the SIER model but permits analytical solutions. This is an example of the use of *toy models* to provide decision makers with a more intuitive understanding of model results than can be achieved through numerical or computational analysis, while still exhibiting many of the features of solutions of the SEIR model. It is found that the use of compartmental models in infectious diseases is analogous to the use of Lanchester modelling in combat. Many of the advantages and restrictions in the Lanchester approach[4] are also present in modelling of viruses.

To investigate how decision makers direct and frame their mathematical requirements, we draw a historical comparison between the use of mathematics by decision makers and the use of intelligence, based on a recent research[5] on the development of the National Security Council (NSC). We conclude that the main challenges are organisational rather than technical or data driven.

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MOTHS: Mathematics of Online Threats and Harms

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We are developing novel mathematical theories and algorithms within network science and related data scientific fields with immediate and transformative impacts in combatting online terrorism, radicalisation, grooming, scamming, hate speech and fake news.

Catalytic Mathematical Challenges

Over the past two decades the mathematical sciences have overseen a massive growth in the development of network theory, and its underpinning relationship to social systems. That theory was pulled along by waves of available data, from diverse applications: internet links, the growth of peer-to-peer social media, digital transactions, and mobile communications [1]. Both characterizing and reverse engineering networks, leveraging a variety of concepts and underlying structures, have proved most fruitful. Three seminal early paradigms were introduced about twenty years ago: scale-free models [2]; small world models [3]; and community "modular" structure models [4]. New ideas followed, including range-dependency [5]; core-periphery and stochastic block structure [6]; ever-more subtle community detection methods [7]; multiplex models, overlaying distinct networks on the same vertex set [8]; and trophic analysis [9]. Indeed, calibration of competing concepts to given data, enabling classification of observable structures, as well as rigorous contrasts between distinct networks, represents an important goal within many applications. Classical ideas about node rankings begat Google's PageRank and other forms of "spectral" centrality, at around the same time. More recently the generalisation of Katz (walk-based) centralities to non-backtracking-walk-centralities has enhanced both the elegance of the concept and the precision of algorithms at little extra computational cost [10].

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Such theoretical research typically applies new concepts to commonly understood datasets, allowing rigorous benchmarking. However, over-focussing on smallish niche examples (of little interest to real users) overlooks some very important challenges.

Accordingly , here we will address some aspects of the following challenges.

Observations at truly massive scales – so large as to render many current methods of calibration, structural inference, and decomposition, impractical.

The uncertain partial observation of networks, nodes and edges (whether due to practicalities, noise, or sampling) [12].

The presence of false positives and negatives – favouring spectral methods over path-following and/or spanning algorithms, sensitive to edge addition/deletion [13].

The need for development of local (node-based) and global (structural classifier) metrics within common frameworks.

Stochastic, dynamic evolution in online social systems, including inverse problems [14,15] and fully coupled social systems [16], where time-dependent states defined at the nodes control the existence of edges (through *homophily* [17]) while present edges allow interactions between adjacent nodes.

The need for "as-live" analysis, making inferences/decisions in close to real-time, as the observable networks evolve; with clear trade-offs betwixt efficiency and accuracy.

The need for algorithms to be applied in an agile way to new threat domain data – with common vocabulary and methodology so understandings may be shared across users.

Beyond the **meso-scale** network perspective there are two further *perspectives*, provoked by **micro-scale**, individual (nodal), behaviour within large social systems, and by **macro-scale** (whole network) structure, trophic web, and *shape*.

One important consequence of networking is the inducement of "behavioural state" changes at certain **micro**, *targeted,* nodes. The possible states at such nodes may be given by a latent discrete classification, each of which characterises certain time-dependent observable behaviour as possibly overlapping distributions over some feature space. This results in hidden Markov models (and generalisations), where some nodes (or subsets of nodes) must go through behavioural state-to-state transitions, to be inferred. This is suitable for a number of pressing

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applications [18]. For example, there is the question of the unreasonable potency of all online grooming: rather than simply a numbers game, this may result from the combination of decision-heuristics and online sampling biases.

At a **macro** level, computational topological data analysis (TDA), and various types of persistent homology, may be applied to data representing point clouds as networks, or else to some underlying networks themselves [19]. The goal is to classify the emergence of structure: the numbers of holes and voids of distinct classes that appertain at distinct scales (filtrations), in the face of high computational costs and a number of robustness (stability) issues for the data. Very little work has been done to date in applying these concepts to time-dependent dynamic networks, but see [20,21].

Many societal and social interactions, economic transactions and citizen-to-citizen communications have moved online to a significant degree. While this has received mass take-up by virtue of 24/7, everything-everywhere, global-online and especially mobile communications, it has presented many opportunities for those with malicious intentions, leaving regular citizens exposed to a number of distinct classes of online threats and potential harms. The appropriate response to such phenomena is not merely a question of increased automated monitoring technologies (that is, policing after the fact). Instead, new ideas, concepts, understandings, models and methods are required to anticipate, infer and forecast in the face of the agility and ingenuity of the perpetrators, at the vast scales of distinct threats.

We are interested in the following seven related fields.

Online grooming, such as radicalisation by extremists, support for terrorism or insurgencies (actual ISIS and AQ grooming manuals exist): this is relevant to UK counter-terrorism effort.

Online child grooming by paedophiles (again grooming manuals exist but possession is illegal): this is relevant to UK efforts in child protection.

Dating/romance scams. In 2016 there were almost 4,000 UK victims of romance fraud totalling £40M. There are common behavioural elements [22].

Other types of grooming where a behavioural-state approach using Hidden Markov Modelling has already been shown to be productive [18].

Echo chamber effects, notably in the international extreme right wing in [23]. This idea extends to other massive evolving communities, such as radical Islam, Irish nationalism and the far left.

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Hate speech, which pollutes discourse, inflicts harms, and exacerbates social divisions. This is addressed by a project at the Turing Inst (funded by UKRI SRF) [24]. We will collaborate with that.

Fake news: designed to undermine the activities of the state; or as conspiracy theories such as the antivax movement (we have a 150,000+ actor network); foreign interference with democratic processes; hostile state activities, such as Russia's responses to the Salisbury attack. Adali and collaborators employ network methods on news content sharing networks [25].

In the paper we shall discuss aspects of these seven use-cases in the light of the mathematical challenges identified above and give some specific examples drawn from recent work on peer to peer communications networks, at scale, derived from online platforms, and the development of problematic gambling behaviour, a kind of self-grooming and consequent addiction.

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Invited Talk - "I need a significant sample size" – Impact with Maths and Stats

Phillipa Spencer OBE (DSTL)

Tukey said that the beauty of being a statistician was getting to play in everyone's backyard. And this is true! Maths and statistics can have impact in any area. This talk is a brief overview of areas maths and statistics have had impact in defence, that I was involved in and the differing scientific language barriers that were overcome in order to have real impact on the problems. From analysing Stone Curlew breeding attempts to recovering Salisbury from the Novichok incident to assessing AI in autonomy, the variety of work undertaken in defence that one can be involved in can develop you, scientifically and professionally.

Invited Talk - Patterns and searches with Bayes.

Dr Sophie Carr (Bays Consulting)

Bayes theorem, which underpins Bayesian searching, was initially explained as a search problem (how to locate balls on a billiard table) and still provides a structured approach to incorporating multiple, often disparate, data sources and associated uncertainty to plan effective and efficient searches. Within the marine environment, one obvious application of Bayesian searching is that of national coast guards looking for those lost at sea. However, there is much wider applicability of both Bayesian searching, and more generally, Bayes theorem in the marine environment.

As autonomous vehicles shape the future of ocean surveys looking for resources such as hydrocarbons, Bayesian searching is increasingly used to ensure an efficient search pattern in a time critical situation which optimises the use of available information and maximises the probability of identifying the location of a seep.

But Bayes theorem isn't only applicable to searching for static seeps. Bayesian statistics can also help protect shellfish stocks by finding patterns in the trace element composition in the shells of the European Mussel as markers of provenance. Working across three harbours on the south coast of England, all were found to have a clear and distinct chemical "fingerprint" based on a combination of elements and this work has been extended to investigate the patterns in freshwater fish.

This talk will present case studies on the in-house research projects on both these areas and discuss the impact of Bayes theorem within searching and data fusion.

Mathematics in the Electromagnetic Environment

Emma Bowley (DSTL)

The UK needs to be able to operate in the electro-magnetic environment (EME) across all domains; space, air, land, sea and cyber. But operating in the EME is becoming increasingly complex as it becomes more congested and contested due to continuing growth in civilian and military demand for spectrum (for example 5G). Although current approaches to solving the challenges of operating in the EME have been successful over many years, this increasing complexity means it is now necessary to explore innovative approaches.

Mathematical research is a powerful way to find innovative solutions to real world problems. Opening up our challenges to mathematicians allows us to tap into a fantastic pool of talent for solving existing and emerging challenges for defence. Mathematicians abstract a challenge from its context and break it down to its essential components before applying general techniques, cutting to the heart of the problem. This enables well-established mathematical techniques from other domains to be applied in new ways, as well as developing new methods.

In this presentation we will show how we have used the power of mathematics for Electronic Warfare challenges. Working with PA Consulting and the Isaac Newton Institute we have held a series of workshops with academic mathematicians to explore challenges for defence in the EME. These have been very successful with around 40 mathematicians attending each event from over 20 different institutions. We are building a community of mathematicians across the UK who are engaged in the challenges we face for Electronic Warfare.

A deep learning approach for through-the-wall radar target tracking in 3D

Gabriele Incorvaia (Dorn Oliver Affiliation: Department of Mathematics, The University of Manchester), (postgraduate research supported by Dstl)

Through-the-wall radar imaging is an application which has gained much attention recently due to its broad applicability and underlying mathematical challenges.
The ability to distinguish the presence of targets of interest inside a building is helpful in rescue operations, hostage situations, surveillance activities etc. Generally, the identification of these targets relies on electromagnetic measurements collected by deploying antennas around the scenario of interest or, alternatively, mounted on drones. However, at the frequency regime of interest, the possible presence of highly scattered fields renders the underlying inverse scattering problem ill-posed and therefore difficult to solve.

In this through-the-wall framework, tracking moving targets in almost real time constitutes a challenging task. In principle, standard Bayesian tracking methods, e.g. Kalman filters and Particle filters, can be applied but they have a few drawbacks. Among them, we mention the following: first, they require solving the underlying inverse problem to approximately localize each object, which can be a quite complex operation in itself; second, they require often prior information on the expected motion in the form of a kinematic model. However, the definition of such a model might be problematic in practice. For example, following people walking, due to the arbitrariness of the possible trajectories, the formulation of a rigorous dynamic model might not even be possible.

We propose a novel data-driven tracking scheme based on deep learning aimed at overcoming the previous limitations. The idea is to use a combination of deep neural networks to map the measurements collected by the receivers directly to the target positions. Hence, by repeating this operation for a sequence of time steps, an estimation of the trajectories followed can be obtained.

The assessment of the reliability of the proposed approach is realized by performing numerical experiments in 3D, where the propagation of the electromagnetic waves is numerically schematized by implementing a finite differences frequency domain (FDFD) approximation of the Maxwell's equations. Although no prior dynamic assumptions are needed, we also present

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a Kalman filter-based generalization of the previous tracking method aimed at those applications where an expected motion model is available. Overall, the numerical results achieved confirm the reliability of our approach and make us confident of its applicability in practical situations.

Modelling and Efficient FEM for Thermo-Mechanical Problems in High Explosives

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This talk discusses efficient and reliable Finite Element Methods to simulate the thermo-mechanical response of high explosives. A key motivation is the modelling of the initiation of shear bands in materials such as HMX. The localised plastic deformation associated with a shear band leads to the formation of hot spots and can subsequently lead to thermal runaway and potentially serious consequences. To prevent and predict thermal runaway it is often typical practise to use standard finite element methods which struggle to accurately resolve the sharp gradients associated with these thermal and mechanical features which may lead to unphysical predictions of the dynamics within high explosives. The numerical methods presented in this talk aim to provide efficient and reliable tools towards modelling the initiation of shear banding and thermal runaway. We consider two approaches: adaptively generated meshes based on mathematically rigorous estimates of the numerical errors, and enriched finite elements. These methods are demonstrated for thermal and elastic problems respectively, as they arise in reduced models when either the thermal or mechanical dynamics can be eliminated in the modelling. We first present results based on adaptive finite elements for non-linear thermal problems. Steep temperature gradients are resolved by appropriate mesh refinement procedures. Steered by indicators for the accuracy of the solution, the algorithm automatically resolves hot spots on a refined mesh, significantly reducing computational costs, see for example [2]. Secondly, we consider space-time enriched finite elements (also known as generalised finite elements) which include a priori physical information into the approximation space. This a priori information could represent localised wave-like features. The modelling can effectively capture features occurring at different spatial and temporal scales [4, 5]. Here we consider a first order formulation of the wave equation [1] and choose plane-wave enrichments [6]. The results show that the space-time enriched method allows for accurate, fast simulation on coarse spatial meshes using large time steps, compared to standard methods. Propagating wave fronts may be efficiently resolved on scales much smaller than the mesh

size, aiding in the computational efficiency of any such scheme. Future work aims to address the full, coupled thermo-mechanical system, as well as to combine the adaptive and enriched approaches of Iqbal et al. [3].

Key words: *Generalized Finite Elements; Adaptive Finite Elements; Space-Time Methods; Plane-Wave Enrichment.*

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A Multiple-Sensor System Based on the Square-Root Decentralized Kalman Filter

Yuri A. Vershinin, (School of Mechanical, Aerospace and Automotive Engineering, Coventry University)

Data fusion techniques are used in many tracking and surveillance systems as well as in applications where reliability is of a main concern. The information from multiple sensors can be fused using the data fusion algorithms on local units. The decentralized Kalman filter can be employed to solve this task. However, the divergence of the filter can occur due to ill-conditioned quantities in finite word length or computational round-off errors. In order to overcome these difficulties the square-root factorization of the decentralized Kalman filter is proposed in this paper.

Mathematical Modelling of Shear Bands and Applications to the Safe Handling of High Explosives

David Torkington, G.; Lacey, A. A.; Gimperlein, H. (Maxwell Institute for Mathematical Sciences; Department of Mathematics, Heriot-Watt University; School of Mathematics, University of Edinburgh)

Under extreme shearing loads, many materials undergo intense deformation that is localised to narrow regions called shear bands. The localisation of the shear entails that the heating due to mechanical dissipation is in turn localised. As such, shear banding has been identified as a likely generator of hotspots, and has been empirically observed to cause local heating within explosives. Thus, from a safety standpoint, it is critically important to understand this phenomenon.

Numerical models of shear bands are challenging due to the extreme strain rates involved and the fine scale of the band. This motivates the present work, which develops an analytical model that contains the essential aspects of shear band formation and growth. The model rigorously computes the evolution of the shear band width with time and the variation of shear stress within the band as a function of space and time, thereby determining the local heating due to mechanical dissipation within the band as well. The obtained results predict these quantities and their dependence on material parameters, offering the opportunity for comparison against experiment.

We take a novel approach wherein the localised material softening that triggers shear banding, which is believed to be a thermal phenomenon in reality, is built into the rheology, through use of a bi-viscous rheology that reduces the material's viscosity once a threshold stress is reached. This acts to decouple the thermal and mechanical problems, thereby simplifying the mathematics and material modelling. This gives a parabolic free-boundary problem for the diffusion of shear stress, where regions in which stress exceeds the threshold are identified with shear bands in the model.

Precise asymptotic methods detect distinguished length and time scales whose existence are not *a priori* apparent – this is another strength of the asymptotic analysis over a direct numerical

method. The asymptotic analysis is continued at each of these spacetime scales, yielding precise expansions for both the evolution of the shear band width with time, and the variation of shear stress within the band as a function of space and time. The expansions involve key model parameters. As such, if empirical values for these constants are found for a given material, the model can predict the dynamics of the shear band geometry, the variation of shear stress within the band over time, and thus the variation of dissipated heat release throughout the band over time for that material.

Improvements on previous analytical work in the area are made by not presupposing the velocity field within the sample, but instead by solving for it using the momentum and constitutive equations, and by using a rheology that is not perfectly plastic (that is, in our model, shear stress is permitted to evolve within plastic regions – the shear bands). We expect good agreement of the model with empirical results, as soon as such data can be obtained. Moreover, we believe that such data could serve to highlight areas of improvement that initially might not have been apparent to mathematicians, thereby providing opportunity for further strengthening of the model.

Keynote Talk - "The quantum revolution is spawning innovations in mathematics offering new capabilities for defence and security."

Dr Ned Allen (Former Lockheed Martin Chief Scientist)

Quantum computers (QC) will not replace digital computers; they will extend analysis because classical circuits, rooted in deterministic logic, joined to quantum ones, rooted in the intrinsic uncertainties of nature, and tuned to mimic complex systems will "solve" by direct analog quantum simulation (DAQS) much that is not "solvable" today. Analogous to the unsettling discoveries that closed-form solutions are rare in the world of analysis, and that logically consistent and complete formal systems may not be possible (Gödel), the emerging character of quantum computing machines suggests, to me, that human thinking might be a small, parochial neighborhood of a larger firmament of thought and information. We are already using these brilliant new QC analog tools in quantum chemistry and nuclear chemistry. Co-evolving with the QC revolution, a new logic tool called Category Theory is emerging to map and guide our search for fruitful analogies between seemingly distinct and immiscible maths. In one important case, self-synchronization of coupled clocks, noticed by Christiaan Huygens three centuries ago, and the beta decay of radioisotopes studied by Enrico Fermi in the twentieth century are, considering modern theories of dynamics and chaos, mathematically homeomorphic. Category theory is extending simulation methods to ever-wider varieties of problems in powerful ways. One consequence of today's QC revolution, then, is that it is building further value into our High-Performance Computing Centers as they incorporate quantum components into their protocols and develop hybrid quantum-classical processing capabilities. But another, more profound, is that the QC may change the character, extent, and power of mathematics and thinking itself and the way we use it. There are many papers outlining the scope for computing when quantum and classical computers are joined because all of today's NISQ quantum machines (Noisy Intermediate Scale Quantum computers) are run by and incorporate pre- and post-processing from classical machines. The quantum revolution is not, then, in the emergence of purely quantum computers, but instead in the emergence of hybrid quantum-classical machines, quassical machines, not purely quantum ones and, more generally, into quassical ways of thinking. There is less discussion and less awareness, though, of the metamorphosis of maths and thought from their classical chrysalis into a beautiful new, quantum-enriched creature of contemplative understanding. Recounting that story and distilling the essence of that insight is my objective in this talk.

Poster Submissions

Biosurveillance and data fusion for early outbreak detection and classification:

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Riskaware, together with the University of Liverpool and Dstl, who also funded and managed the project, have developed an information fusion algorithm for bio-surveillance (BSV), which fuses data from multiple healthcare and non-healthcare sources, such as hospital data, primary healthcare provider data and social media data. The algorithm aims to alert healthcare providers to the presence of a disease outbreak and identify it before they become aware of it through traditional means, such as diagnostic testing. This will allow healthcare providers to respond faster and prevent the further spread of infection. The capability can consider infections that are endemic in the population, such as influenza, as well as those that could be the result of a biological attack, such as anthrax or pneumonic plague.

The algorithm combines a particle filter, an epidemiological model and a Bayesian network. The particle filter performs sequential Bayesian inference to provide a probabilistic estimate for the populations in the compartments of the epidemiological model, which is an extended SEIR (Susceptible, Exposed, Infected, Recovered) model, to infer whether there is an infection present in the population. Both Marginal and Fixed-Lag Sequential Monte Carlo (FLSMC) particle filters have been trialled within the system, with the FLSMC algorithm providing improved results by overcoming the problematic effects of "model-lag", where the signal is detected days after the initial outbreak. If an outbreak is present in the population, the algorithm estimates the probability of each considered infection being the source of the observed signal in the data. The Bayesian network is used to account for seasonal effects and long-term trends in the observations. Probabilistic forecasts can then be generated from the results of the algorithm, to determine the likely future progress of the outbreak.

Applications for the algorithm exist in both military and civilian scenarios and include detecting the effects of biological weapons, as well as the emergence of new strains of a communicable disease. The incorporation of a feature that allows for changing populations allows the algorithm to function correctly in the presence of large population changes due to troop deployment and recall.

The algorithm has already been tested on synthetic data and shown to be able to identify and track infected populations through the different stages of their infections. Testing is currently being conducted using real-world data obtained from Public Health Wales. Evaluation metrics used include probability of detection, probability of false alarm and conditional expected delay. Receiver operating characteristic (ROC) curves have been plotted to explore the effects of different parameters on algorithm performance.

Riskaware now has a licence agreement with Dstl to develop and distribute BSV.

Secure blockchain-based system for IoT devices implementing post-quantum cryptography

Mr. Bakhtiyor Yokubov, Dr. Lu Gan, Dr. Cong Ling

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Blockchain is a distributed ledger maintained by network nodes, which records transactions executed between nodes (in the form of messages sent from one node to another). Information inserted in the blockchain is public, and cannot be modified or erased. Smart contracts are self-executing contracts (generally saved on a blockchain) whose terms are directly written into lines of code. Blockchain infrastructure is built with several elements of network protocols, cryptographic concepts, and mining hardware. All these elements depend on each other in some sense. If we look into the layered architecture of blockchain, each layer is dependent on its upper and lower layers for some input/output. Thus, there are many infrastructure dependencies in blockchain. For instance, the data from the smart contract layer is an input to the transaction layer that outputs actual transactions; the data from the consensus layer results in an input to the network layer through a communication protocol; and the data from the network layer data is sent to the database through database storage management. These dependencies must be taken into account while building a comprehensive blockchain framework for any use case; otherwise, some of the blockchain functionalities will not be fulfilled. Blockchain technology has gained significant prominence in recent years due to its public, distributed, and decentration characteristics, which was widely applied in all walks of life requiring distributed trustless consensus. However, the most cryptographic protocols used in the current blockchain networks are susceptible to the quantum attack. Recent advances in quantum computing pose a severe threat to classical cryptography, as most of the widely used cryptography is based on the hardness of some problem which can be efficiently solved using quantum computers. Thus, research in the Post-Quantum cryptography has taken a massive leap. The security impact of breaking public key cryptography by quantum computers would be tremendous. Elliptic curve cryptography (ECC), which is an approach to public key cryptography, is mostly used in blockchain applications. Using a variant of Shor's algorithm, a quantum computer can easily forge an elliptic curve signature that underpins the security of each transaction in blockchain and so breaking of ECC will affect blockchain in terms of broken keys, hence, digital

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signatures. In our research, we first give an overview of the vulnerabilities of the modern blockchain networks to a quantum adversary and some potential post-quantum mitigation methods. Then, we implement secure blockchain system over existing classical channels for IoT devices based on a new lattice-based signature scheme. Furthermore, we construct new blockchain mechanism that has comparably efficient public key addresses and comparably small digital signatures as the currently used ones, but that is based on Post-Quantum cryptographic schemes.