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ABSTRACT BOOK

Contents

Gibbs ensemble for the Ablowitz-Ladik lattice, circular beta-ensemble and double confluent Heun equation

Professor Tamara Grava (University of Bristol, UK and SISSA, Italy)

We consider the discrete defocusing nonlinear Schrodinger equation in its integrable version, that is called Ablowitz Ladik lattice. We consider the Generalized Gibbs ensemble for the Ablowitz Ladik lattice. In this setting the random Lax matrix of the Ablowitz Ladik is related to the circulant beta-ensemble at high temperature. We obtain the free energy of the Ablowitz-Ladik lattice and the density of states of the random Lax matrix by establishing a mapping to the one-dimensional log-gas. The density of states is obtained via a particular solution of the double-confluent Heun equation. Joint work with Guido Mazzuca.

The linear and nonlinear instability of the Akhmediev breather

Petr Grinevich (L. D. Landau Institute for Theoretical Physics, Russia)

The Akhmediev breather (AB) and its M-soliton generalization, hereafter called AB_M, are exact solutions of the focusing NLS equation periodic in space and exponentially localized in time over the constant unstable background; they describe the appearance of M unstable nonlinear modes and their interaction. It is therefore important to establish the stability properties of these solutions under perturbations, to understand if they appear in nature, and in which form.

There is the following common believe in the literature: let the NLS background be unstable with respect to the first N modes; then i) if the M unstable modes of the AB_M solution are strictly contained in this set ($M In this paper we argue instead that the AB_M solution is always unstable, even in the saturation case $M=N$, and we prove it in the simplest case $M=N=1$. We prove the linear instability, constructing two examples of x-periodic solutions of the linearized theory growing exponentially in time. In the know literature these solutions were missed.

Complex geometric optics solutions to Dirac problems

Professor Christian Klein (Institut de Mathématiques de Bourgogne, UBFC, Dijon, France)

Complex geometric optics (CGO) solutions to d-bar problems appear in many applications, for instance in Electrical Impedance Tomography and in the scattering theory of integrable PDEs in two dimensions. We discuss various numerical approaches to construct CGO solutions to d-bar systems. Of particular interest is the limit of large values of the spectral parameter where the solutions have an essential singularity. This is work in collaboration with K. McLaughlin, J. Sjöstrand and N. Stoilov.

Dispersive Riemann problem for the Benjamin-Bona-Mahony equation

Thibault Congy, Gennady El, Mark Hoefer and Michael Shearer (Northumbria University, UK)

The Benjamin-Bona-Mahony (BBM) equation $u_t + uu_x = u_{xxt}$ as a model for unidirectional, weakly nonlinear dispersive shallow water wave propagation is asymptotically equivalent to the celebrated Korteweg-de Vries (KdV) equation while providing more satisfactory short-wave behavior in the sense that the linear dispersion relation is bounded for the BBM equation, but unbounded for the KdV equation. However, the BBM dispersion relation is nonconvex, a property that gives rise to a number of intriguing features markedly different from those found in the KdV equation, providing the motivation for the study of the BBM equation as a distinct dispersive regularization of the Hopf equation. The dynamics of the smoothed step initial value problem or dispersive Riemann problem for BBM equation are studied using asymptotic methods and numerical simulations.

Emergent wave phenomena for the dispersive Riemann problem can be roughly split into two categories: classical and nonclassical. Classical phenomena include dispersive shock waves and rarefaction waves, also observed in convex KdV-type dispersive hydrodynamics. Nonclassical features are due to nonconvex dispersion and include the generation of two-phase linear wavetrains, expansion shocks, solitary wave shedding, dispersive Lax shocks, DSW implosion and the generation of incoherent solitary wavetrains.

Reference: T. Congy, G. A. El, M. A. Hoefer and M. Shearer, “Dispersive Riemann problem for the Benjamin-Bona-Mahony equation,” arXiv:2012.14579 [nlin.PS]

The Dynamics of Waves in the Neighbourhood of the Benjamin-Feir Instability.

Daniel Ratliff (Northumbria University, UK)

The dynamics of dispersive nonlinear waves remains a problem that attracts significant interest, in part for their interesting stability properties. Arguably the most famous case is the Benjamin-Feir (BF) instability, where uniform wavetrains undergo a stability transition due to a nonlinear frequency correction term ω_2. This transition occurs when $\omega_0''\omega_2 = 0$, where ω_0 is the linear dispersion relation and primes denotes differentiation. There are several emergent behaviours, such as an increase in the wave’s wavelength (frequency downshifting) or resonant wave bursting, which have been observed but elude mathematical insight as to why they occur.

In order to understand such phenomena, we explore the wave dynamics of from the standpoint of Whitham modulation theory and phase dynamics, ultimately uncovering that each of the possible transitions (either $\omega_0'' = 0$ or $\omega_2 = 0$) admits a different set of nonlinear dynamics governing the wave quantities whose solutions can be used to understand the wave behaviours near the BF threshold. Moreover, this work illustrates the role of mean flow is significant and is central to the emergence of permanent frequency downshifting and localised wave bursts one observes. We use this reasoning to explain, at least qualitatively, the experimental observations from wave-tank experiments in water waves and fluid conduits.

Global bifurcation of capillary-gravity waves with overhanging profiles and arbitrary vorticity

Jörg Weber (Lund University, Sweden)

While the research on water waves modeled by Euler's equations has a long history, mainly in the last two decades traveling periodic rotational waves have been constructed with mathematical rigor by means of bifurcation theorems. In this talk, I will present a new reformulation of this traveling periodic water wave problem in two dimensions and in the presence of surface tension, gravity, and a flat bed. Using conformal mappings and a new Babenko-type reformulation of Bernoulli's equation, the problem is equivalently cast into the form "identity + compact", which is amenable for Rabinowitz' global bifurcation theorem. The main advantages of this new reformulation are that no simplifying restrictions on the geometry of the surface profile and no simplifying assumptions on the vorticity distribution (and thus no assumptions regarding the absence of stagnation points) have to be made. Within the scope of this new formulation, local and global solution curves, bifurcating from laminar flows with a flat surface, are constructed. Moreover, I will further discuss the condition for local bifurcation and the possible alternatives for "endpoints" of the global curve.

This is joint work with Erik Wahlén.

Breaking the snaking in natural doubly diffusive convection

Joanna Tumelty (University of Leeds, UK)

Fluids subject to gradients in both temperature and salinity can form a variety of patterns via doubly diffusive convection. Among these are spatially localised states of convection within a background of quiescent fluid, known as convectons. We consider these states in a vertical slot, where horizontal temperature and salinity variations provide competing effects to the fluid density while allowing the existence of a conduction state. In this configuration, convectons have been studied with specific parameter values and have been found to lie on a pair of secondary branches that undergo homoclinic snaking below the onset of linear instability. We extend our understanding of these localised states by presenting how the structure of the secondary branches change as the Prandtl number is reduced, and the primary bifurcation changes from being subcritical to supercritical. In particular, we show how interpreting the spatial dynamics of convectons helps to understand this complex behaviour, and leads to finding them even when the primary bifurcation is supercritical.

Existence of Localised Cellular Patterns

Dan Hill (University of Surrey, UK)

Localised cellular patterns - for example: hexagons, squares, rhomboids, - have been observed in a variety of experiments and numerical simulations. However, our analytical understanding of such structures remains extremely limited.

In this talk, we present analytical results regarding the existence of small-amplitude localised cellular patterns in the planar Swift-Hohenberg equation (SHE). We perform a truncated Fourier decomposition in order to approximate the planar SHE as a finite-dimensional system of nonlinear radial ODEs. Extending the theory of local invariant manifolds and geometric blowup coordinates for radial equations, we prove the existence of small-amplitude localised cellular patterns, subject to an algebraic matching condition. The matching condition depends on the truncation order N and the lattice m of our Fourier decomposition. Finally, we explicitly solve the small-truncation matching problem (for $N = 1; 2; 3$), and apply numerical continuation codes to find larger patches of localised cellular patterns.

Unstable water waves: a new Evans function approach

Professor Vera Hur (University of Illinois at Urbana-Champaign, USA)

I will discuss a new periodic Evans function approach for cylindrical domains, and its application to the spectral instability of Stokes waves in the finite depth. Numerical evidence suggests instability whenever the unperturbed wave is ‘resonant’ with its infinitesimal perturbation waves. This has not been studied analytically except the Benjamin-Feir instability, near the origin of the spectral plane, when (wave number)x(depth)>1.3627…. I will discuss an alternative proof of the Benjamin-Feir instability and, also, the first proof of spectral instability away from the origin, when 0.86430…<(wave number)x(depth)<1.00804…, elucidating the numerical findings. I will discuss capillary-gravity waves and Stokes waves in constant vorticity flows.

Global existence results for the 2D Kuramoto-Sivashinsky equation

David M. Ambrose, PHD (Drexel University, USA)

Most global existence results for the Kuramoto-Sivashinsky equation are in one space dimension. Among the limited results in two dimensions, many are for thin domains; this anisotropy keeps 2D solutions close in a sense to the 1D solutions. With Anna Mazzucato, we have developed some global existence theory for the 2D problem without relying on anisotropy. The dynamics of solutions depend in part on the size of the spatial domain. The theorems to be discussed include the cases when the domain admits no linearly growing Fourier modes, or when the domain admits one linearly growing Fourier mode in each direction. The method of proof is to decompose the set of Fourier modes into three classes: linearly growing modes, linearly decaying modes which serve as energy sinks for the growing modes, and strongly linearly decaying modes. The low and intermediate modes are treated by means of a Lyapunov function, while the high modes are treated with operator estimates in suitably chosen function spaces.

Post-blowup Dynamics in the Nonlinear Schrödinger Equation

José Escorcia and Alexei Mailybaev (IMPA, Rio de Janeiro, Brazil)

The Nonlinear Schrödinger equation (NLS) arises in different physical models, e.g., the nonlinear optics, Bose-Einstein condensate and fluid dynamics. We study the 1D NLS with the critical-case nonlinearity. It is known that solutions, whose mass exceeds a specific critical value, form a singularity in finite time. This singularity is regularized by adding a small dissipative term to the system. The dissipative term becomes important near the singularity, therefore, extending the solution to post-blowup times. In our work, we consider the nonlinear damping term, which corresponds to multiphoton absorption in nonlinear optics. Our goal is to study the asymptotic form of post-blowup solutions in the limit of small dissipation coefficients.

Our study addresses the well-known approximation based on the adiabatic theorem. This approach shows that pre-blowup solutions tend to a locally universal profile. Since small dissipation causes a slow change of the solution, the adiabatic theorem is often considered in the literature at the post-blowup region. Our findings contradict this expectation. We show theoretically that, when the total mass of the solution is strictly larger than the critical value, the adiabatic theorem breaks down strongly at the beginning of the post-blowup region. This breakdown is caused by the emergence of high-wavenumber oscillations. Our conclusions are verified by accurate numerical simulations

Nonlinear mirror image method for the nonlinear Schrödinger equation with time-dependent integrable boundary conditions

Carlos Mbala Dibaya (University of Leeds, UK)

We consider the initial boundary value problem (IBVP) for the nonlinear Schrödinger equation (NLS) on the half-line with a family of integrable time-dependent boundary conditions with two parameters. We use the nonlinear mirror image method, introduced by Bikbaev and Tarasov, which allows the use of a Backlund transformation to construct solutions of a half-line IBVP with integrable boundary conditions from solutions of the full line initial value problem (obtained using the inverse scattering transform). We construct the Backlund transformation needed to generate the time-dependent boundary conditions in two steps, where each step involves a Backlund transformation of Robin type. As a particular case of our results, we recover formulas for the pure multisoliton solutions obtained recently using the dressing of the boundary method.

Resolution of the zero-mass contradiction for the Boussinesq-Klein-Gordon and coupled Boussinesq equations

Matthew Tranter (Nottingham Trent University, UK)

In this talk I will discuss the so-called "zero-mass contradiction" that can emerge when constructing weakly-nonlinear solutions to Boussinesq-type equations for periodic functions on a finite interval. Firstly I will overview the recent results for the Boussinesq-Klein-Gordon equation, where a solution was constructed that takes account of this issue and numerical results will justify this approach. I will then consider the coupled Boussinesq equations. Solutions will be constructed for the cases when the characteristic speeds in the equations are close and when they differ significantly. Solitary and cnoidal wave initial conditions will be used to generate radiating (or generalised) solitary waves or Ostrovsky wave packets. The interaction of these solutions will be explored within this framework and conservation of energy within this system will also be examined. These equations can model the propagation of long nonlinear longitudinal bulk strain waves in a two-layered elastic waveguide with a soft bonding between the layers, where the material in the layers determines the type of equation. This is joint work with Karima Khusnutdinova.

Internal Waves and Related Flows

Professor Emilian Părău (University of East Anglia, UK)

We consider stratified inviscid fluids in the presence of surface tension or of ice plates. Solitary waves, generalised solitary waves and gravity-capillary internal hydraulic falls over obstacles are computed using numerical methods based on boundary integral or spectral techniques. Experimental results on the interaction of internal solitary waves with an ice cover will also be discussed.

Transformation of envelope solitons on a bottom step causes extreme events

Prof. Alexey Slunyaev (HSE University; Institute of Applied Physics of the Russian Academy of Sciences, Russia)

The transformation of surface envelope solitons travelling over a bottom step in water of a finite depth is studied. Relying on the weakly nonlinear theory, the analytic formulae are derived which describe the maximum attainable wave amplitude in the neighbourhood of the step and in the far zone. Solitary waves may be greatly amplified when propagate from relatively shallow water to the deeper domain due to the constructive interference between the newly emerging envelope solitons and the residual quasi-linear waves. The theoretical results are in a good agreement with the data of direct numerical modelling of soliton transformation. In particular, more than double wave amplification is demonstrated in the performed simulations. (A joint work with G. Ducrozet and Y.A. Stepanyants)

Bound-waves due to sea and swell trigger the generation of freak-waves

Dr David Andrade (Technion - Israel Institute of Technology, Israel)

In this talk we present a mechanism for the generation of freak-waves in the ocean. Our mechanism is based on an instability of narrow-banded homogeneous spectra to inhomogeneous disturbances. We show that when the sea state is in the presence of a swell, averaged bound-waves, that arise from their mutual interaction, drive the system away from the equilibrium. Then, by studying the non-linear evolution of the underlying sea state we find a significant increase in the probability of freak wave occurrence. This is joint work with prof. Michael Stiassnie from the Technion.

Internal-Capillary Solitary Waves in A Three Layer Formulation

Oleg G. Derzho (Institute of Thermophysics, Russia)

The study addresses the propagation of plane capillary gravity solitary waves of permanent form in a three layer formulation. The intermediate fluid is assumed to be stratified, while the upper and lower ones are homogeneous and infinitely deep. The interfaces separating these layers are subject to capillarity. The research can be applied to the case of two deep fluids when one of these fluids is stratified near the interface. Therefore, we address a capillary-gravity wave motion beyond the well-examined cases of a free surface or two fluid flows. It is shown that in the considered formulation capillary-gravity solitary waves of finite amplitude obey an integro-differential equation. This equation contains both the differential and the integral dispersions along with a specific nonlinearity, which depends on the properties of the stratified layer. Capillary (KdV type) dispersion dominates if the thickness of the stratified layer is $d \ll d_*$. When $d \gg d_*$, the gravitational (BO type) dispersion determines the flow. The value d_* depends on the mode number, gravitational acceleration and capillarity effects. Analytical solutions for the amplitude function and the streamline patterns are presented.

Quasi-parallel propagation of solitary waves in magnetised non-relativistic electron-positron plasmas

Professor Michael Ruderman (The University of Sheffield, UK)

We study the propagation of nonlinear waves in non-relativistic electron-positron plasmas. The waves are assumed to propagate at small angles with respect to the equilibrium magnetic field. We derive the equation describing the wave propagation under the assumption that the waves are weakly dispersive and also can weakly depend on spatial variables orthogonal to the equilibrium magnetic field. We obtain solutions of the derived equation describing solitons. Then we study the stability of solitons with respect to transverse perturbations.

Auto-setting breather mode-locked fibre laser

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In addition to their growing use as sources of ultrashort pulses for many applications, mode-locked fibre lasers constitute an ideal platform for the fundamental exploration of complex nonlinear wave dynamics. However, harnessing pulse generation from a fibre laser is a challenging task as reaching a specific mode-locked regime generally involves adjusting multiple control parameters, in connection with a wide range of accessible pulse dynamics. Machine- learning strategies and the use of evolutionary and genetic algorithms, which are well-suited to the global optimisation problem of complex functions, have recently shown promising for the design of smart lasers that can tune themselves to desired operating states [1, 2]. Yet, existing machine-learning tools are mostly designed to target laser generation regimes of parameter-invariant, stationary pulses, while the intelligent excitation of evolving pulse patterns in a laser remains largely unexplored.

Breathing solitons form an important part of many different classes of nonlinear wave systems, manifesting themselves as localised temporal/spatial structures that exhibit periodic oscillatory behaviour. Recently, they have also emerged as an ubiquitous mode-locked regime of ultrafast fibre lasers [3, 4]. These nonlinear waves are attracting considerable research interest by virtue of their connection with a range of important nonlinear dynamics, such as exceptional points, the Fermi-Pasta-Ulam paradox and rogue wave events [5]. In this talk, we demonstrate an evolutionary algorithm for the self-optimisation of the breather regime in a fibre laser cavity mode-locked through a four-parameter nonlinear polarisation evolution [6]. Depending on the specifications of the merit function used for the optimisation procedure, various breathing-soliton states are obtained, including single breathers with controllable oscillation period and breathing ratio, and breather molecular complexes with a controllable number of elementary constituents. Our work opens up a novel avenue for the exploration and optimisation of complex dynamics in nonlinear systems.

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Spontaneous patterns from Ablowitz-Ladik equations: cavity boundary conditions, instabilities, and mean-field theory

J. M. Christian and T. T. Moorcroft (Joule Physics Laboratory, University of Salford, UK)

In physics, the discrete nonlinear Schrödinger (dNLS) equation plays a key role in modelling wave propagation in periodic systems. Optical architectures typically involve light confined to a set of waveguide channels with nearest-neighbour coupling and whose dielectric response has a local cubic nonlinearity. While the widely-used dNLS model is non-integrable, it possesses an exactly-integrable counterpart—the Ablowitz-Ladik (AL) equation—which is often of greater interest in applied mathematics contexts. The trade-off for introducing integrability is a nonlinearity in the AL equation that remains cubic but which becomes nonlocal in a way that eludes straightforward physical interpretations. Despite their subtle differences, both models share common asymptotic properties in the long-wavelength (continuum) limit.

Here, the pattern-forming properties of the AL equation are explored in detail. Linear analysis in conjunction with periodic longitudinal boundary conditions—mimicking feedback in an optical cavity—is deployed. Threshold spectra for static patterns are calculated, and simulations test those theoretical predictions in AL systems with both one and two transverse dimensions. Subject to perturbed plane-wave pumping, we find the emergence of stable cosine-type and hexagon patterns, respectively. We conclude with an excursion into mean-field theory, where the AL equation takes on the guise of a discrete Ginzburg-Landau model.

Free surface flows in electrohydrodynamics with prescribed vorticity.

Dr Matthew Hunt (University of Warwick, UK)

Free surface flows have classically used the assumption of irrotationality in the derivation of various different models which includes the celebrated Korteweg-de Vries(KdV) equation. In recent years there has been interest in the inclusion of constant vorticity to the model. This has lead to the use of various potentials to try and get a model described the free surface.

The approach taken here is to isolate the vertical component of the velocity and use that as the master equation. This will allow the introduction of not only a constant vorticity but a variable one. The results presented today will be a derivation of the method, linear and weakly nonlinear theory for a variety of different vorticity distributions for both linear and weakly nonlinear cases.

One-Dimensional Discrete-Time Droplet-Faraday Wave Interactions

Eileen Russell (University of Bath, UK)

A droplet of fluid with bounce indefinitely on a suitably vibrating bath. Increasing the forcing acceleration of the bath, the droplet will 'walk' across the surface of fluid receiving a 'kick' in momentum proportional to the gradient of its wavefield. For a sufficiently large forcing acceleration, the droplet's motion will destabilise into chaos. We consider the droplet-bath system in a discrete-time one-dimensional system, where the droplet's motion is restricted to the x-axis. Analysing this discrete-time dynamical system, interesting characteristic of the droplet's motion emerges. In addition to the steady state walker and bouncer, we determine a highly unstable steady state self-orbiter. In the chaotic regime, the droplet's motion alternates between this periodic orbiter and the steady state walker. We analyse the probability distributions associated with the droplet's motion in the chaotic regime, and present frameworks to simplify the droplet's motion to a lower dimensional stochastic model.

Modelling and numerical simulation of a floating object

Professor David Lannes (University of Bordeaux, France)

We present here how the standard Boussinesq equations frequently used to describe the propagation of nonlinear dispersive water waves can be used to model wave-structure interactions. We show that in order to describe these interactions, it is sufficient to solve a transmission problem for the Boussinesq equations, coupled with a set of nonlinear forced ODEs. We propose a second order numerical scheme to solve these equations and validate this simulations with some explicit solutions that can be computed in particular configurations.

Brave new 'Legoland' of essentially 2d nonlinear evolution equations in boundary layers

Prof. Victor I. Shrira (Keele University, UK)

The overwhelming majority of nonlinear evolution equations used in various branches of science are one-dimensional (1d) differential equations. Here, to describe dynamics of 3d longwave perturbations in boundary layers, we present a way of asymptotic derivation of essentially two-dimensional nonlinear evolution equations with pseudodifferential (rather

than differential) operators. The equations are derived making use of the small parameters in the distinguished limit: nonlinearity is in balance with dispersion and dissipative effects. The dispersion might be due to different factors (e.g. finite wavelength, weak stratification in the boundary layer, stratification, rotation outside the boundary layer). The key feature of the novel evolution equations is that these factors enter the equation in additive way: they sum up. Thus we get almost infinite variety of combinations of different 2d dispersion operators. These novel evolution equations describe dynamics of coherent patterns in boundary layers, under certain conditions they predict finite time blow ups with formation of a point singularity.

Dissipative models of swell propagation across the Pacific

Dr. John Carter (Seattle University, USA)

Ocean swell plays an important role in the transport of energy across the ocean, yet its evolution is not well understood. In the late 1960s, the nonlinear Schrodinger (NLS) equation was derived as a simplified model for the propagation of ocean swell over large distances. More recently, a number of dissipative generalizations of the NLS equation based on a simple dissipation assumption have been proposed. These models have been shown to accurately model wave evolution in the laboratory setting, but their validity in modeling ocean swell has not previously been examined. We study the efficacy of the NLS equation and four of its generalizations in modeling the evolution of swell in the ocean. The dissipative models perform significantly better than conservative ones and are overall reasonable models for swell amplitudes, indicating dissipation is an important physical effect in ocean swell evolution. The nonlinear models did not out-perform their linearizations, indicating linear models may be sufficient in modeling ocean swell evolution over large distances.

Mode Two Solitary Waves in Three-layer Flows

Dr Alex George Doak (University of Bath, UK)

There is an extensive literature on modelling, computation and observation of horizontally propagating waves in stratified flows. The vast majority of this work, particularly when it concerns nonlinear structures and solitary waves, focuses on “mode one”, that is, the fastest wave in the system whereby all the pycnoclines are deflected with the same polarity. The simplest model for mode one waves is the two-layer flow of a lighter fluid above a heavier one bounded above and below by rigid boundaries. In that case the mode one wave is the only wave present, and, in the long wave limit, the KdV and mKdV arise as weakly nonlinear models, and MCC as a strongly nonlinear model. Mode two waves minimally require an additional layer in order for two interfaces to deflect with opposite polarity (mode two), or the same polarity (mode one). Mode two waves are increasingly believed to be of great scientific importance for their role in ocean transport. We shall consider the three-layer problem in this talk using KdV and MCC-like models, and the full Euler equations. We shall describe the problem and suggest an answer to the question: do mode two solitary waves exist in the Euler equations?

Wavefronts and modal structure of long surface and internal ring waves on a parallel shear current

Dr Karima Khusnutdinova (Loughborough University, UK)

There exists a linear modal decomposition (separation of variables) in the far-field set of Euler equations [1], more complicated than the known decomposition for plane waves. Here, we re-derive the modal equations from the formulation for the tangent plane waves, which opens a way to obtaining some important characteristics of the ring waves and to constructing 'hybrid solutions' consisting of an arc of a ring wave and two tangent plane waves [2]. The modal equations constitute a new spectral problem and lead to the need to construct a singular solution of a nonlinear first-order ODE responsible for the adjustment of the speed of the ring wave in different directions. The global and local measure of the deformation of wavefronts are introduced and evaluated.

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Nonlinear interfacial waves on the microscale

Professor Demetrios Papageorgiou (Imperial College, UK)

As system size goes down, physical effects that can be safely ignored in classical wave studies (e.g. viscosity or surface tension) must be brought back in, and indeed can dominate the dynamics. Recent technological applications include microfluidics and other micro-engineering processes, where the main concern is to either identify and harness naturally occurring instabilities, or drive the system out of equilibrium using external forces, in order to facilitate goals such as mixing, heat and mass transfer, and liquid fragmentation, for example. The result is a rich multiphysics environment to be modelled mathematically and understood via analysis and computations.

This talk will provide an overview of nonlinear interfacial instabilities in microscale flows where inertia may be absent or moderate. Several problems involving single-, two- and three-fluid multilayer flows will be discussed and the nonlinear coherent structures that can emerge will be illustrated. In addition, a brief overview of the effect of additives (surfactants) and external fields (e.g. electric fields) will be given with emphasis on the potential of achieving feedback and optimal control of nonlinear interfacial problems.

Stability of a sheared liquid film loaded with soluble surfactant

Dr Mark Blyth (The University of East Anglia, UK)

We investigate the stability of a semi-infinite fluid in a shearing motion over a fluid film above a solid wall. The film is loaded with a soluble surfactant and the motion is assumed to occur at zero Reynolds number. While it is known that this configuration is unstable in the presence of an insoluble surfactant, we show that surfactant solubility has a stabilising influence. As the solubility increases, large wavelength perturbations are stabilised first, leaving open the possibility of mid-wave instability for moderate surfactant solubilities, and the flow is fully stabilised when the solubility exceeds a critical value. Asymptotic expansions performed for long-wavelength perturbations turn out to be non-uniform in the insoluble surfactant limit. In keeping with the findings for insoluble surfactant obtained by Pozrikidis & Hill (2011), the presence of the wall turns out to be a crucial factor in the instability.

Deformation and dewetting of liquid films under gas jets

Dr Dmitri Tseluiko (Loughborough University, UK)

We study the deformation and dewetting of liquid films under impinging gas jets using experimental, analytical and numerical techniques. We first derive a reduced-order model (a thin-film equation) based on the long-wave assumption and on appropriate decoupling of the gas problem from that for the liquid. To model wettability we include a disjoining pressure. The model not only provides insight into relevant flow regimes, but also is used to guide the more computationally expensive direct numerical simulations (DNS) of the full governing equations, performed using two different approaches, the Computational Fluid Dynamics package in COMSOL and the volume-of-fluid Gerris package. We find surprisingly that the model produces good agreement with DNS even for flow conditions that are well beyond its theoretical range of validity, and we analyse under which conditions dewetting is dominated by the gas jet and/or the receding contact line motion. We additionally compare the computational results with experiments and find good agreement.

Groundwater Flow Through A Porous Medium and Modeling Ground Heat Exchangers

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Vertical or Borehole Ground Heat Exchangers (GHEs) constitute a major form of Geothermal Energy Systems (GES). When groundwater flows in the sub-layers past the borehole, the heat injection rates of the GHE can be considerably affected. Here, a mathematical model is constructed for regimes with or without groundwater, allowing for the presence of porous media regions. The problem is solved through a Finite Element Method in the FlexPDE software environment, which is first validated with experimental data from a Thermal Response Test (TRT) carried out in Lakatameia, Cyprus. The validated model is then employed to study the thermal behavior of vertical GHEs and the effect of factors such as (a) BH radius, (b) U-tube diameter, (c) U-tube leg and BH centers distance, (d) grout thermal conductivity and (e) groundwater velocity.

Exponential asymptotics for steady parasitic gravity-capillary waves

Josh Shelton (University of Bath, UK)

It is well known observationally that under the action of both gravity and surface tension, ripples of small wavelength form on the forward face of a steep propagating water wave. In the simplest steady, inviscid, and irrotational formulation, these high-frequency parasitic ripples appear to be a perturbation (for small surface tension) about a leading order gravity wave.

An early theory for these was proposed by Longuet-Higgins in 1963, however his work contained several critical asymptotic errors. We will resolve these though the use of specialised techniques known as exponential asymptotics.

The key difficulty involved is the exponentially-small nature of these ripples. This requires the understanding of singularities in the analytic continuation of the leading order Stokes-wave solution, as these singularities lead to divergence in the asymptotic series, which is required to resolve the Stokes phenomenon.

We derive a solvability condition which demonstrates that these perturbation solutions do not exist for certain values of the surface tension. These analytical results are compared to fully nonlinear numerical solutions and show excellent agreement.

On the Maxwell-Bloch System in the Sharp-Line Limit Without Solitons

Sitai Li (University of Michigan, USA)

We study the Cauchy problem for the Maxwell-Bloch equations (MBEs) of light-matter interaction via asymptotics, under assumptions that prevent the generation of solitons. Our analysis clarifies some features in which physically-motivated initial/boundary conditions are satisfied, including: (i) A boundary layer phenomenon is fully explained in which, even for smooth initial data, the solution makes a sudden transition over an infinitesimal propagation distance. At a formal level, this phenomenon has been described by other authors in terms of a self-similar solution related to the Painleve-III (PIII) transcendents. We make this observation precise and also identify this self-similar solution appearing exactly as the leading-order terms in the asymptotics. We show that such PIII functions are identical to the ones discovered recently to play an important role in several limiting processes involving the focusing nonlinear Schrodinger equation. (ii) Our analysis of the asymptotic behavior of solutions reveals slow decay of the electric field in one direction that is actually inconsistent with the simplest version of scattering theory. (iii) The asymptotic results validate a previous proposed causality requirement for MBEs, and demonstrate that it is a build-in mechanism of the Riemann-Hilbert problem studied in this work. (iv) Finally, the spontaneous decay process of an initially unstable medium is proved via a large family of incident optical pulses. This is joint work with Peter D. Miller.

Soliton resolution and asymptotic stability for the sine-Gordon equation

Bing-Ying Lu (University of Bremen, Germany)

We study the long-time dynamics of the sine-Gordon equation

$$\partial_{tt}f-\partial_{xx}f+\sin f=0,\ \left(x,t\right)\in\mathbb{R}\times\mathbb{R}^{+}.$$

Firstly, we use the nonlinear steepest descent for Riemann-Hilbert problems to compute the long-time asymptotics of the solutions to the sine-Gordon equation whose initial condition belongs to some weighted Sobolev spaces. Secondly, combining the long-time asymptotics with a refined approximation argument, we analyze the asymptotic stability for multi-soliton solutions to the sine-Gordon equation in weighted energy spaces. It is known that the obstruction to the asymptotic stability of kink solutions to the sine-Gordon equation in the energy space is the existence of small breathers which is also closely related to the emergence of wobbling kinks. Our stability analysis gives a criterion for the weight which is sharp up to the endpoint so that the asymptotic stability holds.

Impact On Localised Mode Scattering By Impurities In A Two-Dimensional Hexagonal Crystal Lattice

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Recently studies of collisions of nonlinear localised modes (discrete breathers) in a two-dimensional hexagonal crystal lattice show a rich variety of states [1]. These results demonstrate the full 2D energy scattering by mobile breathers, thus further adding to the discussion about the dark line formation mechanisms in mica by nonlinear lattice excitations. The breathers are thought to be involved in a phenomenon called hyperconductivity [2]. The chosen molecular dynamics system represents an idealized model of mica, which combines a Lennard-Jones interatomic potential with an egg-box harmonic potential well surface. Numerical studies have shown that such mobile discrete breathers are quasi-one-dimensional and long-lived, with their properties highly dependent on the ratio of the well depths associated with the interaction and on-site potentials [1; 3]. In this current work we further extend our studies by allowing the breathers to scatter on crystal impurities. In some cases we get interesting localized modes pinned to the impurity, and some of these show rotational behaviour.

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On Monotonic Patterns in Periodic Boundary Solutions of Cylindrical and Spherical Kortweg-De Vries-Burgers Equations

Professor Alexey Samokhin (Moscow State Technical University of Civil Aviation, Russia)

For the Kortweg-de Vries-Burgers equations on cylindrical and spherical waves, the development of a regular profile starting from an equilibrium under a periodic perturbation at the boundary is studied. The regular profile at the vicinity of perturbation looks like a periodical chain of shock fronts with decreasing amplitudes (a sawtooth part). Further on, shock fronts become decaying smooth quasi-periodic oscillations. After the oscillations cease, the wave develops as a monotonic convex wave, terminated by a head shock of a constant height and equal velocity. This velocity depends on integral characteristics of a boundary condition and on spatial dimensions. In this presentation the explicit asymptotic formulas for the monotonic part, the head shock and a median of the oscillating part are found.

A vorticity wave packet breaking within a rapidly rotating vortex

Professor Philippe Caillol (University of Bío-Bío, Chillán, Chile)

This study considers the critical-layer like interaction between a free vorticity wave packet and a rapidly rotating vortex in the quasi-steady regime, a long time after the initial, unsteady and strong interaction. We study a singular, nonlinear, amplitude modulated and helical mode inside a linearly stable, columnar and axisymmetric vortex on the f-plane.

The interaction generates a vertically sheared three-dimensional mean flow of higher amplitude than the wave packet. The chosen envelope regime assumes the formation of a mean radial velocity of the same order as the wave packet amplitude, deviating the streamlines in a spiral way with respect to the rotational wind. The neutral mode enters resonance with the vortex on a spiralling critical surface. The singularity in the modal equation on this surface strongly modifies the flow in its neighborhood, the 3D helical critical layer, the interacting zone.

The knowledge of the wave amplitude, the leading-order mean axial and azimuthal velocity and vorticity evolutions can be simply determined from three first-order quasi-linear differential equations whose main outcome is that the wave packet/vortex interaction leads to a vorticity wave breaking.

Time-dependent Kelvin cat-eye structure due to a linear shear current

Roberto Ribeiro (Federal University of Paraná, Brazil)

In this talk we discuss some numerical results concerning the flow structure beneath rotational water waves. We first consider linear surface waves and a study on the formation of a Kelvin cat-eye structure in the presence of bottom topography is presented. In some cases an initial disturbance is prescribed, while in others the waves are generated from the rest. Submarine particle dynamics numerically captures the horizontal critical layer, defined by closed orbits separating the fluid domain into two disjoint regions. In the wave's moving frame, these recirculation regions are structured in the form of Kelvin cat-eyes. Subsequently, we consider the weakly nonlinear, weakly dispersive model given by the Korteweg-de Vries equation to investigate the flow beneath solitary waves collisions when a linear shear current exists. The first part of the talk is a joint work with André Nachbin (IMPA) and Marcelo Flamarion (UFRPE), and the second part is an ongoing work with Marcelo Flamarion (UFRPE).

Generation of Linear and Nonlinear Periodic Traveling Waves in a Viscous Fluid Conduit

Yifeng Mao (University of Colorado, USA)

Conduits generated by the buoyant dynamics between two miscible, Stokes fluids with high viscosity contrast have been studied due to their remarkable nonlinear wave behavior. Applications in geological and geophysical contexts include the dynamics of channelized flows in magmatic and glacial systems. The wavemaker problem consisting of a time periodic boundary condition initiated from rest at t=0 is studied. Laboratory measurements of glycerin viscous fluid conduits are compared with theoretical predictions from the analysis of IBVPs for the conduit equation. In the linear regime, large t asymptotics of a trigonometric boundary condition result in four distinct space-time regions with different wave dynamics due to the bounded, nonlocal conduit dispersion relation. Experimental measurements of linear wave profiles quantitatively agree with the solutions to the conduit equation. A downshift of the critical frequency is observed and explained by the full two Stokes fluid system. When the boundary condition corresponds to the temporal profile of a nonlinear periodic wave solution of the conduit equation, weakly nonlinear and cnoidal-type waves are observed close to the injection boundary that agree with the conduit nonlinear dispersion relation. This study is an important precursor to experimental investigations of more general boundary value problems in nonlinear wave dynamics.

Spin-Injection-Generated Shock Waves and Solitons in a Ferromagnetic Thin Film: A Spin Piston Problem

Mingyu Hu (University of Colorado, USA)

The hydrodynamic interpretation of ferromagnetic magnetization dynamics was first proposed by Halperin and Hohenberg in 1969, revealing an analogy between magnetization dynamics and fluid dynamics. In this talk, unsteady magnetization dynamics of an initial-boundary value problem are studied in an easy-plane ferromagnetic channel subject to spin injection at one edge and a uniform, perpendicular-to-plane constant applied magnetic field. In the hydrodynamic framework, the applied field sets the initial density of a strip of quiescent magnetic “fluid” and the spin injection acts as a moving piston that excites dynamics within the strip. A full classification of the solution types over short enough timescales—where the induced dynamics can be approximated by the dissipationless limit—is provided using the nonlinear wave (Whitham) modulation theory. When the system is convex (defined by the strict hyperbolicity and the genuine nonlinearity of the modulation equations), the solution structures are identified to be cnoidal dispersive shock waves (DSWs) and rarefaction waves. When the system exhibits nonconvexity, the solutions are contact DSWs, composite waves, or no longer simple waves. When the magnetic “fluid” is supersonic, one signature is a stationary soliton at the injection site that persists in the long-time steady state, where dissipation is evident.

Nonlinear spectral synthesis of breather gas in focusing NLS equation: a numerical approach

Giacomo Roberti (University of Northumbria, UK)

Soliton gas was introduced by Zakharov [Sov. Phys. JETP 33, 538 (1971)] as an infinite ensemble of interacting KdV solitons randomly distributed in velocity and positions. This concept has been extended by El and Tovbis [PRE, 101, 052207 (2020)] in their development of the spectral theory of soliton and breather gases in the framework of the focusing Nonlinear Schrodinger equation. Moreover, it has been shown in a recent work by Gelash et al. [PRL, 123, 234102 (2019)] how the spectral soliton gas formalism could lead to a new understanding of the evolution of random processes in integrable systems, the so-called integrable turbulence. In this context, the ability to numerically build the soliton and breather gas solutions from the nonlinear spectral plane is a key element for testing the mathematical model and investigate its physical applications. In this work, we focused on the numerical synthesis of breather gases from the corresponding finite-gap spectrum. We were able to verify the theoretically predicted values of the phase shift in two-breather interactions and the effective velocity of a trial breather propagating through a uniform breather gas. This is joint work with P. Suret, S. Randoux, G. El and A. Tovbis. [PRE, 103, 042205 (2021)

Undular bores generated by fracture

Curtis Hooper (Loughborough University, UK)

Undular bores, or *dispersive shock waves*, are non-stationary waves propagating as oscillatory transitions between two basic states, in which the oscillatory structure gradually expands and grows in amplitude with distance travelled. We demonstrate for the first time, using high-speed pointwise photoelasticity, the generation of undular bores in solid (polymethylmethacrylate) pre-strained bars by natural and induced tensile fracture [1]. For the distances relevant to our experiments, viscoelastic extended Korteweg - de Vries (veKdV) equation is shown to provide very good agreement with the key observed experimental features for suitable choice of material parameters, while some local features at the front of the bore are also captured reasonably well by the linearisation near the nonzero pre-strain level. Similar waves have been observed in impact tests (e.g. `*coda*' [2]), but they have not been described using the KdV-type models. This is joint work with Pablo Ruiz, Jonathan Huntley and Karima Khusnutdinova.

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Model-free modelling of nonlinear pulse shaping in optical fibres

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Machine learning is transforming the scientific landscape, with the use of advanced algorithmic tools in data analysis yielding new insights into many areas of fundamental and applied science. Photonics is no exception, and machine-learning methods have been applied in a variety of ways to optimise and analyse the output of optical fibre systems [1]. In parallel with these developments, pulse shaping based on nonlinear propagation effects in optical fibres has developed into a remarkable tool to tailor the spectral and temporal content of light signals [2], leading to the generation of a large variety of optical waveforms [3, 4]. Yet, due to the typically large number of degrees of freedom involved, optimising nonlinear pulse shaping for application purposes may require extensive numerical simulations based on the integration of the nonlinear Schrödinger equation (NLSE) or its extensions. This is computationally demanding and potentially creates a severe bottleneck in using numerical techniques to design and optimise experiments in real time.

Here, we present a solution to this problem using a supervised machine-learning model based on a feedforward neural network (NN) to solve both the direct and inverse problems relating to pulse shaping, bypassing the need for numerical solution of the governing propagation model [5]. Specifically, we show how the network accurately predicts the temporal and spectral intensity profiles of the pulses that form upon nonlinear propagation in fibres with both anomalous and normal dispersion. Further, we demonstrate the ability of the NN to determine the nonlinear propagation properties from the pulses observed at the fibre output, and to classify the output pulses according to the initial pulse shape. We also expand our analysis to the case of pulse propagation in the presence of distributed gain or loss, with a special focus on the generation of self-similar parabolic pulses [6]. The results show that the network is able to accommodate to and maintain high accuracy for a wide dynamic range of system parameters. Although demonstrated here in a fibre optics context, the principle of using NN architectures to solve wave equation-based inverse problems is expected to apply to many physical systems.

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Linear stability spectra of a novel long wave-short wave system

Marcos Caso-Huerta (Northumbria University, UK)

Several integrable models have been proposed throughout the years to describe the interaction between long and short waves, among which the so-called Yajima-Oikawa and Newell models are possibly the most prominent. In the present work, a new, more general, integrable long wave-short wave model is proposed, encompassing the aforementioned systems as particular choices of the coefficients. The linear stability of plane waves for this novel system is carried out following the algebraic method proposed by Degasperis, Lombardo & Sommacal (2018), previously applied to study the vector NLS system. The Lax pair is obtained and employed to construct the locally perturbed plane wave solutions. The instabilities are investigated and classified with respect to the whole parameters space, which includes parameters featured by the system and by the solutions. The stability spectra and the associated eigenfrequencies are explicitly computed, leading to identifying a relation between the topology of the spectra and the gain of the system. Preliminary analysis indicates that, similarly to the vector NLS case, the classification of the stability spectra allows one to predict regions of existence for rogue wave type solutions. Based on joint work with A. Degasperis, S. Lombardo and M. Sommacal.

Chaotic scattering problems with polygons and polyhedra: exit basins and uncertainty fractal dimension

J. M. Christian (Joule Physics Laboratory, University of Salford, UK)

The Gaspard-Rice (GR) problem provides a paradigm for studying ballistic scattering in a plane, and it exhibits the phenomenon of sensitive dependence on initial conditions. The classic incarnation comprises a point-particle projectile reflecting specularly from three hard-edged discs located at the vertices of an equilateral triangle. Between reflections, the projectile travels at constant velocity. In this presentation, a regular-polygon generalization is proposed and its impact on properties such as exit basins, their boundaries, and time-delay functions are explored through computations. Consideration is also given to chaotic scattering from regular polyhedra, placing particular emphasis on the tetrahedron and hexahedron with hard spheres centred on their vertices. The ray-tracing algorithm implemented for GR-type systems can be readily adapted to accommodate motion in three spatial dimensions. Our analysis focuses on the calculation of exit basins, and on probing the uncertainty fractal dimension of their boundaries as a function of scatterer configuration.

Previous investigations of polyhedra have been primarily concerned with exploring the topology of basin boundaries through the similarity and box-counting dimensions. Here, we measure the fraction of phase space associated with uncertain outcomes, allowing us to establish the susceptibility of these purely deterministic scattering experiments to small fluctuations in their initial conditions.

Characterization of envelope solitary waves in nonlocal dispersive hydrodynamic systems

Sathyanarayanan Chandramouli (Florida State University, USA)

Envelope solitary waves (ESWs) are nonlinear, localized wavepackets traveling in a dispersive medium with an envelope propagation speed that differs from the phase speed associated with their carrier wave oscillations. In the frame moving with the envelope speed, ESWs are time-periodic. Hence, they represent a class of special solutions to nonlinear, dispersive wave equations that generalize periodic and solitary traveling wave solutions. They occur in optical fibers, ultracold atoms, magnetic materials, and deep water waves. In this work, ESW solutions of the conduit and Benjamin-Bona-Mahoney (BBM) equations, both of which exhibit nonconvex dispersion due to nonlocality, are directly computed using a Newton conjugate gradient scheme applied to a (1+1)D space-time boundary value problem. For each, a two-parameter family of bright ESWs is obtained that limit to bright soliton solutions of the nonlinear Schroedinger (NLS) equation for small amplitude and localized elevation wave defects on large amplitude cnoidal-type carrier waves. Higher-order NLS reductions are developed and identified as an appropriate framework for studying the emergence of the periodic, resonant background in the weakly nonlinear regime. Preliminary experimental results in a viscous fluid conduit, accurately modeled by the conduit equation, show promise for the realization of strongly nonlinear dark envelope solitary waves that cannot be approximated by the NLS and its higher-order generalizations. While strongly nonlinear ESWs have been obtained for the discrete p-Schroedinger equation, their existence for the physically relevant conduit equation represents a first for continuum models.

Furthermore, this study of ESWs may prove insightful for the class of equations wherein nonconvex linear dispersion results from nonlocality, the Ostrovsky equation, and its fully nonlinear counterpart the Miyata-Choi-Camassa equations being examples.

Contribution statement: S. Chandramouli was mentored by Dr. M. Hoefer for formulating the problem, selecting and implementing the NCG scheme, and analyzing the numerical results. S. Chandramouli also received mentoring and advice from Dr. Z. Musslimani in the implementation of the NCG scheme and the development of an in-house spectral renormalization scheme for (1+1)D problems. Y. Mao was mentored by Dr. M. Hoefer in the development of the higher-order NLS models, and the experimental investigation.