MATHEMATICS MATTERS

Designing the Future:

How mathematics is redefining architecture

The roof truss of the Emirates Stadium

Try and picture some of the most iconic buildings of modern times. You might be thinking of skyscrapers such as the Gherkin, the Shard or stadiums like the O2 Arena. These kinds of buildings are far too complicated for an architect to design solely on paper, so CAD (Computer Aided Design) software is used to create digital models of their structures.

You may be surprised to learn that, until relatively recently, building designs were first sketched on paper and then manually traced-over to get them into CAD software. In many ways this is highly inflexible – if an engineer wants to change the size of a roof beam then an architect would have to manually update all the other parts of the design which were affected by the proposed change. Adjusting one single beam could lead to the manual alteration of hundreds of other elements.

Parametric Models

Mathematics is revolutionising modern building design. Mathematicians are now able to supply architects and engineers with software capable of creating what are known as 'Parametric Models', meaning that the different elements of a design are specified by equations and relationships inside the CAD software.

In a sports stadium for example, a curved roof beam might actually be a small section of a circle, meaning that it can be described within CAD using the equation for a circle. If the length of a roof beam is changed, the software is able to automatically update the overall design using the same equation, without the need for lots of manual manipulation.

"Mathematics is revolutionising modern building design"

This has enabled architects and engineers to experiment more freely with different options, leading to exciting new designs, as well as saving on costs. Parametric Models have now been used in the design of many famous buildings such as Dublin's Aviva Stadium and the roof of the British Museum's Great Court.

Optimisation

Now that architects can change parts of their design so easily, it is now also possible to optimise, looking for the best design solution for a given situation. Mathematicians have designed software which can help architects and engineers optimise their designs automatically, for example by determining what is the best possible length of a roof beam or what is the best possible angle for the roof beam to be placed at. The software is able to perform this optimisation by taking into account a large number of constraints. For example, a building may have to be built within a certain boundary and stay under a certain maximum height. There are also other subtle constraints to take into account, for instance each piece of roof beam may need to fit within the length of the lorry that will be delivering it to the site.

Computer optimisation is an extremely exciting development, enabling architects and engineers to improve their designs according to a huge variety of different criteria. Often they might want to design a building so that it uses the least amount of material, reducing costs and benefitting the environment. However, in the case of a sports stadium, they might also want to optimise the roof so that the pitch receives the most sunlight, or improve the angle of the seating so that every spectator gets the best view possible.

Researchers at the University of Bath have used optimisation in the design of the Aarhus Botanical Garden in Denmark, winning a major international competition with their design. They optimised the shape of the greenhouse so that

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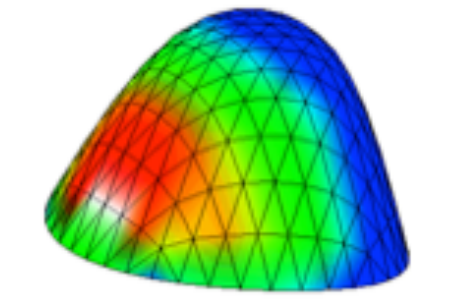
it would capture the largest amount of the sun's rays throughout the year, enabling it to maintain its tropical climate inside with minimal artificial heating.

Computer optimisation is also being used to influence the design of more every day buildings such as office blocks. Some companies are now focusing on producing pre-fabricated parts, where one building can be highly optimised and then the same design built in several different locations. Other companies are focusing on optimising each project individually, as the computer software means that it doesn't cost much more to produce a bespoke design that has been optimised for a particular location.

Researchers are also looking at how buildings can be optimised once they are in operation, making them highly responsive to their environments. For example, data about traffic around an office block could result in automatic window closing during rush hour, or information about when workers exit a building could



A computer model showing how the sun's rays hit the Aarhus Botanical Garden

The Aarhus Botanical Garden finished building

lead to a response in the traffic signals on surrounding pedestrian crossings.

New construction processes for buildings are also being investigated, and researchers at the University of Bath are currently exploring how advanced mathematics could be used to design and optimise structures which could be 3D printed using flying robots. If this was shown to be feasible, it could revolutionize construction techniques, make building sites much safer, and help reduce construction costs.

TECHNICAL SUPPLEMENT

Genetic Algorithms

An architect might want to optimise a roof truss (a large beam made from lots of shorter struts joined together) by finding the best combination of two different variables, e.g. the number of segments in the truss (x) and the height of the truss (y). The optimal result will have the lowest cost, while still producing a truss which doesn't bend too much or buckle. This is a highly simplistic example as an architect will usually want to compare dozens of variables simultaneously.

One of the ways in which this optimisation can be done is by using genetic algorithms. A genetic algorithm mimics the process of natural selection by 'breeding' different combinations of variables together. In the example above you might have several different starting options, stored in a list where the first number represents the number of segments in the truss (x) and the second number represents the height of the truss (y).

There will be an associated cost with each combination $\binom{x}{y}$.

In this very simple example we begin with three possible options in what is known as our Gene Pool:

Option A: $\binom{5}{40}$ Cost = £10k

Option B: $\binom{6}{50}$ Cost = £2k

Option C: $\binom{7}{80}$ Cost = £3k

According to our genetic algorithm we will select the best two options and 'breed' them together. Let's take Option B and breed it with Option C by cutting them up and swapping the parts, creating the following: $\binom{\boxed{6}}{50}\binom{7}{\boxed{80}} \longrightarrow \binom{6}{80}$. There might also be a 'mutation' along the way, where one number is randomly changed, resulting in a new Option D: $\binom{6}{85}$ which has an associated cost of £1.5k. This new Option D has a cheaper cost than all of the others, therefore the most expensive Option A is eliminated from the gene pool and Option D is now included.

The process is then repeated and the gene pool gradually improves. After many iterations, the genetic algorithm can be expected to result in the most optimal choice for the number of segments and height of truss.

References

Shepherd, P., Hudson, R. & Hines, D., 2011, "Aviva Stadium: A parametric success", International Journal of Architectural Computing, 9(2), pp167-185, doi:10.1260/1478-0771.9.2.167.

Shepherd, P. & Richens, P., 2012, "The Case for Subdivision Surfaces in Building Design", Journal of the International Association for Shell and Spatial Structures, 53(4), pp237-245.

Web link to Paul Shepherd's website page about a talk he gives to schools on Optimisation and Genetic Algorithms:
http://people.bath.ac.uk/ps281/maths_talk/best/

Expert

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