Editorial

Being asked to write about my favourite area of mathematics for this editorial posed two initial problems: firstly what topic to choose and secondly overcoming my nerves to write for a magazine I've enjoyed reading my whole working career. It turns out the latter problem can be overcome with chocolate, editorial support and a strong talking to myself. Choosing what topic to write about was much harder mainly because there are areas of mathematics I'm not that keen on or frankly not that good at. However, that still leaves an enormous treasure trove of options to choose from.

So, what is my favourite area of mathematics? When thinking about this, I realised that I don't have a 'single' favourite in any area of my life from chocolate to films to pizza topping. I do have some inherent biases in choosing favourites (always some form of dark chocolate; is the film part of a franchise or not; pizza toppings always include goats cheese) but my actual favourite is conditional on factors such as where I am or who I'm with.

In mathematics, I am slightly biased towards probability (but my first real interest was fluid mechanics) and the best way I know to pull together all the information I have available accounting for uncertainty and bias is with Bayes' theorem. A theorem which at its simplest can be expressed with: three terms, two letters, one multiplication and a division. Used across science, engineering, academia and industry, Bayes' theorem combines observations to make predictions. It is the cornerstone of many areas of mathematical modelling and the quality of the data impacts the accuracy of any predictions made.

Harold Jeffreys in his 1961 book *The Theory of Probability* [1, p. 1] wrote that

> The fundamental problem of scientific progress, and a fundamental one of everyday life, is that of learning from experience. Knowledge obtained in this way is partly merely description of what we have already observed, but part consists of making inferences from past experience to predict future experience.

What Bayes' theorem does that is so amazing is just this. It replicates something so intuitive that you do it without noticing – updating probability every time you get new information. This innate skill of reassessing your viewpoint every time information becomes available is something you do all the time, whether it is crossing the road or searching the internet. However, Bayes' theorem is also an area of mathematics which generates an enormous amount of debate about the validity of the approach.

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The Bayesian versus frequentist debate is not new and simply put both apply approximation methods to understand the complex problems they are seeking to answer. The philosophical difference comes down to how measurements and probabilities are considered. Frequentist statistics approaches probability in terms of repeated measurements – probabilities are expressions of the frequency of events. From a Bayesian view, probabilities relate to what we know and understand about an event and are expressed as a distribution. This simple difference leads to distinct approaches and entrenched examples of both sides saying that their side is the *right* way, the *best* way, or at worst, the *only* way.

Arguments in any sphere of life can get heated, and often don't lead to any headway being made. What is sad about this is that in mathematics there is so much we all agree upon. The rigorous nature of our field means that the definitions, axioms, conjectures, proofs, corollaries, lemmas, propositions and theorems we work with create a common foundation.

What can lead to huge advances are open, lively debates and it is here that the vibrant nature of mathematics comes to life. There are fundamental questions we talk about – the philosophical questions about what mathematics is and its usefulness. There is also the cutting-edge work, the papers and conference presentations where we learn about where the latest developments in mathematics are taking us. Communicating our work is at times harder than actually developing mathematics. To this end, I agree wholeheartedly with Professor Andrew Wiles, who described how cautious mathematicians are in the words they use because once something has been shared publicly it can't easily be taken back.

By developing a strong and vibrant mathematical community in which we share and debate our work, we develop the core skills of expressing ourselves and active listening.

To keep up to date and advance our knowledge we have to see other people's perspectives: why they have chosen certain approaches and how the results are interpreted. Following the latest advances requires us to pay attention – to listen so that we can understand where the presenter is coming from and offer feedback. Sometimes this can be immediate, by offering thoughts at the end of a presentation or writing a response to a publication. Both approaches enable us to open up constructive debates helping everyone to expand their knowledge and increase their enjoyment of working in mathematics. Occasionally, we realise that what we need to do is defer judgement. That is we need to think a little more and improve our understanding before we respond. This can be much harder to do.

It is often too easy to dismiss results or work that does not fit in with our preferred ideas, to ignore work that challenges our status quo, even if there is mounting evidence that we may be in the wrong. After all, old habits are hard to break and it can take a lot of effort to objectively review ideas presented on paper or at a meeting.

But it is in just such moments that I have some of my fondest working memories: the keenness and passion that has come flowing out in favour of one approach or another. Of listening to people on opposite sides of the table (literally and metaphorically) talk, explain, cajole, eye-roll and at times despair but ultimately

work together. Whilst my experience is mainly in the frequentist and Bayesian discussions surrounding statistics, I know there are equivalent debates in every other field of mathematics.

Looking outside of mathematics, examples of those with opposing views who can still respect and work with each other is exemplified by the friendship between Justices Antonin Scalia and Ruth Bader Ginsburg.

When asked about how she would like to be remembered, the notorious RBG said [2]:

> Someone who used whatever talent she had to do her work to the very best of her ability. And to help repair tears in her society, to make things a little better through the use of whatever ability she has. To do something, as my colleague David Souter would say, outside myself. 'Cause I've gotten much more satisfaction for the things that I've done for which I was not paid.

I am excited to see what advancements in all areas of mathematics are going to be made in 2021. To really make an impact then as a community we need to all play our part: actively listen and support each other so we can work to the best of our ability. If we can do this, we'll inevitably go a long way to helping society because there isn't an area of our lives not touched by mathematics. Happy listening.

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Acknowledgement: Pizza © Yauheni Labanau | Dreamstime.com

References

1 Jeffreys, H. (2003) *The Theory of Probability*, 3rd Edition, Oxford University Press.

2 Carmon, I (2015) Exclusive Justice Ruth Bader Ginsburg interview: Full transcript, MSNBC, tinyurl.com/RBG-interview