Satellite Communication

Satellite communication is vital in the aftermath of a natural disaster. Yet without mathematics, the satellites would quickly move out of formation and render the whole system useless.

In the modern world, we've become accustomed to being connected all the time. We think nothing of whipping out our smartphones and being instantly able to make a call. But it isn't always so simple. Imagine the situation in the aftermath of a natural disaster, where a hurricane, tsunami or earthquake has laid waste to vast swathes of the communications infrastructure. Those responding to the emergency still need to be able to talk to each other in the field and co-ordinate their efforts. They are able to do so by making use of the hive of communication satellites which orbit our planet, making satellite phones a crucial piece of kit in disaster management. They also allow the broadcast media to report on the situation. But without mathematics, the system wouldn't work at all.

One of the leading satellite communication systems is run by Inmarsat, a company based on 'Silicon Roundabout' in London. They have a fleet of 11 satellites in orbit which provide satellite communication coverage to the whole planet. The satellites are in what's known as 'geosynchronous' orbit – in which the spacecraft orbits the world in exactly the same time as it takes the Earth to rotate – which means that, relative to someone on the ground, the satellite always appears in the same spot in the sky. It is the same technique deployed by television satellites to ensure there is always a satellite overhead for your TV dish to communicate with. It is why, in the Northern hemisphere, dishes all point towards the south (in the Southern hemisphere they all point North) – the fixed position occupied by the geosynchronous satellites.

"Station-keeping is a two stage process and mathematics is vital in both"

They never stay exactly where they are, however, because of our planet's complex shape. The Earth isn't perfectly spherical, and so its gravitational pull is not entirely symmetrical. The varying gravity of our planet has been mapped by satellites such as the European Space Agency's Gravity Field and Steady-State Ocean Circulation Explorer (GOCE), but there are still uncharted variations. In addition, the relative positions of the Sun and the Moon with respect to the Earth change over time. Sometimes they pull together; sometimes they work against each other. These factors combined mean that the forces pulling on the satellites are not all completely predictable. The positions of the satellites can only be forecast for a few weeks at a time – eventually they will start to move out of the formation which allows them to provide global communications coverage. So it is imperative to get them back into formation as quickly and efficiently as possible – an endeavour known as 'station keeping'.

Institute of mathematics & its applications

This is a two stage process, and mathematics is vital in both. First, the station keepers have to know the position the satellite has drifted to. This orbit determination is achieved via radio ranging, where a beam of radio waves is transmitted from a ground station and reflects off the satellite before returning to Earth. The longer it takes the signal to return, the further away the satellite. It isn't quite that simple, however. This technique yields a long string of data points from which the station keepers have to determine an orbital path. This is more complicated than drawing 'a line of best fit' through the data as we might do during experiments at school, plotting the line by eye so as to ensure that the number of data points above and below the line are approximately equal. Instead, the mathematical technique of weighted least squares fitting is deployed. This can yield an orbit accurate to within 20 metres.

Once the station keepers know where the satellite is, they can set to work getting it back into position. Each satellite is equipped with thrusters which can be fired in order to manoeuvre the machine. It is not only important that the satellite then ends up where it should be, but also that it does so having used the minimum amount of fuel. These multi-million dollar satellites cannot be refuelled, and so careful fuel management is vital in ensuring it stays operational for as long as possible. To achieve this, the station keepers turn to the 'Rocket Equation'. Its formulation is often attributed to the early twentieth century work of Russian rocket scientist Konstantin Tsiolkovsky. However, there is evidence to suggest the same idea was independently found by British mathematician William Moore as early as 1813 when he was working at the Military Academy in Woolwich. His equation uses a mathematical quantity known as a logarithm to relate the change in speed of the satellite to both the exhaust velocity and the change in mass of the satellite due to lost fuel. It is this mathematical relationship that's crucial in getting the satellite back on track as efficiently as possible. Without this mathematics, the satellites would continue to drift out of formation and emergency workers would find it harder to communicate, putting additional lives on the line.

TECHNICAL SUPPLEMENT

Rocket equation

The rocket equation, first formulated by British mathematician William Moore in 1813 is as follows: $\Delta v = v_e \ln(m_0/m_1)$ where Δv is the maximum change of velocity of the satellite, v_e is the effective exhaust velocity, $\ln$ is the natural logarithm and m_0 and m_1 are the initial total mass and final total mass respectively. It is derived by starting with Newton's Second Law of Motion (which relates force to mass and acceleration) and eventually integrating the solution.

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