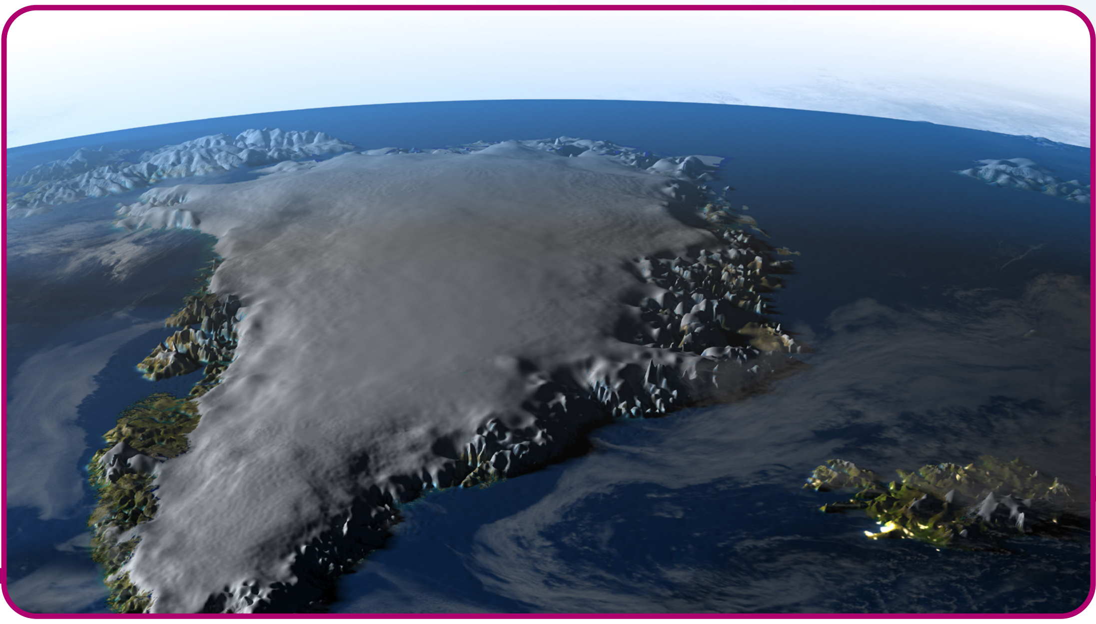
MATHEMATICS **MATTERS**

What did the Greenland ice sheet used to look like?

Using maths to understand climate change



As average global temperatures increase, one of the biggest impacts of climate change can be seen in the form of rising sea levels. Sea level changes have the potential to cause devastation to some of the world's most densely populated areas, meaning that it is vital to obtain accurate estimates of these increases. Unfortunately making predictions in this area is difficult, and scientists have made estimates for sea level rises which contain a great degree of variation. One of the reasons that prediction is difficult is that the earth has not gone through such a significant warming in the recent past, meaning that scientists don't have the data to explain how the ice sheets will melt and how sea levels might rise.

The Last Interglacial Period

This is why researchers have been looking back in time 125,000 years to the Last Interglacial Period when the earth was significantly hotter. At this time the earth was likely to have been 1-2 degrees warmer than before the industrial revolution and the polar regions were up to 5 degrees warmer, meaning that the Last Interglacial Period can serve as a good guide for how our earth might look in the future.

In particular, researchers have been looking at what the shape of the Greenland ice sheet might have been like after melting during the Last Interglacial Period. The shape is important, because different shapes result in hugely different global sea levels. Until now, researchers have been estimating very different shapes for the Greenland ice sheet, resulting in associated sea level rises which vary between 0.3 metres and 5.5 metres. A team of researchers at the British Antarctic Survey (BAS) have been working with mathematicians at the University of Leeds to come up with a novel approach to reduce this variation and produce a much more accurate model of what the Greenland ice sheet would have looked like.

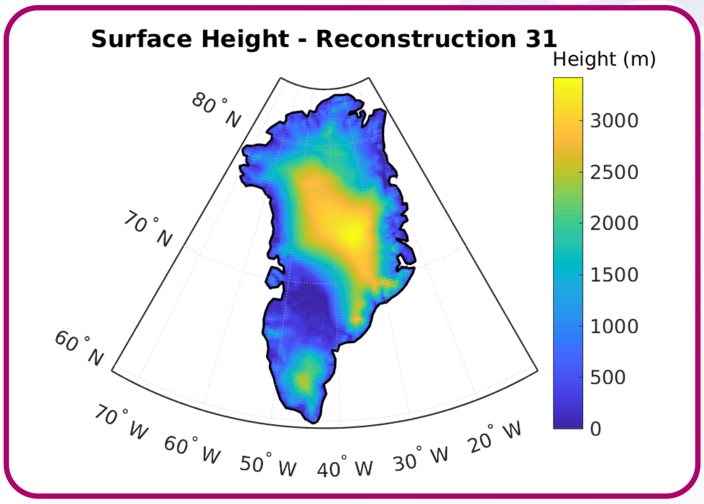
How can you know what the Greenland ice sheet looked like?

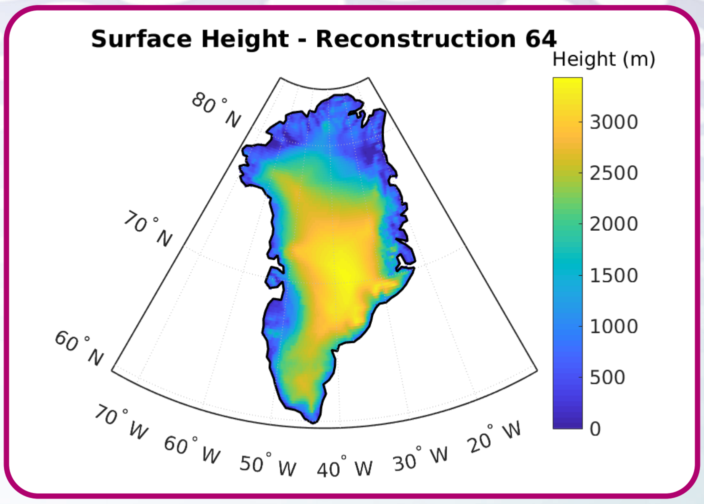
Scientists have extracted seven ice cores from the Greenland ice sheet which contain ice from the Last Interglacial Period. Chemical analysis of this ancient ice allows scientists to get data about the climate at that time. As a result of this data different groups of scientists have produced diverse possible pictures of what the Greenland ice sheet might have looked like in this period.

> It has been an important collaboration between climate scientists and mathematicians, where both skills were vital to the outcome of the research.

The researchers from BAS and the University of Leeds have taken 14 different versions of these Greenland ice sheet shapes and combined them using a mathematical method called *Principal Component Analysis* to produce 64 new shapes which are made up of different combinations of the original 14 pictures.

Institute of mathematics & its applications





Examples of some of the 64 Greenland ice sheet shapes generated using Principal Component Analysis. In each picture, the colours represent the height of the surface (increasing height from blue to yellow).

Climate models

When anyone is predicting the weather or climate they will use a mathematical climate model which can be run on a computer. These are the types of tools which are used to generate your weather forecast, and similar models can be used to predict the general climate.
The researchers from BAS and the University of Leeds took the 64 possible shapes of the Greenland ice sheet and ran them through a climate model to predict what should be observed at the different ice core sites, given the shape that had been used.

On the basis of the observed results they then used another mathematical tool, known as interpolation, which could be used to predict the climate model response corresponding to any other ice sheet shape. Comparisons with the real ice core data were then carried out and, if the results matched, then the researchers knew that it was likely that they were dealing with a realistic shape for the Greenland ice sheet.

Narrowing down the possible shapes

In order to narrow down the possible shape further, the researchers first had to do the opposite and increase the number of shapes which they were considering, in order to ensure that no possibilities were missed. In a very similar way to the one used to generate the 64 shapes, they could generate a much larger set of possible shapes and predict the climate model outputs for each of them, through interpolation. This is the first time that a comprehensive approach has been taken where all possible shapes have been considered. The researchers then compared these predictions with the ice core data, finding that only 1% of these shapes were in agreement. This means they eliminated wide swathes (99%) of all possible shapes as being 'unlikely'.

This research has resulted in a more accurate prediction of what shape the Greenland ice sheet would have been in the Last Interglacial Period. It has been an important collaboration between climate scientists and mathematicians, where both skills were vital to the outcome of the research. Improved knowledge about how the Greenland ice sheet might melt will mean that scientists will be able to predict more accurately what could happen as our earth warms up over the next 50 to 500 years, as well as enabling careful monitoring of the parts of the Greenland ice sheet which are deemed to be most vulnerable.

TECHNICAL SUPPLEMENT

At the beginning of the project, there were 14 different shapes for the Greenland ice sheet which had been generated from previous scientific work, using the evidence which had been found in the ice cores. There were significant variations in these 14 possible shapes, and therefore it was important to understand how these shapes differed and to discover what was the most likely shape of the Greenland ice sheet during the Last Interglacial Period.

The researchers used a technique from multivariate statistics called Principal Component Analysis (PCA) to compare the 14 shapes. Each of the Greenland ice sheet shapes was split into eight 'principal components' – meaning that an eight dimensional vector described each of the shapes $(x_1, x_2, \ldots, x_8)$, where for example x_1 might describe the distance from east to west on the ice-sheet.

The use of Principal Component Analysis was able to detect the regions over Greenland where the 14 ice sheet shapes differed the most from each other, and to order them according to how much they varied from one another. As a result of this analysis the researchers were initially able to construct 64 different possible shapes for the Greenland ice sheet on which the climate model could be run. Principal Component Analysis was originally invented in 1901 by Karl Pearson, and has gained in popularity since computers can now be used to more easily compute principal components.

Links and References

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