Editorial

It's the sort of question about which books are written and movies are made. It's even the sort of question, which forms myths and legends. So, I think it's a question that's worthy of some of our time, if only for the length of this editorial: 'If you had one wish, what would it be?'

Of course, in many cases, the implied 'and why?' is actually more interesting than the wish itself. Comedies and tragedies arise if the stated wish does not match the intent. You only need think of King Midas, whose touch turned everything to gold, to be reminded of that. A more recent example may be the way 'reward hacking' allows algorithms developed using machine learning to satisfy the stated goal, but not the original intent.

One obvious answer to the question is to ask for more wishes. From a narrative perspective, this rather spoils the plot, so it doesn't tend to be allowed. However, I wonder if there's something interesting hidden within the 'many wishes' notion. Suppose, for a minute, that a wish was like currency, so like there are 100 pence to the pound, we could split our wish into, say, 100 'unicorns', or 'wishingtons', or whatever the appropriate subdivision is.

Then, our protagonist could choose between a single really big wish, perhaps ending world poverty, and lots of smaller wishes, maybe sunshine for the last day of a test match. If, rather than being discrete, 'wishingtons' were continuous then, through the magic of mathematics, an infinite number of wishes could be received, each reducing in size. The infinite sum $1/2^n$ illustrates one way to do this.

To make this wish-splitting approach work, we'd need some way of valuing a wish. If you're in the business of wish fulfilment then, I guess, you'd have some idea of how difficult it would be to grant a particular wish. Sadly, I'm not in that business, so we need inspiration from other sources. There are, I think, two main things to consider.

The first thing relates to time. More specifically, whether the wish involves an instantaneous change (e.g. winning a sizeable sum of money) or whether it requires continuous, or at least repeated, intervention (e.g. maintaining a consistent weight, regardless of diet or exercise). In general, instantaneous changes would seem to be easier to achieve.

The second thing relates to the number of people affected. This seems to be a (very) rough analogue for space, so that our two considerations together cover space-time, which seems appropriate. Obviously, the more people affected, the more expensive the wish.

We may choose to add another people-related factor, specifically, the number of people who would wish something different. Asking for good weather for a family barbecue is unlikely to upset many people; wishing your team wins the FA Cup is a different story. To address this, we could create a market to price different wishes. But, from my perspective, making wishes a marketable, perhaps even tradeable, commodity seems to diminish the notion.

Typically, those granted wishes are not presented with a long list of caveats. Some limitations might appear if a particular type of wish is asked for, or even hinted at: the Genie saying he can't make Jasmine fall in love with Aladdin (at least in the animated Disney version of the tale) is one example. So, I think it's an interesting idea to restrict wishes to a certain sphere. More particularly, if you had one mathematical wish, what would it be?

You may wish, for example, for something related to governance. Perhaps, your desire is for our subject to have a unified voice, which is listened to by those in power. Or, on a more personal front, you might like to be elected to the IMA Council. Interestingly enough, if it is successful then the Big Mathematics Initiative should address the first of these. As for the second, nomination and election are all that's needed.

If governance is not your thing, maybe public perception is. The excellent Mathematics Matters series (see page 52) has helped make hidden mathematics more visible. There are also examples in popular culture, including the TV show *Numbers*, which features a mathematician working alongside the FBI, and the film *Hidden Figures*, which demonstrated the important role played by female African-American mathematicians in the space race. Despite this progress, there is still some way to go before mathematicians are revered in the same way as, say, Premiership footballers. There's also some way to go before these two groups receive similar remuneration!

Although I can see the attraction of wishes related to governance or public perception, if I had a mathematical wish then I'd probably choose for a problem to be solved. One of the things that makes our subject so interesting is there's a wealth of important problems I could pick from. For example, at the turn of the century, the Clay Mathematics Institute (CMI) of Cambridge, Massachusetts, posed seven problems. These were judged to be the deepest, most difficult mathematical problems of the time. To date, only one of the seven problems has been solved: Grigori Perelman proved the Poincaré Conjecture, which is concerned with the connectedness of a three dimensional sphere.

Of the remaining six problems, how should I target my wish? Perhaps, the Riemann Hypothesis, which concerns the distribution of prime numbers. Or, maybe, a proof as to whether solutions exist for the Navier–Stokes equation (which governs fluid flow) and, if so, whether they are unique. As well as being of great mathematical significance, solutions to either of these

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problems would also be of practical importance, for example, in cryptography and aerodynamics.

Despite these benefits, I would probably choose the $\mathrm{P} =? \mathrm{NP}$ problem, which (in essence) asks, 'if it is easy to check that a solution to a problem is correct, is it also easy to solve the problem?' There's currently a very strong belief that this should be answered in the negative, that is $\mathrm{P} \neq \mathrm{NP}$. So, maybe a proof of this won't advance the art of problem solving, although it would undoubtedly include some very interesting mathematics.

However, since we're wishing, I guess I'm allowed to wish for a proof that $\mathrm{P} = \mathrm{NP}$. Some might argue this is going too far. They might believe, and they may be right, that this is asking the wish granter to alter a fundamental property of our universe. But, since all good wishes alter things in unimaginable ways, I think the wish is valid. Knowing that (in the universe in which my wish is fulfilled) $\mathrm{P} = \mathrm{NP}$ would be of tremendous importance. It would show us that there are efficient ways of finding solutions for whole classes of problems.

I've already stretched the wish a long way but, if you'll permit me, I'm going to stretch it just a little bit further. In particular, I'm going to ask that the wish-given proof of $\mathrm{P} = \mathrm{NP}$ is an existence proof, rather than a constructionist one. For example, it could start from the premise that $\mathrm{P} \neq \mathrm{NP}$ and reach a contradiction. That would give us the tantalising realisation, in my wish-given universe, that there was an efficient way of solving NP problems. Of course, we'd still have to find it.

The CMI millennium problems, and the many other extant problems in mathematics today, demonstrate there's lots more work to be done. But, a proof of the type asked for in my wish would clearly demonstrate that mathematics is a gift that keeps on giving. That's certainly been my experience. And, I think, it's one that should be uppermost in the mind of those responsible for governance, funding, promotion, development, pursuit and, most of all, enjoyment of the subject.

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