



**Institute** of  
**mathematics**  
& its applications

## BOOK OF ABSTRACTS

### 1<sup>st</sup> IMA Conference on Maths in Music

Royal College of Music, London, UK  
13-15 July 2022

*In cooperation with*



**ROYAL COLLEGE OF MUSIC**

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## Organizing Committee

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## Public Lecture

*Numbers and Notes; Patterns and Progressions*  
Robin Wilson (Open University) & Rob Sturman (University of Leeds).

## Conference Schedule

	Wednesday 13/7/2022	Thursday 14/7/2022	Friday 15/7/2022
08:40-09:00	Registration	Key note: Michelle Phillips <i>Mathematics and music: the perspective of the listener</i>	Key note: Geraint Wiggins <i>Mathematics and the Source of Music</i>
09:00-09:20			
09:20-09:40	Welcome	Dorothy Ker <i>Meeting in the Shadows</i>	Thomas Goodman <i>Assessing the feasibility of real-time analysis of timing and coordination between ensemble musicians</i>
09:40-10:00	Key note: Emily Howard <i>Orchestral Geometries: Torus, sphere, Antisphere</i>		
10:00-10:20		Coffee-break	Coffee-break
10:20-10:40			
10:40-11:00	Lasse Rempe <i>Points of convergence - A dialogue between music and dynamical systems research</i>	Eyal Buks <i>Musical tonality and synchronization</i>	Alan Shepherd <i>Applying Information Theory and Statistics to Numbers in Music</i>
11:00-11:20	Alberto Alcalá-Alvarez <i>A Framework for Topological Music Analysis (TMA)</i>	Ismael Patriota <i>Math and religion in Arvo Pärt's De Profundis</i>	Paras Patel <i>Heart Rate Variability and the Impact of Music Listening on Autonomic State</i>
11:20-11:40			
11:40-12:00	Paul Archbold <i>Some compositional implications of symmetries in microtonal pitch class sets</i>	Lane Hughston <i>Well-Tempered Complexity</i>	Léon Snyman <i>The Complex Sum of Simple Parts: how <math>LV/xi</math> produces an immersive whole through simultaneous variation</i>
12:00-12:20			
12:20-12:40	Lunch	Lunch	
12:40-13:00			
13:00-13:20	Key note: Moreno Andreatta <i>The Music of Maths: a 'mathematical' journey</i>	Public lecture: Robin Wilson & Rob Sturman <i>Numbers and Notes; Patterns and Progressions</i>	
13:20-13:40			
13:40-14:00	Zani Ludick <i>Investigating the relationship between linguistic measures and self-similarity matrices in assessing musical structure</i>	4 Poster mini-talks (10min each)	
14:00-14:20			
14:20-14:40	Coffee-break	Coffee-break	
14:40-15:00			
15:00-15:20	Marco Buongiorno Nardelli <i>Network topology of generalized musical spaces</i>	4 Poster mini-talks (10min each)	
15:20-15:40			
15:40-16:00	David Hewett <i>SCALES - Rhythms of fractals and collective movement</i>	Break	
16:00-16:20			
16:20-16:40	Concert		
16:40-17:00			
17:00-17:20	Conference dinner		
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17:40-18:00			
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# 1 Abstracts of Invited Keynotes

## The music of maths: a 'mathemusical' journey

Moreno Andreatta<sup>a,b</sup>

a. IRMA, University of Strasbourg, France

b. Music Representation Team, IRCAM, France

In this presentation, I will provide an overview of some of the most active research axes of the SMIR (Structural Music Information Research) Project I'm leading at the University of Strasbourg. The project, hosted by IRMA (Institut de recherche mathématique avancée), has been carried out since 2017 in deep collaboration with computer science researchers from the Music Representation Team at IRCAM in Paris. Ongoing research axes include the use of Mathematical Morphology, Formal Concept Analysis and Persistent Homology in the automatic classification of musical styles; the categorical formalisation of transformational music analysis; the interplay between algebraic and geometrical approaches in the construction of tiling rhythmic canons and their connection to Homometry theory and Fuglede Spectral Conjecture. After discussing the "mathemusical" dynamics underlying all these research axes, I will offer several music-theoretical examples showing how to creatively use some formal and computational tools in mathematically-based popular music compositions. I will end by shortly discussing some recent research directions in music cognition and perception we have explored in the last two years within a subproject supported by CNRS and entitled ProAppMaMu (Processes and Learning Techniques of Mathemusical Knowledge).

Some useful links:

- The SMIR Project: <http://repmus.ircam.fr/moreno/smir>
- The Tonnetz web environment: <https://morenoandreatta.com/software/>

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## Orchestral Geometries: Torus, sphere, Antisphere

Emily Howard<sup>a</sup>

a. Royal Northern College of Music, UK

The transformation of mathematical notions into musical ideas has become an important research methodology within my compositional practice. Whilst never a direct translation, it is precisely by attempting to carry out this impossible task that something is gained. I find that this approach often reveals new questions from unusual vantage points that result in unexpected ways to organise sound. Over the past seven years, the development of a series of geometry-inspired orchestral works has been a central preoccupation. For each of these *Orchestral Geometries*, an abstract mathematical shape was the imaginative cornerstone for the creative process. Each work is titled after the shape in question, and it is as though this foregrounded shape was a filter through which myriad decisions about the piece were made. The series is ongoing and the three existing works explore different curvatures: Euclidean geometry (*Torus*, 2016), elliptic geometry (*sphere*, 2017) and hyperbolic geometry (*Antisphere*, 2019). *Torus* is concerned with musical implications that arise from considering the significant mathematical result that there are two ways around a torus (a mathematical doughnut) that do not intersect; musical parameters are torus-shaped by design. When writing *sphere*, I was influenced by the consideration of local 'musical' space informing notions of higher dimensional global 'mathematical' space. Musical responses to two different models of hyperbolic space (the Poincaré disc and pseudosphere or antisphere) occur simultaneously in *Antisphere*; notions of negative curvature and shrinkage (for the angles in a triangle add up to less than 180° in hyperbolic space) have led to musical parameters being transformed as though through a saddle-shaped lens. In this lecture I will introduce this research by presenting musical examples alongside a commentary illuminating thought processes and transformational methodologies. Scores and recordings of *Torus*, *sphere* and *Antisphere* can be found here: <http://www.emilyhoward.com/works.php>.

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## Mathematics and music: the perspective of the listener

Michelle Phillips<sup>a</sup>

a. Royal Northern College of Music, UK

There are many ways in which we may find mathematics in music - by analysing a musical score, by a songwriter or composer talking about the techniques that they use to write music, by the notation itself having a mathematical title or mathematical annotations throughout, etc. However, this paper is concerned with another way in which we encounter music - as we hear it unfold in real time. Research regarding the perception of music has blossomed over the last 30-40

years, and has revealed aural perception to be a complex, multi-faceted, and flexible process. How we hear music depends on many factors, including not only the features in the music itself, but also our own personal tastes and characteristics, and how we respond to the music. Moreover, the environment in which we listen to music (and how our attention is divided), and the particular performance we are hearing (no two live performances are ever completely the same) contribute to this complicated web of factors which influence our sense of how we hear music as it happens. This notion that our sense of how long sections of music are may vary according to multiple aspects, for example, music perception research has shown very convincingly that our sense of musical duration changes according to how much we are enjoying the music, or how familiar it is, gives us a useful critical lens through which to view discussions of whether we can hear mathematical relationships in music. For example, Howat's (1983) statement that "[Debussy' proportional systems] they show ways in which the forms are used to project the music's dramatic and expressive qualities with maximum precision" ([1], page 1) could be interpreted as suggesting that Debussy's structures are available to perception in some way. This paper will discuss how music perception research can help us to examine such scholarship, and provide us with tools to explore mathematics and music which necessarily take account of the latest research regarding music perception.

## References

[1] R. Howat, *Debussy in Proportion: A Musical Analysis*, Cambridge University Press, 1983.

## Mathematics and the source of music

Geraint A. Wiggins<sup>a,b</sup>

*a.* Vrije Universiteit Brussel, the Netherlands

*b.* Queen Mary University of London, UK

Sometimes, we mistake mathematical models of physical phenomena for rules that define those phenomena. For example, we sometimes talk about formulae that describe the relationships between physical quantities as though they themselves were the "Laws of Physics". I think this arises partly from a desire (which is probably a Western world-view) to quantify and understand our world in direct, straightforward and incontrovertible terms: a sort of Ockham's Razor of expression. This desire extends beyond physical phenomena into cultural ones. Over the past few hundred years, Western music theory has attempted to codify what does and does not happen in (Western) music. In many cases, it does so by naming common phenomena, and placing constraints on what may occur in a performance. For example, certain chord progressions "should" be used to end a piece of music. Even when transformational creators, such as Arnold Schoenberg, set out to change the world, they have tended to do so in terms of adapting or denying existing accepted rules, and often replacing them with new ones. More incremental transformers, such as Beethoven, gently and steadily changed the rules by example, over an extended period. But throughout this evolution, indeed since the Ancient Greeks, Western philosophers of music have attempted to describe music as it appeared in their world in quasi-formal, quasi-mathematical terms. And in more recent years, some music theorists have begun to study the relationship between musical structures and mathematical structures such as groups, directly.

Our desire to understand external phenomena, in terms of rules with which we can calculate, is an important part of our modern world, without which scientific progress could not happen. But it can sometimes be misleading. In particular, to describe music as it currently exists in terms of rules begs the question of why the rules now are different from the rules that existed 800 years ago. Why, for example, did chromatic harmony not arise in Medieval Italy (except in the music of Gesualdo, who is generally dismissed as an insane outlier), and why is the English cadence beloved of Thomas Tallis no longer in use? An adequate theory of music should not only be able to describe such musical structures, but also to explain how and, ideally, why they may become more or less important over time and also how and why they can change.

However, the vast majority of music theory does not address these questions, except perhaps via the lineage of ideas passed from teacher to pupil. In particular, the vast majority of music theory focuses on the surface form of the music: what are the notes, what rules do they follow or break and so on. Only rarely does one read about the effect of music-theoretic structures on the listener, and then often only in terms of semiotic connotation: Romantic notions of musical "semantics" conveying figurative ideas (for example, "hunting the stag"). Structural theories of this kind, therefore, run a scientific risk: without a clear idea of how change in music can happen, a theory that adequately describes, say, all the music from Pérotin to Maxwell Davies, risks being so general that it distinguishes nothing from anything, and thus becomes meaningless.

The reason that attempts to describe music based on its surface are doomed to failure, or at least incompleteness, is that the surface of music is an effect, and not a cause, of the phenomenon. To see this, consider what happens in a room containing musical scores, recordings and musical instruments, but no people, and ask, "is there music in this room?" The answer is definitively, "No." There are representations of the surface of music (in the form of notes on scores, perhaps, and of digital representations of sound waves on CDs). But in the absence of a listener (who might also be player), these representations mean little: it is the interpretation of listener and/or player that turn the information into music. Furthermore, it is the

ability of the listener to hear sound (or imagine it from reading a score) and to understand musical structures – which are not explicitly present in the musical surface – that actually creates music itself.

Thus, music is fundamentally a psychological phenomenon, entirely dependent on the ability of the human mind to perceive and process sound. It is, of course, also a cultural, artistic, social, sociological, economic, and wonderful phenomenon, and many more things, too. But it is psychological first. Without the involvement of a living brain (usually, but not always, human), there is no musical activity, by definition. A complete music theory, that is capable of describing not just the surface of music but also its source in the mind, must therefore include psychology and psychological phenomena, for it is these which ultimately delimit what is musically possible.

Mathematics has its place in this psychological study too. Mathematical and computational models of perceptual and cognitive process can help us understand the mechanisms that underlie musical hearing and understanding, by permitting rigorous experimentation. With such models we can perhaps explain the general tendency towards complexity in music, for example, or why it is easy for a Greek child to clap in complex rhythms that a British professional musician might have to think twice about. In this talk, I will describe a series of models based in the mathematics of information theory, which seem to capture particular aspects of human musical experience, at levels beyond, and syntactically independent of, the surface, particularly well.

Without such a deep view of music, we attempt to study the movement of an iceberg by looking only above the waterline. It is the deep currents of mind, not the superficial glistening of notes and chords, that ultimately determine the nature of music and musical culture. To understand the iceberg of music, then, we must dive down, beyond the surface, into the musical mind.

## 2 Abstracts of Contributions

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### Some compositional implications of symmetries in microtonal pitch class sets

Paul Archbold<sup>a</sup>

*a.* School of Advanced Studies, University of London, UK

In his seminal book *The Structure of Atonal Music* [1], Allen Forte draws particular attention to the interval vector of a set and places particular emphasis on the  $Z$  relation. In microtonal pitch class sets, several sets may be connected by the  $Z$  relation but retain other distinctive properties. In this paper I explore some of these symmetrical features in microtonal sets and illustrate their application in some of my recent music.

#### References

[1] A. Forte, *The Structure of Atonal Music*, Yale University Press, 1977.

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### A framework for Topological Music Analysis (TMA)

Alberto Alcalá Alvarez<sup>a</sup>, Pablo Padilla Longoria<sup>a</sup>

*a.* Institute for Applied Mathematics (IIMAS), National University of Mexico (UNAM), Mexico

In the present article we describe and discuss a framework for applying different topological data analysis (TDA) techniques to a music fragment given as a score in traditional Western notation. We first consider different sets of points in Euclidean spaces of different dimensions that correspond to musical events in the score, and obtain their persistent homology features. Then we introduce two families of simplicial complexes that can be associated to chord sequences, and calculate their main homological descriptors. These complexes lead us to the definition of dynamical systems modeling harmonic progressions. Finally, we show the results of applying the described methods to the analysis and stylistic comparison of fragments from three *Brandenburg Concertos* by J.S. Bach and two *Graffiti* by Mexican composer Armando Luna.

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### Musical tonality and synchronization

Eyal Buks<sup>a</sup>

*a.* Technion, Israel Institute of Technology, Haifa, Israel

The current study is motivated by some observations of highly nonlinear dynamical effects in biological auditory systems. We examine the hypothesis that one of the underlying mechanisms responsible for the observed nonlinearity is self-excited oscillation (SEO). According to this hypothesis the detection and processing of input audio signals by biological auditory systems is performed by coupling the input signal with an internal element undergoing SEO. Under appropriate conditions such coupling may result in synchronization between the input signal and the SEO. We present some supporting evidence for this hypothesis by showing that some well-known phenomena in musical tonality can be explained by the Hopf model of SEO, and the Arnold model of synchronization. Moreover, some mathematical properties of these models are employed as guidelines for the construction of some modulations that can be applied to a given musical composition. The construction of some intriguing patterns of musical harmony is demonstrated by applying these modulations to known musical pieces (examples can be downloaded from <https://buchs.net.technion.ac.il/MuH/>).

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### Network topology of generalized musical spaces

Marco Buongiorno Nardelli<sup>a,b,c,d</sup>

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The abstraction of musical structures (notes, melodies, chords, harmonic or rhythmic progressions, etc.) as mathematical objects in a geometrical space is one of the great accomplishments of contemporary music theory. Building on this foundation, I generalize the concept of musical spaces as networks and derive functional principles of compositional design by the direct analysis of the network topology. This approach provides a novel framework for the analysis and quantification of similarity of musical objects and structures and suggests a way to relate such measures to the human perception of different musical entities. The analysis of a single work or a corpus of compositions as complex networks provides alternative ways of interpreting the compositional process of a composer by quantifying emergent behaviors with well-established statistical mechanics techniques.

In particular, in this talk I will also introduce the concept of dynamical score networks for the representation and analysis of tonal compositions: a score is interpreted as a dynamical network where every chord is a node and each progression links successive chords. This network can be viewed as a time series of a non-stationary signal, and as such, it can be partitioned for the automatic identification of tonal regions using time series analysis and change point detection without relying on comparisons with pre-determined reference sets or extensive corpora. I demonstrate that the essential features of tonal harmony, centricity, referentiality, directedness and hierarchy, emerge naturally from the network topology and its scale-free properties. Finally, solving for the minimal length path through a route optimization algorithm on these graphs provides an abstraction of harmonic sequences that can be generalized for the conception of generative models of tonal compositional design.

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## Assessing the feasibility of real-time analysis of timing and coordination between ensemble musicians

Tom Goodman<sup>a</sup>, Nori Jacoby<sup>b</sup>, Min Susan Li<sup>c</sup>, Maciek Tomczak<sup>d</sup>, Maria Witek<sup>c</sup>, Ryan Stables<sup>d</sup>, Alan Wing<sup>c</sup>, Max Di Luca<sup>c</sup>, Mark Elliott<sup>a</sup>

*a.* University of Warwick, UK

*b.* Max Planck Institute for Empirical Aesthetics, Germany

*c.* University of Birmingham, UK

*d.* Birmingham City University, UK

When musicians play in ensemble, they continuously adapt to each other to ensure that the group keeps time together. The musicians' timing correction can be captured by a linear phase correction model, where each player has a correction gain to other players – representing how much they adjust to one another. In addition, such models incorporate two noise components – timekeeper variance and motor variance.

The Augmented Reality Musical Ensemble (ARME) project [1] is building a system for solo musicians to practise with a virtual ensemble of AI players that will interact with the individual, and keep timing in a natural manner. To achieve this, onset detection, parameter estimation, motion capture, AI, and augmented reality techniques all need to run in real time. The work presented in this Paper analyses the feasibility of estimating the parameters of the linear phase correction model at the required speeds.

The model parameters can be estimated using the work of Jacoby et al. [2], who used the Cramér-Rao Lower Bound (CRLB) to identify the need for an additional restriction (that the timekeeper variance is strictly larger than the motor variance). This is used alongside generalised least squares estimation to extract the three model parameters for any time-series of note onsets from players.

Simulations of real-time estimation performance show that the mean error remains negligible even with little information (e.g., less than  $\sim 20$  note onsets), but the variance increases – potentially rendering a real-time system unusable. We provide insights on the effect of initial correction parameters on estimation performance, suggest a number of solutions to potentially facilitate real-time estimation, and provide comparisons with similar methods. Through these, we demonstrate the feasibility of real-time estimation required to build the interactive agents for virtual musician ensembles – such as the one the ARME project is working to build.

## References

- [1] A.R.M.E. (Augmented Reality Musical Ensemble) Project, <https://ar-me-project.co.uk/>.
  - [2] N. Jacoby, P.E. Keller, B.H. Repp, M. Ahissar, and N. Tishby, Lower Bound on the Accuracy of Parameter Estimation Methods for Linear Sensorimotor Synchronization Models, *Timing & Time Perception*, (2015).
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## SCALES - Rhythms of fractals and collective movement

Bernhard Schimpelsberger, **David P. Hewett**<sup>a</sup>, Angelika Manhart<sup>a</sup>

<sup>a</sup>. University College London, UK

In this talk we describe our recent maths-music collaboration, supported by a UCL Culture award funded by the Engineering and Physical Sciences Research Council (EPSRC). As a percussionist and composer, Bernhard Schimpelsberger is guided by the mantra “rhythm is everywhere”. At the age of 15 he embarked on an in-depth study of Indian classical rhythms, and many years in India under the tutelage of rhythm master Suresh Talwalkar exposed him to arguably the most logical, complex and comprehensive rhythm system in the world, inherently based on number patterns, and capable of creating an infinite array of rhythmical compositions that dazzle with mathematical complexity. India’s rhythm language, Konnakol, allows rhythms to be sung before being played on any instrument. Bernhard’s expertise in Konnakol and his fascination for number patterns were the foundation for his collaboration with UCL mathematicians David Hewett and Angelika Manhart, who work respectively on acoustic and electromagnetic wave scattering by fractals (David) and collective motion in biology (Angelika). What connects their work is a mutual interest in understanding how small-scale structures and interactions give rise to large-scale wave patterns. They were attracted to work with Bernhard, sensing an exciting opportunity to explore how the formal mathematical ideas in their research might map onto a creative and emotional field such as music. The collaboration involved (i) an exploration of how mathematical structures and concepts in wave scattering by fractals and collective motion of bacteria (such as self-similarity and multiscale dynamics) might translate into rhythmical motifs, structures and textures; (ii) the composition of the piece SCALES (Schimpelsberger, 2021); (iii) the development and delivery of interactive workshops in which we shared our findings and explored excerpts from the composition with a group of amateur musicians aged 8-80; and (iv) the production of a Youtube “Rhythm Diary” documenting our interactions [1].

### References

- [1] *Rhythm Diary*, Rhythms of fractals and collective movement, A collaboration between percussionist/composer Bernhard Schimpelsberger and mathematicians Dr Angelika Manhart and Dr David Hewett from University College London, *Scales* (composed by Bernhard Schimpelsberger, 2021), commissioned by Performance Lab (UCL Culture), <https://www.youtube.com/watch?v=xexC7b39gZg>, 2021.

## Well-tempered complexity

Jeffrey R. Boland<sup>a</sup>, **Lane P. Hughston**<sup>b</sup>

<sup>a</sup>. Syndikat LLC, Los Angeles, USA

<sup>b</sup>. Goldsmiths University of London, UK

For centuries, musicians and mathematicians alike have struggled with the vexing challenges of temperament and tonality. The system of equal temperament, in which semitones are tuned in the irrational ratio of  $2^{1/12} : 1$ , is best perhaps seen as a serviceable compromise, sacrificing purity for flexibility. Just intonation, in which the ratios of all notes are taken to be rational, based on powers of 2, 3, and 5, is more natural, but is of limited flexibility. Here we propose a new scheme in which ratios of Gaussian integers form the basis of an abstract tonal system. The tritone, so problematic in just temperament, is in our scheme represented by the complex ratio  $1 + i : 1$ . The just major and minor tones, given by intervals of  $9/8$  and  $10/9$ , can each be factorized into products of complex semitones, giving us a major complex semitone  $3/4(1 + i)$  and a minor complex semitone  $1/3(1 + 3i)$ . The perfect third, given by the interval  $5/4$ , can likewise be factorized into the product of a complex tone  $1/2(1 + 2i)$  and its complex conjugate. Once augmented with these supplementary tones and semitones, the resulting scheme of complex rational intervals leads very naturally to the construction of a complete system of major and minor scales in all keys.

## Meeting in the shadows

**Dorothy Ker**<sup>a</sup>

<sup>a</sup>. University of Sheffield, UK

What does it feel like to move around on a Möbius strip? How do I know I am on one? What does this beguiling mathematical phenomenon afford a composer for conceptualising musical materials, behaviours and structural possibilities? I found myself exploring these questions while writing *Fauna of shadows* (2022), a duo for contrabass clarinet and bass flute. They point to wider questions about how we relate mathematical objects to the tangible, body-oriented space of

what may be commonly regarded as ‘musical’. In considering how abstract mathematical conceptions (e.g. unbounded spaces, higher-dimensional topologies, symmetries, infinities, or irrational numbers) might be creatively fruitful to explore musically, it is pertinent to ask what it is that draws us to them, and how they relate to the body. In their book *Where mathematics comes from: how the embodied mind brings mathematics into being* [1], Lakoff and Núñez posit the origins of mathematical notions within conceptual metaphors. In this presentation I explore whether their insights may help to illuminate the vital connection between mathematics and music that drives a composer’s instinct for mathematically-guided compositional exploration. Other key reference points are the writings of Henri Poincaré and J.L. Borges, and earlier works of mine, including *Amelia and the Mapmaker*, which explores the exhilarating mathematics of the Poincaré Conjecture.

## References

- [1] G. Lakoff, and R. Núñez, *Where Mathematics Come From: How The Embodied Mind Brings Mathematics Into Being*, Basic Books, Perseus Books, 2001.

## Investigating the relationship between linguistic measures and self-similarity matrices in assessing musical structure

Zani Ludick<sup>a</sup>, Wilben Pretorius<sup>a</sup>, Trudie Strauss<sup>a</sup>

<sup>a</sup>. University of the Free State, South Africa

Generating music with long-term structure is an ongoing challenge which has enjoyed much recent attention in deep-learning literature. However, assessments of the musical structure of generated music often rely on subjective evaluations, such as time-consuming listening tests. Automatic assessment techniques for identifying elements of structure in compositions provide an objective alternative which may enhance the assessment and generation of structure in music.

Self-similarity matrices (SSMs) can be used to capture and represent structural patterns in music. Identified blocks, paths and corners in SSMs correspond respectively with areas of homogeneity, repetition and novelty. However, the ability of SSMs to capture structural information depends on the choice of features (such as pitch, rhythm, or dynamics) and the size of the time window used to calculate the feature vectors.

If one considers music and language to be similar in structure, several linguistic measures, such as measures of repetitiveness, complexity, and power law characteristics, may lend themselves to the describing of features in musical structure. This study investigates the meaning of these linguistic measures when interpreted in terms of musical structure and aims to describe links between the use of these measures and self-similarity matrices. This should contribute to automatic music structure detection and possibly also the generation process of music.

This research encompasses aspects of computational linguistics, music structural analysis and automatic music generation by suggesting an approach that augments the structure detection abilities of self-similarity matrices with computational linguistic measures. It is argued that in introducing such an augmented approach the need for panels and human interference is reduced by allowing for the automatic assessment of musical structure.

## Heart rate variability and the impact of music listening on autonomic state

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Heart rate variability(HRV), the variation in the time interval between consecutive heartbeats, is a proven measure for assessing changes in autonomic activity. Music has been shown to affect the limbic system, respiratory rate, and blood pressure. Significant changes in HRV have been reported for musical characteristics such as tempo, genre, and valence. There have been few empirical investigations on the effect on HRV compared to mean heart rate(HR). Most studies have been experimental rather than interventional.

This pilot study aimed to evaluate the impact of short duration music listening on the autonomic nervous system response of healthy adults. Six participants were tested to investigate the effect on HR and HRV. Electrocardiographic(ECG) data was recorded at a sampling rate of 1000 Hz using an eMotion Faros 360 device while participants listened to four pre-selected songs in a random order separated by a relaxation period of 5 minutes. Data was cleaned and processed through *Kubious\_HRV\_2.0* software. Statistical analysis using the Wilcoxon signed rank test was performed. For all but one song shorter than 3 minutes (song 1), we observed a statistically significant increase in standard deviation of the RR intervals (SDRR) (song 1:  $P = .125$ ,  $r = .333$ ; song 2:  $P = .023$ ,  $r = .575$ ; song 3:  $P = .014$ ,  $r = .635$ ; song 4:  $P = .014$ ,  $r = .635$ ) and in the Low Frequency (LF) component of the cardiac spectrogram (song 1:  $P = .300$ ,  $r = .151$ ; song 2:

$P = .038$ ,  $r = .514$ ; song 3:  $P = .014$ ,  $r = .635$ ; song 4:  $P = .014$ ,  $r = .635$ ) with a large effect size  $r$ , indicating increased HRV. No significant change in mean HR was observed (song 1:  $P = .173$ ,  $r = -.272$ ; song 2:  $P = .058$ ,  $r = -.454$ ; song 3:  $P = .125$ ,  $r = -.333$ ; song 4:  $P = .232$ ,  $r = -.212$ ). Songs of 3 minutes 30 seconds or longer were associated with significant increases in HRV measures, especially SDRR and LF. Music thus has the potential to overcome autonomic nervous system(ANS) dysregulation, benefitting health and wellbeing.

## Math and religion in Arvo Pärt's *De Profundis*

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Arvo Pärt is an Estonian composer who began his career under the influence of twelve-tone music and extensive use of dodecaphonism. From 1968 to 1976, he isolated himself to search for a new path as a composer. It was in this period that Pärt converted to Russian orthodoxy. He changed his way of composing to what he called *tintinnabuli*, a musical technique that combined triadic chords with simple melodies. Recent analysis on Pärt demonstrates that mathematics is an influence before and after his secluded period.

The present discussion deals with the relationship between math and religion in his work *De Profundis* (1980). We will base our analysis on the *Guidelines for Style Analysis* by Jan LaRue [1], which, in a general sense, proposes a three dimensions analytical method. The first dimension will consider the work as a whole, and elements such as the division between movements and entire sections are analysed. This approach tries to take a picture of the total and grasp its significance. In a second analytical stage, the investigation will be more profound by considering medium aspects, like relevant sentences, paragraphs, and parts of a section. The final phase examines the minimum elements of the piece as motives, sub phrases, and minute fragments. Our focus is on the text, a very important psalm for the Christian tradition, patterns used by the composer, and the role of the organ, which presents a rhythm with the *tintinnabuli* system. The analysis of these elements demonstrates that Pärt uses maths as a formal element, and the composer combines musical parameters to execute a personal and innovative interpretation of the text.

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## Points of convergence - A dialogue between music and dynamical systems research

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In 2015, Emily Howard (now Professor at the Royal Northern College of Music) was a Leverhulme-funded Composer in Residence at the University of Liverpool's Department for Mathematical Sciences. During this residency, we had a number of discussions about some of my recent research in the area of Dynamical Systems. This led to a number of compositions, and in the longer-term contributed to changes in Prof. Howard's practice as well as the establishment of the Centre for Practice & Research in Science & Music (PRiSM) at the RNCM.

I will speak about the experience of this collaboration from a mathematician's point of view. I will focus on two aspects that I believe contributed to the success of this dialogue between cutting-edge mathematical research and music. The first was having a shared language (Prof. Howard has a degree in mathematics and computing, and I am an amateur musician), while the second concerns specific features of the piece of research we discussed that made it suitable for fruitful conversations.

## Applying information theory and statistics to numbers in music

Alan Shepherd

There is a large amount of literature claiming to find numerical significance in musical works, particularly that of J. S. Bach. There are two main areas, one being the use of numeric alphabets to find coded words or messages and the other the theory that some composers structured their works to have certain proportions within the lengths such as 1:1 or 1:2 or the golden section.

For the first area I use information theory to define some criteria for including or discovering a message, and use basic statistics, partitioning and permutations to show the extent of ambiguity between encoding and decoding and the effect of using different numeric alphabets.

For the second area I introduce further statistical methods – probability, hypothesis testing, Monte Carlo simulation, Bayesian Networks – and show how these can be applied to proportions.

Taking Ruth Tatlow's work [1] as a basis and using combinatorics to find all possible ways of dividing a collection into proportions, I have developed a computer program to analyse the possibilities. This finds all occurrences of a given proportion over multiple layers and uses a technique of signatures to find patterns in the results. It can also generate random samples of equivalent collections to investigate how the actual work compares with these.

The presentation will summarise the techniques and some of the results from the book [2].

## References

- [1] R. Tatlow, *Bach's Numbers – Compositional Proportion and Significance*, Cambridge University Press, 2015.
- [2] A. Shepherd, *Let's Calculate Bach – Applying Information Theory and Statistics to Numbers in Music*, Springer Nature, 2021.

## The complex sum of simple parts: how LV/xi produces an immersive whole through simultaneous variation

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*LV/xi* is an eleven language immersive sound installation for performance practice-based research exploring an adapted Implicit Body Framework (Nathaniel Stern, [1]).

Similar installations are commonly limited to using a single language and often contain nonverbalised sounds. The eleven languages sung across 66 speakers makes this work unique in its conception.

The diverse sources for and complex arrangement of the work and the incorporation of digital voices necessitated the use of unifying elements to fuse the work into a cogent whole.

Mathematical ideas were developed, adapted and possibly even corrupted both in the design of the machine to perform the work and in the musical content of the work itself. Elements such as a simple scalar geometry result in a complex outcome through varied simultaneous repetitions or permutations which were used to bring order to the chaos that could otherwise ensue.

Several versions were explored and different approaches attempted before the machine was built or the musical content developed. Several avenues of development were abandoned before the final version was produced.

Despite expecting the completed work to be a wall of white noise as each language is simultaneously centred on a discrete pitch class, the geometry determining the design of the machine and the aleatoric and algebraic solutions controlling and defining the pitch classes and durations within each voice result in movements within the eleven minute duration that have a discernible form and structure and harmonies and vocal lines that are sometimes coincidentally even beautiful (explored through Dmitri Tymoczko's *Geometry of Music* [2]).

Without the mathematics used in the conception, development and building of both the machine and the music it performs, the experience of the sound installation would be substantially inferior and more incoherent. The underlying mathematical principles have allowed the machine to be visually satisfying and the musical content to sing.

## References

- [1] N. Stern, *Interactive Art and Embodiment: The Implicit Body as Performance*, Gylphi, 2013.
- [2] D. Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice*, Oxford University Press, 2011.

### 3 Abstracts of Posters

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#### Music development with the use of neural networks: an approach to Iannis Xenakis's music

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The role of Artificial Intelligence in music is increasingly gaining interest among researchers. However the use of A.I. for the analysis of avant-garde music remains limited. This work explores how musical patterns in Iannis Xenakis's music can be identified, learned and reproduced using Deep Learning algorithms, such as Neural Networks. The final outcome is a set of sound "generators", each of them reflecting a musical pattern of Iannis Xenakis.

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#### Mathematics behind the Multiverse of Music

Pranav Bhatt

Music, a disguised form of mathematics, whilst most interpret the vibrations as a symphony of art, the mathematician views it as a set of frequencies that creates a physiological impact far deeper than just divertimento.

My paper aims to explore the true depths of the mathematical application in the broad world of music. Mathematics from Thales of Miletus to Simon Singh. Music from the Bible till the latest radio hits. The most ancient forms of civilisation built around mathematics and music, they are the two ancient languages that coexist in a beautiful harmony.

First I explore the historical research of the mathematics behind the art of music and reflect on how the experience of history has shaped this journey. I then move on to evaluate these ideas and build on them based on new mathematical discovery and innovations in musical technology. Something I found interesting in my research was just how much music has evolved and changed over the decades even up to the last three years,

Vibration theory in mathematics skims over, infinite series, complex function and Fourier integral transforms. As well as the Physical aspects which can be expressed through Newtonian mechanics and general data analysis, stress discomfort and the deeper human psychology.

I want to specialise in the particularity of Asian instruments whilst looking at the mathematics from India and other oriental countries and how they believe that music is infinitely powerful, from chants such as Om, to ancient beliefs connected to religion; I strive to explore the aspects of this topic that have remained hidden; the culture of music and history of mathematics, from the mountains in Macau to the ravines of Kashmir. My paper explores The Mathematics Behind the wide Multiverse of Music, from hundreds of years ago till less than a week ago.

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#### Using group theory to identify musical patterns

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Music and group theory share similar features and it is tempting to apply group theory, or properties of group theory, to musical composition. In group theory a finite group is a set with a finite number of elements on which a binary operator is defined. The set of numbers with the binary operator have certain properties that define them as a group. As an example, a group can be formed by a set of twelve integers under the operator addition module twelve. By identifying each set element with a note in the musical scale of twelve semitones, one can translate the various operations performed on the group into music. Using the same principle, one can encode an existing composition into a set of numbers and then study those numbers from the perspective of group theory. Musical concepts such as translation, inversion, retrogression and rotation (in time as well as in pitch) become symmetry operations on the mathematical group. We demonstrate the latter by applying group theory to two compositions: *Art of Fugue* by Johann Sebastian Bach and the Queen anthem *We Are The Champions*. Doing this highlights the compositional techniques used and provides a measure of the richness of musical patterns within each composition. Also, an analysis like this shows if the creator of the music has a predilection for certain compositional techniques and hence may provide a signature associated with a certain individual, musical group or musical genre.

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## Approaching multi-pitch estimation with geometric and combinatorial modelling

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Multi-Pitch Estimation (MPE) is one of the first steps toward Automatic Music Transcription (AMT) – arguably Music Information Retrieval's (MIR) closest parallel to the total Turing test. Recently, the overwhelming majority of approaches have relied on Deep Learning methods, which – whilst often incredibly effective – generally elicit little information or understanding of the underlying structures being approximated.

We present a non-Machine Learning model in which the problem of MPE is reduced to a problem of distinguishing between 'real' and 'false' fundamentals on a discretised infinite cylinder. On this cylinder, which is a product of the circle of fifths (pitch chroma) and the integers (pitch height), a fundamental and its first three harmonics traces out one of two shapes – a turnstile (⊥), or a gamma (Γ). From this perspective, MPE can be seen as the decomposition of a cluster of cells on the cylinder into its constituent turnstiles and gammas, and 'false' fundamentals are harmonic tones that present with these shapes due to overlap.

Initially we explore the model from a discrete viewpoint — one that is generally understudied in the field — before incorporating the notion of intensity and assigning Real values to tones. We further provide an in-depth characterisation of precisely the ways in which so-called edge cases can occur, looking in particular at the notion of 'basic' edge cases — ones in which the constituent parts of a false fundamental are satisfied precisely once. From there, we reduce their occurrence to eight basic edge types (and a ninth type, which we prove is the only irreducible non-basic type).

Computationally, we present results of analysing simulated data on the model, highlighting the prevalence of the various types with respect to the number of simultaneous fundamentals present. Finally, we provide insight into applying the model to 'real world' MPE, and suggest a number of future extensions and investigations.

## Rhythmic complexity effects on ensemble timing linkage with a violin duo

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How do music ensembles maintain relative timing in the absence of a conductor? The Linear Phase Correction Model developed for classical string quartets proposes that each player corrects the timing of their next note in proportion to the asynchrony with fellow players on the previous note [1]. We applied the model to investigate how violin-playing participants synchronise to a violin duo to explore the effects of melodic similarity, rhythm complexity and constraints on interpersonal information flow. Participants played the melody part in synchrony with a violin duo, in which we manipulated the performance mode of the melody player (live/recording) and the metrical structure of the accompaniment player (simple/complex rhythm). Participants then rated the perceived influence on their playing of the two duo members. Timing performance revealed lower correction gain to the accompaniment player estimated with the phase correction model. This effect was more pronounced when the melody player was live. Complementing this finding, ratings indicated greater perceived influence of the melody than the accompaniment, especially in the simple rhythm condition. These patterns of correction and perceived influence indicate that players favour taking correction cues from the score part that is more similar to their own in terms of melody and rhythmic structure.

### References

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## Discrete Fourier Transform unveils decreasing diatonicity and increasing fragmentation in Debussy's piano music: a diachronic corpus study

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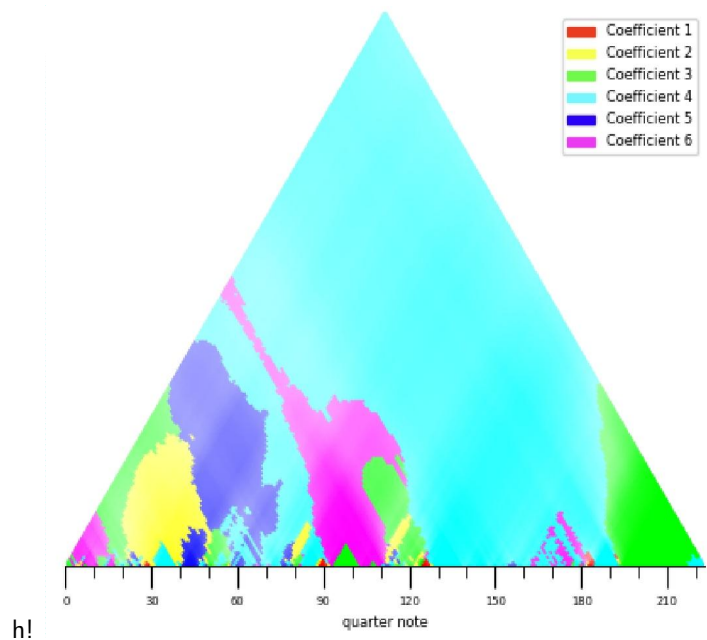


Figure 1: Summary wavescape of Debussy's Prelude "Ondine". Each point in the plot is a node corresponding to a segment of the piece: the root node (top) is associated with the entire piece, the leaves (bottom) with quarter-note segments. The hue of each node reflects the strongest DFT component in the corresponding segment, the opacity reflects its magnitude.

In this study, we propose a quantitative, corpus-based approach to investigating the diachronic evolution of Debussy's style. Specifically, we exploit the music-theoretical interpretability of the Discrete Fourier Transform (DFT), which decomposes a pitch-class distribution into components that resonate with different scalar prototypes [2, 1, 7]. Based on musicological literature, we hypothesise a decrease over time in diatonicity (5th component) in favour of symmetric pitch-class materials such as octatonic (4th) or whole-tone (6th) scales, and an increase in fragmentation in terms of the use of different types of pitch-class materials at different time-scales within individual pieces [3, 4, 6]. Drawing from representations previously introduced as wavescapes [5], we map each segment of a piece into a node of a hierarchical graph, the summary wavescape (Figure 1). Each node of the summary wavescape is labelled with the index of the DFT component with highest magnitude in the corresponding segment, and we consider connected regions with identical labels to form a partition of the summary wavescape.

We computed the relative size and the moment of inertia of the wavescape's regions labelled with a given DFT component as measures of the prevalence and hierarchical prominence of the corresponding type of pitch-class material in the piece, respectively. Both measures were found to decrease over time for the 5th component, whereas an opposite trend was found for the 4th component. We further computed the degree of fragmentation of a piece as the measure-theoretic entropy of the summary-wavescape's partition, which was found to increase over time. Overall, these results provide quantitative support for music-analytical insights and contribute to understanding the analytical value and limitations of DFTs and wavescapes as tools for computational corpus studies.

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## Musica universalis and Harmonices Mundi - on mathematical and physical aspects of Renaissance and Baroque music and the context of space on the basis of historical treaties

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*Musica universalis*, also called the music of the spheres or the harmony of the spheres, is a philosophical concept that treats about the proportions in the movements of the celestial bodies - the Sun, the Moon and the planets – as a form of music. This theory, which originated in ancient Greece, was the principle of Pythagoreism and later developed it 16th-century astronomer Johannes Kepler.

The concept of “music of the spheres” includes the principle that mathematical relationships express qualities or “tones”, energies that are manifested in numbers, visual angles, shapes and sounds – everything is combined in the formula of proportion. Pythagoras suggested that the Sun, Moon and planets emit their own unique noise based on their orbital rotation, and that the quality of life on Earth reflects the tone of the heavenly sounds that are physically imperceptible to the human ear.

In 1619, Kepler published *Harmonices Mundi*, developing the concepts he introduced in *Mysterium* and assuming that musical intervals and harmonies describe the movements of the six known planets of the time. In this work he tried to explain the proportions in the world (especially in astronomical objects) in terms of music, referring to the Pythagorean music of the spheres.

The musical treatise *Musurgia Universalis* by Atanasius Kircher turned out to be the most exhaustive of his time. Besides the historical and analytical parts that recreate many entire compositions, there are long chapters on more complex areas of knowledge. Kircher attaches great importance to the numerical basis of intervals, scales, and costumes, thus demonstrating that music is essentially a branch of mathematics, and thus an image of God's creation of “all” things in number, measure and weight. Kircher in his treatise explains the harmonious organization of the human body and elements and the solar system itself.

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## A network science approach to efficient voice-leading

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We study the neo-Riemannian principle of parsimonious voice-leading using tools and techniques from classical graph theory and the modern field of complex networks. We quantify the relative importance of particular chords within this framework. The graph-theoretic notion of eccentricity suggests that when working in a harmonic scheme dictated by any common musical scale, no triad is any more isolated than any other. Complex network theory refines this idea, and in this context provides measures of how important particular triads might be for the flow of chord progressions through the harmonic network. We review and compare several different such measures of centrality and communicability.