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CONFERENCE ABSTRACT BOOK

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Learning Consistent Discretizations of the Total Variation

Speaker – Thomas Pock
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In this work, we study a general framework of discrete approximations of the total variation for image reconstruction problems. The framework, for which we can show consistency in the sense of Gamma-convergence, unifies and extends several existing discretization schemes. In addition, we propose a piggy-back primal-dual algorithm for learning discretizations of the total variation in order to achieve the best possible reconstruction quality for particular image reconstruction tasks. Interestingly, the learned discretizations significantly differ between the tasks, illustrating that there is no universal best discretization of the total variation.

Regularising Inverse Imaging Problems Using Generative Machine Learning Models

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We consider the use of generative models in a variational regularisation approach to inverse problems. Generative models learn, from observations, approximations to high-dimensional data distributions. The considered regularisers penalise images that are far from the range of a generative model that has learned to produce images similar to a training dataset.

In contrast to other data driven approaches, these regularisers do not require paired training data and are learnt independently of the forward model.

Their success depends on the quality of the generative model and so we propose a set of desired criteria to assess models and highlight avenues for future research.

Convergence analysis of Tikhonov regularization with oversmoothing penalty for nonlinear statistical inverse learning problems

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ABSTRACT. In this paper, we study the Tikhonov regularization scheme in Hilbert scales for the nonlinear statistical inverse problem with a general noise. The regularizing norm in this scheme is stronger than the norm in Hilbert space. We focus on developing a theoretical analysis for this scheme based on the conditional stability estimates. We discuss the high probability estimates of the direct and reconstruction error in terms of sample size. The rates of convergence are established over regularity classes defined through appropriate source conditions using the reproducing kernel approach. Our results improve and generalize previous results obtained in related settings.

This is the joint work with Prof. Peter Mathe.

Deep Learning – accelerated Bayesian inference of our Universe

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Bayesian inference of parameters that describe our cosmological model is the standard approach to the solution of the inverse problem encountered in the analysis of astronomical data. Upcoming experiments will provide us with an unprecedented amount of such data, capable of revolutionizing our understanding of the Universe. However, the challenges posed by the size of these datasets dangerously hinder the feasibility of their Bayesian analysis. In my talk I will present a Bayesian framework for orders-of-magnitude accelerated inference of cosmological parameters, based on Deep Learning surrogate models tested on both currently available as well as simulated, next-generation astronomical datasets.

Inverse Problem of aerosol property retrieval from light scattering measurements

Romana Boiger, Paul Scherrer Institute, Switzerland
Joint work with: Rob L. Modini, Alireza Moallemi, David Degen, Martin Gysel-Beer, Andreas Adelmann

Understanding the influence of atmospheric aerosols on the climate is highly relevant to assess the human impact on climate change.
To capture aerosol properties, in-situ multi-angle light scattering measurements are done. The inverse problem is now to retrieve the aerosol properties from those measurements. In this work we propose a new method based on invertible neural networks. This method proves to be quick and precise and it is capable of simulating also the forward direction, without further training. Applicability and performance of the method is shown with simulated data, that mimic laboratory and field measurements.

Metal object characterisation using harmonic generalised polarization tensors and symmetry groups

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January 5, 2022

Hidden metal objects can be located with electromagnetic fields, eg mines and unexploded bombs. The generalized polarization tensor (GPT) gives coefficients of an asymptotic expansion of the response. Many mortar bombs and cluster bomblets are symmetric under the action of a group of rotations and reflections. We introduce harmonic generalized polarization tensors (HGPT) using a basis of products of harmonic polynomials. We show the number of independent coefficients needed to characterise objects can be reduced using the symmetry group of the object and propose an approach for determining the subspace of harmonic polynomials fixed by the group. Thus we determine the independent coefficients for different groups.

Characterisation and Classification of Metallic Threat Objects for Security Screening

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Based on an asymptotic expansion of the perturbed magnetic field for eddy current problems, we have derived an economical magnetic polarizability tensor (MPT) object description. The MPT is a function of the object's properties and the exciting frequency. We employ a finite element method, accelerated by a reduced order model and scaling results, to efficiently compute a large dictionary of invariants of MPT spectral signature characterisations of realistic threat and non-threat objects relevant for security screening using metal detection. We will discuss the performance of probabilistic and non-probabilistic machine learning classifiers for identifying hidden threat objects learnt from our dictionary.

Title: Seeing through light: sparse and learning methods for super-resolution fluorescence microscopy

Luca Calatroni (CNRS & Nice University, France)

Abstract: Super-resolution fluorescence microscopy techniques overcome the physical barrier of light diffraction allowing for the observation of indistinguishable sub-pixel entities of high relevance in several biological imaging applications. State-of-the-art methods achieve adequate spatio-temporal resolution under challenging experimental conditions by means either of costly devices and/or specific fluorescent molecules. Over the last decade, a major effort in the field has thus been made in order to develop `democratic' super-resolution techniques able to adapt to common microscopes and conventional fluorescent dyes. From an inverse problem perspective, encoding prior structural assumptions on the samples observed and/or data-driven information can be beneficial in this respect. In this talk, we focus on a model for 2D and 3D super-resolution microscopy based on sparsity and non-convex optimisation. Two approaches are considered: in the former, the sparsity assumption on the fluorescent molecules as well as their temporal and spatial independence are integrated in a covariance domain, where a sparse and non-convex optimisation problem is formulated for retrieving molecule locations precisely. In the latter approach, a Generative Adversarial Network (GAN) is used to estimate directly intensity information by comparisons with training data and by means of a physically-inspired generative network.
The proposed approaches are able to retrieve noise and background information as well as sample intensities, a valuable piece of information for 3D super-resolution imaging. Automatic parameter selection strategies based on algorithmic restarting and discrepancy-type techniques are discussed and several results both for simulated and real data are reported as well as comparisons with state-of-the-art approaches.

Unifying convergence analysis for degenerate preconditioned proximal point algorithms

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We tackle degenerate preconditioned proximal point algorithms, to solve monotone inclusion problems. We show that it is possible to make use of positive semi-definite preconditioners instead of positive definite ones. We establish weak convergence results under mild assumptions that can be easily employed in the context of splitting methods for monotone inclusion and convex minimization problems. Moreover, we show that the degeneracy of the preconditioner allows for a reduction of the variables involved in the iteration updates.

We show the strength of the proposed framework, giving a new perspective on existing splitting schemes simplifying the analysis of convergence, and paving the way for new algorithms.

SGD in Banach spaces

Zeljko Kereta

In this work we investigate and develop the mathematical framework forstochastic gradient descent in Banach spaces. Since the gradient of a Banachspace objective belongs to the dual space the resulting methods are non-linearand require the use of duality mappings, which depend on the ambient Banachspaces. We focus on methods for solving inverse problems using Banach norm basedobjectives and study their convergence properties. We discuss adaptations thatare needed for non-linear problems, which require assumptions on conditionalstability of the inverse problem.

Automatic balancing parameter selection for Tikhonov-TV regularisation

Dr Silvia Gazzola

This talk presents a new algorithm for regularising linear ill-posed inverse problems whose solutions
can be represented as sums of smooth and piecewise constant components. The regulariser is expressed as the sum of a 2-norm Tikhonov term and a total-variation (TV) term, which are balanced by a scalar parameter. We solve such regularised problems by applying the alternating direction method of multipliers (ADMM), which incorporates an optimal adaptive choice of the balancing parameter according to statistical principles and a novel way to estimate noise in the data. The new method is validated on tomography and geophysical imaging test problems.

Inverse problems in tomography – from theory to practice

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Tomographic imaging plays an important role in many applications, ranging from medical CT, materials science, archeology, cultural heritage, and industrial quality control. The classical inverse problem of inverting the X-ray transform has been well-studied and serves as a testbed for algorithm development. In many applications a conventional tomographic reconstruction does not suffice, and more comprehensive approaches are needed. This may include a more complex forward model, or specific prior information. In this talk, I will give an overview of several inverse problems arising from tomography, including foreign object detection, calibration, experimental design, multi-object reconstruction, and dynamic tomography.

Photoacoustic and Ultrasonic Tomography for Breast Cancer Imaging

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New high-resolution 3D imaging techniques can probe the breast without delivering harmful radiation. In particular photoacoustic tomography (PAT) and ultrasound tomography (UST) promise to give access to high-quality images of tissue parameters with important diagnostic value. However, the involved image reconstruction problems are very challenging from an experimental, mathematical and computational perspective. In this talk, we illustrate some of these challenges based on a clinical feasibility study that used a prototype scanner for combined PAT-UST. After reviewing our current results, we will discuss future developments that will broaden the range of clinical scenarios in which these techniques are viable.

On the Frequentist Accuracy of Bayesian Uncertainty Quantification Procedures in Inverse problems in Imaging
Speaker: David Thong, The Maxwell Institute and Heriot-Watt University

One of the main strengths of Bayesian imaging strategies is their ability to quantify uncertainty in the solutions delivered and support inferences such as hypothesis tests. If the marginal distribution the unknown image was known and used as a prior distribution in a Bayesian model, the probabilities from a Bayesian analysis would also be valid in a frequentist sense (i.e., Bayesian probabilities would coincide with the frequencies obtained over many repetitions of the same experiment). For example, if a large number of replicate experiments were performed, the 95% Bayesian credible region would cover the true image in 95% of the experiments. However, the prior distributions used in Bayesian imaging models are not specified this way. There are many methods for assessing the adequacy of a prior in terms of a point estimate (e.g. a MAP estimate), but no methods exist to quantify a prior's reliability in terms of the credible regions obtained. We propose a procedure for estimating the coverage probability of a given prior's credible region, and thus a measure of the prior's reliability with respect to uncertainty quantification.

We conducted a study in which the coverage properties, under a TV prior, a TGV prior, and a Plug-and-Play (PnP) prior, were estimated through Monte-Carlo. Using large image datasets as a sample from the distribution of the images (or class of images) of interest, credible intervals were obtained from samples of the posterior distribution using the MYULA (or PnP-ULA) algorithm. The regularization parameters (for the TV and TGV priors) were estimated using an empirical Bayes approach. The methodology is illustrated on a range of examples, such as non-blind deblurring and MRI-like reconstruction from a limited number of radial lines. We find that the credible intervals are conservative but highly reliable, in the TGV and TV cases, and include the true image with high probability. With PnP priors the credible intervals were found to be overconfident.

Stochastic Normalizing Flows and the Power of Patches in Inverse Problems

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Joint work with Fabian Altekrüger, Paul Hagemann, Johannes Hertrich (TU Berlin)
Alexander Denker (U Bremen).

We show that patch-based methods in connection with normalizing flows provide a powerful framework for solving inverse problems in situations, where only limited training data are available. Then, to overcome topological constraints and improve the expressiveness of normalizing flow architectures, we introduce stochastic normalizing flows which combine deterministic, learnable flow transformations with stochastic sampling methods and variational autoencoders.
We consider stochastic normalizing flows from a Markov chain point of view which provides a mathematical sound way to combine the different approaches. In particular, we replace transition densities by general Markov kernels and establish proofs via Radon-Nikodym derivatives which allows to incorporate distributions without densities in a sound way.

Lifted Bregman Networks

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Abstract

A combination of gradient-based optimisation method and gradient computation via backpropagation is often the method of choice when it comes to training deep neural networks. In this work, we focus on alternative training strategies that are based on recent developments in distributed optimisation of deeply nested systems (Carreira-Perpinan & Wang 2014). In particular, we propose a novel training approach that is based on proximal activation functions and generalised Bregman distances. The main advantages of this approach compared to previous approaches is that it enables (potentially distributed) training of neural network parameters without having to differentiate activation functions.

A Mathematically Tractable Deep Generative Model for Inverse Problems

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This talk is concerned with the development, analysis and numerical realization of a novel variational model for the regularization of inverse problems in imaging. The proposed model is inspired by the architecture of generative convolutional neural networks. In contrast to conventional neural-network-based approaches, inspired by the Deep Image Prior [Ulyanov et al "Deep image prior." 2018.], the convolution kernels are learned directly from the measured data such that no training is required. Contrary to the original Deep Image Prior, however, our method allows for theoretical results about existence/stability/regularity of solutions and convergence for vanishing noise and a significantly reduced number of parameters without compromising performance. In the talk, we discuss the theoretical analysis and show results of our method applied to different imaging problems, such as inpainting, denoising, deblurring and JPEG decompression.

Sensor Optimisation in Seismic Imaging via Bilevel Learning

Ms **Shaunagh Downing**

As part of the seismic imaging process, waves are emitted from a source into the earth and sensors record the resulting signal. An image of the subsurface is created using Full Waveform Inversion (FWI). In this talk, it is shown that, given prior information about the likely make-up of thesubsurface (in the form of one or more training models), one can optimise the location of thesensors to retrieve the best possible information about the subsurface. This problem is considered inthe framework of bilevel learning, where the upper level is the sensor optimisation problem, and thelower level is the FWI problem.

Tomography of the fast-ion velocity distribution in tokamak plasmas with a physics-based prior

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In this inverse problem, we compute the velocity distribution of fast ions in a small volume of a tokamak plasma from measurements of the Doppler shift of photons emitted from the plasma. This inverse problem is severely underdetermined, and there is a demand for incorporating prior information about the solution in order to stabilize the inverse problem. A successful way to do this is to express the solution in a physics-based basis consisting of so-called slowing-down functions that reflect the statistical behavior of fast ions in the plasma [2].

We demonstrate the usefulness of this approach, and that it is equivalent to using a general-form Tikhonov regularization term [1] that dampens undesired high-frequency components.

1. P. C. Hansen, *Oblique projections and standard-form transformations for discrete inverse problems*, Numer. Lin. Alg. Appl., 20 (2013), pp. 250-258, doi: 10.1002/nla.802.
2. B. Madsen, J. Huang, M. Salewski, H. Järleblad, P. C. Hansen + 19, *Fast-ion velocity-space tomography using slowing-down regularization in EAST plasmas with co- and counter-current neutral beam injection*, Plasma Physics and Controlled Fusion, 62 (2020), 115019, doi: 10.1088/1361-6587/abb79b.

Title: Magnetic Resonance Electrical Impedance Tomography with incomplete data

Kim Knudsen and Hassan Yazdanian,3rd IMA Conference on Inverse Problems from Theory to Application

Abstract:

Imaging modalities such as Electrical Impedance Tomography (EIT), Current Density Impedance Imaging (CDII) and Magnetic Resonance EIT (MREIT) are designated to recover the interior electrical conductivity from indirect measurements. Under favorable conditions, the resulting tomographic problems in CDII and MREIT are fairly well-posed, however, in many practical applications the information is only partial and challenges occur.

In this talk we will focus on CDII and MREIT when only incomplete is available. We systematically investigate the limitations to stability, and we will characterize the artifacts that appear in reconstructed conductivities due to the lack of information.

TOP-CT: Trajectory with Overlapping Projections X-ray Computed Tomography

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TOP-CT (Trajectory with Overlapping Projections X-ray Computed Tomography) is a new class of CT scanning geometries for high throughput industrial CT scanning. In TOP-CT multiple objects move with a constant spacing over the same trajectory between a stationary X-ray source and detector. The projections of multiple objects can overlap, which provides additional flexibility when designing CT scanning geometries. Reconstruction algorithms were developed to reconstruct objects one by one from the overlapping projection data as soon as the objects move out of the field of view of the scanning setup. This makes it possible to make reconstructions while new objects with overlapping projections keep being added.

The forward problem of TOP-CT is linear with a band block Toeplitz structure, and the matrix of the forward problem can be constructed from multiple copies of a non-overlapping CT projection matrix, so existing software toolkits can be used for TOP-CT with only a small modification. Simulation experiments and a real life experiment were performed on a U-turn TOP-CT geometry. One experiment showed that reconstructions from an overlapping projection setup have a slightly higher SSIM (Structural Similarity Index Metric) (0.828 vs 0.811) and similar PSNR (Peak Signal to Noise Ratio) (33.50 vs 33.34) compared to a non-overlapping setup, using the same scan time per object and the same reconstruction algorithm (SIRT (Simultaneous Iterative Reconstruction Technique)). Another experiment showed that a reconstruction algorithm making reconstructions one by one using only local projection data performed without loss of quality compared to a baseline reconstruction method using all projection data.

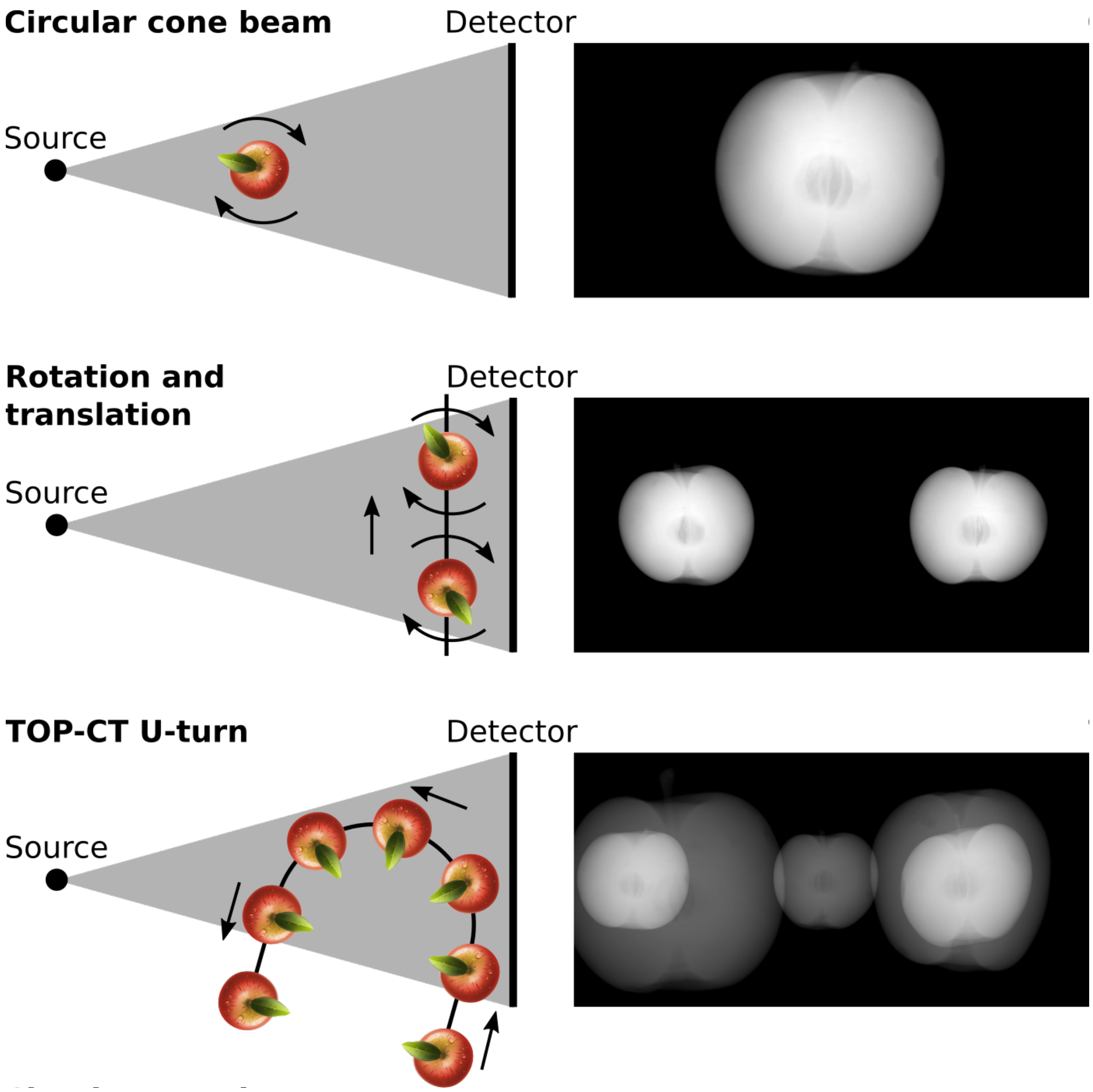

Figure 1. Comparison of different X-ray tomography geometries. For every geometry an illustration of the geometry is given on the left and an example of a projection image obtained from the geometry is given on the right. Top: Standard circular cone beam geometry with a rotating object, Middle: Rotation and translation geometry, where objects rotate while they move over a line. Bottom: One example of the class of geometries presented in this abstract.

Title: Posterior Sampling with AutoEncoding Prior

Oral presentation Mario González Olmedo Departamento de Matemática y Estadística del Litoral Universidad de la República,Uruguay

Abstract: We consider Variational AutoEncoders (VAE) as priors for solving inverse problems in imaging. Markov Chain Monte Carlo (MCMC) methods for sampling from the posterior distribution permit to explore the solutions space and to compute point estimates as well as other statistics about the solutions such as uncertainty estimates. However, the performance of widely used methods like Metropolis-Hastings depends on having precise proposal distributions which can be challenging to define on high-dimensional spaces. Using data augmentation techniques, we develop a Gibbs-like posterior sampling algorithm that exploits the bidirectional nature of VAE networks. To accelerate the burn-in period we explore the adaptation of the annealed importance sampling with resampling method.

Gradient Step Denoiser for convergent Plug-and-Play

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Plug-and-Play methods constitute a class of iterative algorithms for imaging problems where regularization is performed by an off-the-shelf denoiser. Although Plug-and-Play methods can lead to tremendous visual performance for various image problems, the few existing convergence guarantees are based on unrealistic (or suboptimal) hypotheses on the denoiser, or limited to strongly convex data terms. In this work, we propose a new type of Plug-and-Play methods, based on half-quadratic splitting, for which the denoiser is realized as a gradient descent step on a functional parameterized by a deep neural network. Exploiting convergence results for proximal gradient descent algorithms in the non-convex setting, we show that the proposed Plug-and-Play algorithm is a convergent iterative scheme that targets stationary points of an explicit global functional. Besides, experiments show that it is possible to learn such a deep denoiser while not compromising the performance in comparison to other state-of-the-art deep denoisers used in Plug-and-Play schemes. We apply our proximal gradient algorithm to various ill-posed inverse problems, e.g. deblurring, super-resolution and inpainting. For all these applications, numerical results empirically confirm the convergence results. Experiments also show that this new algorithm reaches state-of-the-art performance, both quantitatively and qualitatively.

Bayesian imaging using Plug & Play priors: when Langevin meets Tweedie

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Key-words : Bayesian inferences, Inverse problems, Deblurring, Inpainting, Langevin Algorithm, Markov Chain Monte-Carlo

Remark : This work has been accepted for publication in SIAM Journal Imaging Sciences (preprint available at `https://arxiv.org/pdf/2103.04715.pdf`). Readers might also be interested in [1].

Abstract :

This talk presents theory for Bayesian analysis and computation with Plug-and-Play priors that are implicitly defined by an image denoising algorithm. We study Plug-and-Play Unadjusted Langevin Algorithms for Monte Carlo sampling and minimum mean squared error estimation. We establish detailed convergence guarantees for these algorithms under realistic assumptions on the denoising operators used, with special attention to denoisers based on deep neural networks. We also show that these algorithms approximately target a decision-theoretically optimal Bayesian model that is well-posed and meaningful also from a frequentist viewpoint. This is demonstrated on several canonical imaging problems.

References

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[2] S. V. Venkatakrishnan, C. A. Bouman, and B. Wohlberg. Plug-and-play priors for model based reconstruction. In *2013 IEEE Global Conference on Signal and Information Processing*, pages 945–948. IEEE, 2013.

Cosmological mass-mapping with trans-dimensional trees

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In this work we use a trans-dimensional Markov Chain Monte Carlo sampler for mass-mapping, promoting sparsity in a wavelet basis. This sampler gradually grows the parameter space as required by the data, exploiting the extremely sparse nature of mass maps in wavelet space. The wavelet coefficients are arranged in a tree-like structure, which adds finer scale detail as the parameter space grows. This method produces naturally parsimonious solutions, requiring less than 1% of the potential maximum number of wavelet coefficients. We show how this method is able to recover detailed mass maps with uncertainties on both simulated and real datasets.

Source Separation for gravitational waves using representation learning

Elie Leroy

January 2022

Abstract

The future LISA space-interferometer will simultaneously acquire gravitational waves emitted from thousands of sources. Untangling these signals poses a challenging underdetermined source separation problem. In order to isolate individual events, we introduce a new algorithm that combines sparse component separation with representation learning. Our method makes it possible to efficiently extract physically meaningful signals from the entangled data with a high separation precision. This will be illustrated on realistic simulations of the LISA measurements.

Learned methods for image reconstruction in interferometric imaging with the SPIDER instrument

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The Segmented Planar Imaging Detector for Electro-Optical Reconnaissance (SPIDER) is a concept design for an optical interferometric imaging device that aims to reduce the size, weight, and power consumption of traditional space telescopes. Traditional reconstruction methods for reconstructing images from interferometric measurements use convex optimisation techniques, which have need for handcrafted priors. Using data-driven methods, this prior information can be learned from a dataset instead of using handcrafted priors. We present data-driven methods for reconstructing images from interferometric measurements using learned denoising and learned unrolled iterative algorithms to improve on existing reconstruction techniques in both reconstruction quality and computation time.

Title: Detecting and localising changes in complex environments

Invited Speaker - YI YU

Abstract:

In applications ranging from climate monitoring to surveillance, we often wish to estimate when the distribution generating data in a time series has changed. This task, referred to as change point detection and localisation, is particularly challenging when data is high-dimensional and reflects complex behaviour. In this talk, I will first describe recent advances in this field. Time series data are in the form of sequential data collecting in a one-dimensional chain graph. It is natural to go beyond one-dimensional chain graph to arbitrary-dimensional grid graphs, or even general graphs. I will then present recent results in partition recovery in such more challenging settings, which can be seen as extensions of change point detection and localisation in these general graphs.

Learned operator correction in inverse problems

Andreas Hauptmann

Iterative model-based reconstruction approaches for high-dimensional problems with non-trivial forward operators can be highly time consuming. Thus, it is desirable to employ model reduction techniques to speed-up reconstructions in variational approaches as well as to enable training of learned model-based techniques. Nevertheless, reduced or approximate models can lead to a degradation of reconstruction quality and need to be accounted for. For this purpose, we discuss in this talk the possibility of learning a nonlinear data-driven explicit model correction for inverse problems and whether such a model correction can be used within a variational framework to obtain regularized reconstructions.

Scalable high precision computational imaging with learned priors

M. Terris[1] and Y. Wiaux[1]

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We discuss a new algorithmic framework [1, 2] for high-resolution, high-dynamic range image reconstruction, and its application to radio astronomy. “AI for Regularisation in Radio-Interferometric Imaging” (AIRI) adopts a plug-and-play approach consisting in learning a prior image model by training a DNN denoiser, substituted for the proximal regularization operator of an optimization algorithm. The denoiser is trained at a specific signal-to-noise ratio inferred from the data, with a loss incorporating a nonexpansiveness term ensuring algorithm convergence, and on-the-fly dynamic range enhancement via exponentiation. Simulations suggest that AIRI provides superior imaging quality to the state-of-the-art, and significant acceleration and scalability.

References

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[2] J.-C. Pesquet, A. Repetti, M. Terris, and Y. Wiaux, “Learning maximally monotone operators for image recovery,” *SIAM Journal on Imaging Sciences*, vol. 14, no. 3, pp. 1206–1237, 2021.

Imposing measurement consistency in Deep Neural Networks for Inverse Problems via Optimization Layer and End-to-End Training

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Abstract:
Deep neural networks (DNNs) are state-of-the-art for solving many linear inverse problems (LIPS). One usually One usually trains the network by minimizing the distance between the reconstructed image and ground-truth. In LIPs, however, this fails to guarantee that, during testing, the network output, when passed through the forward linear operator, matches the input measurements. A solution is to project the network output to the space that generates the measurements. Here, we leverage the implicit function theorem to train end-to-end the concatenation of a DNN and a projection problem. Our results on super-resolution and MRI indicate superior performance compared to prior approaches.

Dynamic Tomography Regularization with Sparse 4D Representation Systems

Tommi Heikkilä

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As computed tomography (CT) and in particular benchtop X-ray imaging devices have become more accessible, the need for specialized but viable reconstruction methods for sparsely sampled, limited or dynamic data have become more evident. For example studying nutrient perfusion in plants with X-ray CT[1] leads to undersampled cone-beam measurements from a changing object.

Sparsity based regularization methods using wavelets or shearlets are well known to the inverse problems community for a good reason. Enhancing the key geometric features such as edges in mathematically robust yet computationally efficient manner provides regularization across all three spatial dimensions and time.

This work considers two different 4-dimensional representation systems - dual-tree complex wavelets[2] and cylindrical shearlets[3] - and how both can be used in a simple motion-aware model which is viable with real data.

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Comparing optimal control to physics-informed neural networks for PDE-constrained parameter identification with MRI data

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Cerebrospinal fluid plays a crucial role in development of dementia by removing toxic molecules from the human brain, yet the underlying mechanisms are not clearly understood. The dominating hypothesis is that molecules are transported through the brain by both convection driven dispersion and diffusion.

To test this hypothesis, we assume transport is purely diffusive and use PDE-constrained optimization to find diffusion coefficients describing the spread of molecules in the brain as seen in MRI.

We solve the problem using optimal control and physics-informed neural networks, discuss regularization techniques for noisy MRI data, and compare accuracy and stability.

Analytical inversion formula of a Radon transform on double circle arcs arising in a modality of Compton Scattering Tomography with translational geometry

Cécilia Tarpau, Javier Cebeiro, Geneviève Rollet, Mai K. Nguyen and Laurent Dumas

In this work, we address an alternative formulation for the exact inverse formula of the Radon transform on circle arcs arising in a modality of Compton Scattering Tomography in translational geometry proposed by Webber and Miller (Inverse Problems (36)2, 025007, 2020). The original study suggests a first method of reconstruction, using the theory of Volterra integral equations. The numerical realization of such a type of inverse formula exhibits some difficulties. Here, we provide a suitable formulation for exact inversion that can be straightforwardly implemented in the Fourier domain. Simulations are carried out to illustrate the efficiency of the proposed reconstruction algorithm.

Title: Bayesian model selection in cosmology and astrophysics

Abstract: In the study of cosmology, where we seek to uncover an understanding of the fundamental physical processes underlying the origin, content, and evolution of our Universe, we are not blessed with the ability to perform experiments -- rather, we have only one Universe to observe. To understand the dynamics of astrophysical processes in our Universe we again cannot perform experiments but may be fortunate enough to have a small number of observations of similar processes. In these scenarios, while we are of course interested in estimating the parameters of models describing the physical processes observed, we are typically most interested in selecting the best underlying model, which has given rise to the prevalence of Bayesian model selection in cosmology and astrophysics. Such analyses allow us to address fundamental scientific questions such as "what is the nature of dark energy", "what are the physics governing black hole interactions", and, perhaps less exotically but of no less importance, "what is the best model of the instrumentation observing such processes". While I will motivate recent developments in Bayesian model selection from problems in cosmology and astrophysics, I will mostly focus on new methodological developments. I will discuss new approaches that leverage ideas across statistics, optimization and machine learning to bring to bear the respective strengths of these paradigms to the highly computationally challenging problem of Bayesian model selection. In particular, I will review the learnt harmonic mean estimator for both likelihood-based and simulation-based inference, and the proximal nested sampling framework for high-dimensional model selection.

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Non-blind single-image super-resolution with spatially-varying blur using deep unfolding networks

Charles Laroche, Andrés Alamansa, Matias Tassano

Abstract

Non-blind single-image super-resolution (NBSISR) methods seek to restore a high-resolution image from blurred, subsampled and noisy measurements. Despite their impressive performance, existing techniques usually assume a uniform blur kernel. We address the more realistic and computationally challenging case of spatially-varying blur operators, which are no longer diagonal in the Fourier basis. We propose an ADMM-inspired deep unfolding network that can solve the NBSISR problem. After a single end-to-end training, the network generalizes to a large family of spatially varying blur kernels, noise levels and subsampling rates. Our experiments show the improved effectiveness and computational efficiency of the proposed method.

Hyperspectral super-resolution accounting for spectral variability: coupled tensor LL1-based recovery and blind unmixing of the unknown super-resolution image

Clémence Prévost[*], Ricardo A. Borsoi[‡,§], Konstantin Usevich[†], David Brie[†], José C. M. Bermudez[‡], Cédric Richard[§]

We propose to jointly solve the hyperspectral super-resolution problem and the unmixing problem of the underlying super-resolution image (SRI). We consider tensor LL1 block-term model accounting for spectral variability. Exact recovery conditions for the SRI and its non-negative LL1 factors are provided. We propose two algorithms: an unconstrained one and another one subject to non-negativity constraints, to solve the problems at hand. In this presentation, we will showcase performance of our approach for the fusion and unmixing parts of the problem, using a set of real images.

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On dimension reduction and exact recovery for dynamic spike superresolution

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We consider the super-resolution reconstruction of moving particles from Fourier measurements, where particles are represented in a lifted, R^{2d}-dimensional position-velocity space, with d being the space dimension. To avoid discretization of the resulting high-dimensional variables, we propose a dimension reduction technique based on the Radon transform, which reduces the space-dimension for the unknowns to R^{d+1}. For this setting, we prove that known exact reconstruction results stay true after dimension reduction. Additionally, we prove new error estimates for reconstructions from noisy data in optimal transport metrics, which are of the same quality as one would obtain in the non-dimension-reduced case.

A Hessian-Free Mirror Langevin Algorithm with Applications to Sparse Regression

Lorenz Kuger

We propose a Hessian-free Mirror Langevin Algorithm (MLA) for sampling from a probability distribution.
MLA is a sampling analogue of the celebrated mirror descent scheme, also known as linearised Bregman iterations.
Our method replaces the Hessian matrix of the mirror function, which acts as diffusion coefficient in conventional Mirror Langevin methods, with an iterative approximation, as used in quasi-Newton methods. This way, our algorithm avoids the possibly infeasible evaluation of the Hessian of the mirror function. At the same time, it becomes applicable for non-smooth mirror functions, e.g., sparsity enforcing regularization functionals.

This is joint work with Tim Roith from the University of Erlangen and Leon Bungert from the University of Bonn.

Sampler algorithm for non-convex inverse problem

Pierre Palud[1], Emeric Bron[2], Pierre-Antoine Thouvein[1], Franck Le Petit[2], Pierre Chainais[1]

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Non-convex and multimodal distributions arise in inverse problems with non-linear forward models. Classical MCMC methods generally fail to explore efficiently these distributions and chains get stuck in local minima. Most of the dedicated methods from the literature require running parallel chains or a prior identification of the local minima.

We define a simpler general sampler based on the mixture of a MALA-like kernel dedicated to local moves, and an MTM-like kernel to improve global exploration. It is particularly relevant when the posterior is cheap to evaluate. We apply this sampler to solve a challenging inverse problem from millimeter astronomy.

Distributed sampling for imaging inverse problems in high dimension

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Both optimisation and Bayesian sampling methods are widely used to solve inverse problems. While the former are scalable but only provide point estimates, the latter give statistical information on the solution reliability at a larger computational cost. Recently, scalable optimisation-inspired samplers have been proposed. Among these, the SPlit-and-Augmented (SPA) Gibbs sampler can handle linear operators in parallel without inversion or sub-iterations using client-server communications. In this work, we propose a SPA Gibbs sampler over hypergraphs with a Single Program Multiple Data implementation. We show that the proposed approach is much faster than its client-server counterpart on imaging inverse problems.

Accelerating Markov chain Monte Carlo methods for Bayesian imaging

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Bayesian imaging methods are becoming increasingly important for applications that rely on images to inform important decisions and conclusions, such as medical imaging and astronomy. However, the high dimensionality and the ill-posedness often encountered in imaging inverse problems are a challenge for Bayesian computational methods, particularly state-of-the-art sampling methods based on the Euler-Maruyama discretisation of the Langevin diffusion process.

This talk introduces more sophisticated sampling schemes based on stochastic Runge-Kutta-Chebyshev approximations that combine several gradient evaluations to significantly accelerate the convergence speed, similarly to accelerated gradient optimisation methods. The proposed methodology is demonstrated through a variety of imaging experiments.

Keywords:

mathematical imaging, inverse problems, Bayesian inference, Markov chain Monte Carlo methods, proximal algorithms.

Reference:

Pereyra, M., Vargas-Mieles, L. A., & Zygalakis, K. C. (2020). Accelerating proximal Markov chain Monte Carlo by using an explicit stabilized method. SIAM Journal on Imaging Sciences, 13(2), 905-935.

Approximation properties of two-layer neural networks with values in a Banach space

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Approximation properties of infinitely wide neural networks have been studied by several authors in the last few years. New function spaces have been introduced that consist of functions that can be efficiently (i.e., with dimension-independent rates) approximated by neural networks of finite width. Typically, these functions are supposed to act between Euclidean spaces, typically with a high-dimensional input space and a lower-dimensional output space. As neural networks gain popularity in inherently infinite-dimensional settings such as inverse problems and imaging, it becomes necessary to analyse the properties of neural networks as nonlinear operators acting between infinite-dimensional spaces. In this talk, I will present dimension-independent Monte-Carlo rates for neural networks acting between Banach spaces with a partial order (vector lattices), where the ReLU no linearity will be interpreted as the lattice operation of taking the positive part.

SDE-based MCMC methods for low-photon imaging inverse problems

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This talk describes a Markov Chain Monte Carlo (MCMC) methodology to perform Bayesian inference in inverse problems related to low photon imaging involving Poisson, Binomial and Geometric random noise processes. These problems are severely ill posed and computationally difficult to tackle because their log-likelihoods are not Lipschitz differentiable and involve non-negativity constraints. We propose to address this difficulty by adapting state-of-the-art proximal MCMC methodologies based on the Langevin diffusion process. The proposed methods allow efficiently calculating Bayesian point estimators and performing advanced analyses such as uncertainty quantification or model calibration.

Proximal Iterated Extended Kalman Smoother for regularised non-linear models

Johanna Järvsoo

The extended Kalman smoothing (EKS) algorithm allows for inference in non-linear state space models. Its iterated version, the iterated EKS (IEKS) can be seen as an approximated gradient-descent algorithm equivalent to the Gauss-Newton method. Hence, the IEKS converges to a minimiser of an objective function consisting of a sum of squared l2-norms composed with non-linear functions. We propose a proximal version of the IEKS which considers non-smooth regularisers in the minimisation problem. We investigate the convergence of the resulting prox-IEKS, showing its equivalence with the proximal Gauss-Newton method.

Black box variational inference for Bayesian inverse problems with complex forward models

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The computational costs associated with Bayesian inverse problems for large-scale numerical models can be reduced with (probabilistic) model derivatives. In addition to being intrusive to the forward solver, adjoint formulations are not available for some models, e.g., coupled problems. In these cases, black box algorithms, such as sequential Monte Carlo (SMC), are employed instead. To avoid a prohibitively large number of model evaluations and reduce the overall computational time, we resort to black box variational inference in combination with importance sampling. We demonstrate the resulting algorithm on multiphysics all-solid-state battery models and conduct benchmarks with respect to state-of-the-art SMC algorithms.

Empirical Bayesian estimation for semi-blind inverse problems: Application to image deblurring with total variation regularisation

C. Kemajou Mbakam[1], Marcelo Pereyra[1], Jean-Francois Giovannelli[2], and Yoann Altmann[1]

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Abstract

This work presents an empirical Bayesian methodology for solving semi-blind imaging inverse problems. With the assumption that the forward operator belongs to a known parametric class, we propose a Stochastic Approximation Proximal Gradient method to estimate the unknown parameters of the forward operator by maximum marginal likelihood estimation, followed by a maximum-a-posterior estimation of the unknown image, conditionally on the estimated model parameters. The proposed methodology is demonstrated on a range of semi-blind image deblurring problems, where it is used to accurately estimate the blur operator as well as noise level and regularisation parameters.

Title: Ensemble Inference Methods for Models With Noisy and Expensive Likelihoods

Andrew Duncan

Abstract: In this talk we consider the use of ensemble methods which leverage interacting particle systems to approximate the posterior distribution arising from a Bayesian inverse problem. Multiscale analysis is used to study the behaviour of such algorithms when the forward model is 'polluted' by noise/rapid fluctuations. Through this analysis we identify a dichotomy between different classes of ensemble methods, and show that one particular class, which includes the ensemble Kalman sampler, performs very robustly. On the other hand, Langevin-based methods are adversely affected. Based on these observations, a new class of ensemble method known as ensemble Gaussian process samplers, which combine the benefits of both ensemble Kalman and Langevin methods, are introduced and shown to perform favourably. This is joint work with Andrew Stuart (Caltech) and Marie-Therese Wolfram (Warwick).

Determination of thermal properties of biological tissues

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(Work supported by the EPSRC Mathematical Sciences Small Grant Scheme (**EP/W000873/1) **for the project "Transient Tomography for Defect Detection", 2021-2022)

Biological tissues possess thermos-physical properties that are difficult to measure directly and their estimation is very important in order to decide the appropriate treatment procedure (e.g. place of inoculation and appropriate amount of drug dosage) in case an abnormality is detected. We numerically investigate, for the first time, the determination of several thermo-physical blood-tissue properties of a one-dimensional, multi-layered biological skin tissue subjected to external heating. In contrast to the usual parabolic model that assumes infinitely fast propagation of the heat signal, in the present situation, the bio-heat transfer in such a biological body is mathematically modelled by a hyperbolic partial differential equation, which takes into account that the thermal wave speed is finite. On the skin surface a convective boundary condition holds taking into account the heat exchange with the environment, whilst on the most inner wall of the tissue a uniform temperature or a zero heat flux adiabatic boundary condition applies. Then, in this framework, the piecewise constant thermo-physical properties of interest given by the thermal conductivity, heat capacity and blood perfusion rate are accurately and stably reconstructed from temperature measurements.

A computational approach to link cell history with cell fate at the single level

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During early development of the mammalian embryo, three cell types are specified. How this process is coordinated remains unknown. To gain a better understanding of potential spatio-temporal cues, we use different imaging techniques. However, the matching of cells and data across imaging techniques is non-trivial.

We describe the matching of cells across images as an inverse problem and use MCMC techniques to help describe the correspondence of cells between images. We introduce parameters that describe the weighting of specific cells within the dataset to help account for the presence of spurious data points due to cell death/division between image acquisition.

Joint reconstruction, motion estimation and segmentation of the dynamic foot and ankle problem

Nargiza Djurabekova, Andrew Goldberg, David Hawkes, Guy Long and Marta M. Betcke
University College London

Understanding precise dynamical motion of the foot and ankle complex would be immensely helpful in performing correction surgeries. This is difficult to do, not least due to the number of bones and joints within the structure. Our contribution aims to reconstruct a dynamic phantom of the foot and ankle structure from what we assume to be two full rotation scans. The problem is formulated in a joint variational context, combining motion information enforced by the optical flow constraint with segmentation. We solve this formulation by using an extension of the Proximal Alternating Linearized Minimization algorithm, and its inertial counterpart.

Jacobian of solutions to the conductivity equation in limited view

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We address solutions to the conductivity equation in a limited view setting for a domain in two-dimensional Euclidean space. We consider a Dirichlet boundary condition that is non-zero on a boundary part of control and zero on the remaining boundary. Our work is motivated by the Radó-Kneser-Choquet theorem and generalizations of it which state that under certain conditions on the boundary functions the Jacobian of the solutions is non-vanishing. We extend this theorem from the full to the limited view setting for anisotropic conductivities. This result is useful for inverse problems such as reconstructing the conductivity from interior power densities.

The Henderson problem and the relative entropyfunctional in the thermodynamical limit F

Fabio Frommer

The inverse Henderson problem of statistical mechanics is the theoretical foundation for many bottom-up
coarse-graining techniques for the numerical simulation of complex soft matter physics. This inverse
problem concerns classical particles in continuous space which interact according to a pair potential
depending on the distance of the particles. Roughly stated, it asks for the interaction potential given the
equilibrium pair correlation function of the system. Henderson proved that this potential is
uniquely determined in a canonical ensemble and recently it has been argued that this potential
minimizes a relative entropy.
Here we provide rigorous extensions of these results to the thermodynamic limit.

Fast Task-Based Adaptive Sampling for 3D Single-Photon Multispectral Lidar Data

Mohamed Amir Alaa Belmekki, *Student Member, IEEE*, Rachael Tobin, Gerald S. Buller, Stephen McLaughlin, *Fellow, IEEE*, Abderrahim Halimi, *Senior Member, IEEE*

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Abstract

3D single-photon LiDAR imaging plays an important role in numerous applications such as Defence and environmental sciences. However, corrupted and high-volume data acquired using the LiDAR modality may lead to many challenges such as long acquisition time and/or ill-posed problems. This presentation aims to improve the acquisition and processing of 3D LiDAR data. The acquisition part will be improved using a task-optimized adaptive sampling strategy. The processing part regularizes corrupted and/or incomplete acquired data using a Bayesian model that is carefully built to allow fast per-pixel computations while delivering parameter estimates with quantified uncertainties for different inverse problems (e.g., classification).

Index Terms

3D Multispectral imaging, Single-photon LiDAR, Bayesian estimation, Poisson statistics, Multispectral classification, Inverse problems.

Acknowledgment

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Palamodov Inversion for Bistatic Synthetic Aperture Radar (SAR)

Sean Holman & William Lionheart

Bistatic Synthetic Aperture Radar (SAR) involves a transmitter and receiver that travel along particular trajectories. The transmitter illuminates a scene andthe receiver records the magnitude and time delay of the echoes. For a givenpair of locations for the antennas and time delay, an echo could come fromanywhere on a particular ellipsoid. We use this to show recovery of the Scenefrom isotropic incoherent Bistatic SAR data is equivalent to inverting aMinkowski-Funk transform. We present transmitter and receiver trajectories forwhich the conditions for Palamodov's Inversion formula holds, giving a uniquerecovery of the scene.

Adaptive Undersampling in Spectromicroscopy

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X-ray spectromicroscopy is a powerful tool for studying material distributions, which is extracted from the data using a combination of PCA and cluster analysis. However, the traditional data collection setting has some significant weaknesses (e.g., long scanning times and material degradation due to x-ray radiation).

In this poster, we propose a novel approach for undersampling, reconstructing, and analysing X-ray spectromicroscopic measurements based on low-rank matrix completion. The new method allows the selection of robust undersampling ratios and other matrix completion parameters, while minimising the impact of undersampling on the cluster analysis. Results obtained on real data will be illustrated.

Challenges of dynamic data: MPI reconstruction with motion estimation

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Magnetic particle imaging (MPI) is a relatively new imaging technique with high potential for clinical application. It is non-invasive and doesn't use ionizing radiation, however it has high spatial and temporal resolution. The image reconstruction task poses an ill-posed inverse problem even for static data, but we face even more challenges when reconstructing dynamic images. We expect reconstructions of higher quality when solving the image reconstruction tasks jointly with motion estimation, as both processes endorse each other [1]. We start from a variational formulation and exploit prior knowledge to adapt the formulation. The problem is solved by stochastic primal-dual approaches [2].

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Rich and Non-linear Inverse Problems in SAR

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Synthetic Aperture Radar imaging often makes some basic assumptions about the interaction of radar pulses with the scene, including single reflection from stationary, isotropic scatterers. In many important situations these do not hold, resulting in artifacts and ambiguities. In dynamic scenes, moving objects will not only be defocused but also displaced. In complex scenes, multiple scattering results in artefacts. We discuss some of the resulting (often non-linear) challenges as well as opportunities provided by using the right rich multi-dimensional data – such as multi-static radar – presenting initial results and ongoing work of a recently appointed RAEng Intelligence Community Fellowship.

Artifact Reduction for Magnetic Particle Imaging with an FFL-Scanner and Dynamic Particle Concentrations

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Magnetic particle imaging (MPI) is a relatively new but promising medical imaging technique allowing for the reconstruction of the spatial distribution of magnetic nanoparticles injected into the patient's body. Using a field-free line (FFL) for spatial encoding, corresponding MPI data in the ideal setting can be traced back to the Radon transform of the particle concentration. In the more complicated dynamic setting we adopt the MPI signal equation and build a link to an adapted version of the Radon transform used in dynamic computerized tomography. We show reconstruction results by means of total variation regularization for simulated dynamic data.

A Bayesian Level Set Method for Identifying Subsurface Structures in Stokes Flow

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To understand the movement inside the Earth, we aim to reconstruct the shape of slabs and infer material properties such as density or viscosity from surface observations. Modeling lithospheric flow using incompressible instantaneous Stokes equations, the problem is formulated as an infinite-dimensional Bayesian inverse problem. Subsurface structures are described as level sets of a smooth auxiliary function, allowing for flexible topological changes. To explore the posterior probability distribution, we use a Markov chain Monte Carlo method.

We apply the method to a realistic model problem describing a subduction zone. The aim is to infer the geometry of the subducting plate and its density as well as the viscosity in the hinge zone using measurements of plate velocities and normal stresses. We discuss the benefits and limitations of our method, show trade-offs between different parameters and provide physical interpretations.

Inclusion detection for the Bayesian EIT problem

Aksel Rasmussen

Electrical Impedance Tomography (EIT) gives rise to the Calderón problem of determining the electrical conductivity distribution in a bounded domain given exterior measurements of currents and voltages. A promising approach for the Calderón problem is the Bayesian approach, which serves as a regularization tool and as a way of characterizing uncertainties in the reconstructions. In this poster presentation we look at a Bayesian approach to the inclusion detection problem in the two-dimensional case and discuss well-posedness results. In addition, we compare the theoretical results with a numerical investigation of the posterior distribution arising from observed data in decreasing noise.

Stability for a special class of anisotropic conductivities via an ad-hoc misfit functional

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In recent years there has been an increasing interest in the stability issue for the inverse conductivity problem, known also as the Calderón problem. Inspired by the ideas introduced in [1], we have proved a stability estimate for a special class of anisotropic conductivities. In a nutshell, given a conductive body, we assume that it can be partitioned into disjoint subdomains. The anisotropic conductivity is defined as the product of an unknown piecewise affine scalar function which is discontinuous at the interfaces of the subdomains and of a Lipschitz symmetric matrix-valued function. In this poster, we present the stability estimate of Hölder type in terms of a novel misfit functional. This formulation was firstly introduced in [2] for the inverse problem for the Helmholtz equation and it has been proved successful for the reconstruction of the wave speed. As a corollary, we derive a Lipschitz stability estimate in terms of the classical local Dirichlet-to-Neumann map.

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Deep Composition of Tensor-Trains Using Squared Inverse Rosenblatt Transports

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Many inverse problems are ill-posed and contain random variables. Bayesian inference allows one to characterize the solution via a posterior distribution. However, numerical computations (using e.g. MCMC) can be daunting due to high dimensionality and nonlinear interactions of the variables.

We develop a method that learns a transport map from tractable reference random variables to the sought posterior variables. The method is based on chained Rosenblatt transformations, driven by tensor approximations of tempered probability density functions.
We demonstrate that this method can overcome random walk MCMC by orders of magnitude for posteriors defined by ODE and PDE problems.

Title: Dual forward-backward unfolded network for Flexible Plug-and-Play

Speaker: Audrey Repetti

Authors: Audrey Repetti[†], Matthieu Terris[†], Yves Wiaux[†], Jean-Christophe Pesquet[⋆]
Affiliations: [†] Heriot-Watt University, UK, [⋆] CentraleSupelec, France

Abstract: Proximal methods are extensively used to find maximum a posteriori (MAP) estimates for inverse problems. Recently, they have been mixed with neural networks (NN) to further improve the reconstruction quality. We propose a plug-and-play algorithm using forward-backward iterations, where the proximity operator is replaced by an unfolded NN based on a dual forward-backward structure. This NN mimics the behaviour of a Gaussian MAP estimator, with a parameter in the model enabling to tune the regularisation strength. This enables to adapt the method to multiple statistical models, without training the NN for different noise levels.

Regularization via deep generative models: an analysis point of view

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This paper proposes a new way of regularizing an inverse problem in imaging (e.g., deblurring or inpainting) by means of a deep generative neural networks. Compared to end-to-end models, such approaches seem particularly interesting since the same network can be used for many different problem and experimental conditions, as soon as the generative model is suited to the data.

Previous works proposed to use a synthesis framework, where the estimation is performed on the latent vector, the solution being obtained afterwards via the decoder. Instead, we propose an analysis formulation where we directly optimize the image itself and penalize the latent vector.

We illustrate the interest of such a formulation by running experiments of inpainting, deblurring and super-resolution. In many cases the analysis formulation achieves a clear improvement of the performance and seems to be more robust, in particular with respect to initialization.

References

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Related publication

Oberlin, Thomas, and Mathieu Verm. "Regularization via deep generative models: an analysis point of view." 2021 IEEE International Conference on Image Processing (ICIP).

Learning the Regularisation Parameter for Inverse Problems

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One popular approach to solving linear inverse problems is via variational regularisation, wherein the data-fit is perturbed by a regulariser term, in order to encourage a priori information of the solution in the attained reconstruction. One must also choose the regularisation parameter, a value which determines how much the regulariser term contributes to the problem. A correct parameter value is critical, as one wants to overcome the ill-posedness of the problem, yet still attain a meaningful reconstruction. This work will cover bilevel learning, a framework in which one is able to *learn* appropriate parameter values via a machine learning approach.

Poster Presentation Abstract

Authors: Daniel Ball, Oliver Dorn, Frank Podd

Presenting Author: Daniel Ball

Affiliation: The University of Manchester

Title: Deep Neural Networks for Ground Penetrating Radar in Landmine Detection

Abstract:

Ground Penetrating Radar (GPR) imaging has a long history and has many practically important applications in civil engineering, industry, geophysics or hazard management, including landmine detection and clearance. In this poster presentation, we will discuss some recent results of our research using deep Neural Networks (NNs) for the detection and classification of landmines from back-scattered electromagnetic radar data. In this proof-of-concept study, we will focus on simulated situations in 2D using both, fully connected NNs as well as convolutional NNs. We will compare those results with more classical approaches including shape reconstruction techniques with level sets.

Diverse image super-resolution with hierarchical variational autoencoders

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Image super-resolution is an ill-posed inverse problem in the sense that diverse high-resolution candidates are plausible solutions for each single low-resolution image.

In this work we propose to make use of deep hierarchical variational autoencoders (VAE) to produce diverse super resolution. Hierarchical VAEs have shown impressive results for the task of high-resolution image synthesis and provide a strong prior image model via a self-organized hierarchy of latent variables. We find that these structured latent variables are related with the image information at different scales. Based on this observation, we show that pretrainded hierarchical VAEs can be repurposed to perform diverse super-resolution.

Title: Bistatic two dimensional synthetic aperture radar as a tensor tomography problem

Authors: N.M. Desai, O.T. Graham, W.R.B. Lionheart

In bistatic radar each reflecting object can be described by a radar scattering cross section at each point that is a complex scalar function of an incoming and outgoing direction. For a fixed bistatic angle, assume that the radar scattering cross section can be approximated by a polynomial in angle of some degree. We show that the inverse problem for the radar scattering cross section reduces to the longitudinal ray transform of a tensor field. This operator has an explicit inverse up to a known null space. We explore the implications of this theory for the radar problem in practice.