Editorial

I don't want to perpetuate a cliché about the limited social skills of mathematicians but, in my particular case, I think it's fair to say that I don't always find it easy to navigate social situations or to understand people. So, I'm slightly wary about suggesting how useful simple questions, like 'Are you a glass half full or glass half empty type of person?', can be in determining people's personalities.

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Regardless of how much, or how little, an individual's response might tell you about them, I think that question in itself is rather interesting. For example, from my perspective, it's one of those questions to which there's not really a right answer.

If I had taken the trouble to look, I'm sure I could have found research suggesting that, in general, a positive approach to life is linked to a more active group of friends and better mental health. If so, that would suggest favouring the 'glass half full' response. However, I think a case can also be made for favouring the 'glass half empty' response, too. Being aware of potential downsides and understanding what might go wrong can help produce more robust plans and strategies. When designing safety critical systems, a certain amount of pessimism can literally be a lifesaver.

Given that the event occurred well over two decades ago, I'm sure things have changed a bit since I graduated. But, I still remember the absence of a 'correct answer' being one of the things I most struggled with when I started work. Having been schooled to expect integrals to fall neatly into a nice shape when the correct substitution was used, and to expect formulae to simplify magically when the correct algebraic manipulation was conducted, it took me a while to get used to messy real-world problems.

This meant I often persisted, refusing to believe that my first (or, sometimes, my second or even my third) answer was, in some essential way, correct. In some cases, this persistence paid off because, believe it or not, I don't get everything right first time! In other cases, the persistence allowed me to adopt that favourite trick of mathematicians, changing the problem to one that still provided insight but could more easily be solved.

Of course, there were also occasions where the persistence didn't allow me to progress any further. I hesitate to say that this represented a waste of time, though. By trying different approaches, I often learnt more about the problem at hand. And, even when that didn't occur, I still practised mathematical techniques, keeping my skills honed for the next challenge that might come my way.

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Another thing I think is interesting about the 'glass half full' question is how it naturally describes a world view. By this I mean the respondent is provided with a world in which there are only two options: either the glass is half empty or it is half full. In some sense, this represents a binary or, if you prefer, digital (i.e. 0 or 1) question.

I guess when asked the question an imaginative person could respond 'both', as in the glass is both half full and half empty. Obviously, that makes sense, even if (at least, to me) it feels like one of those logic puzzles whose solution rests on clever use of tautological statements. More generally, perhaps, the 'both' response could also be viewed as adopting a quantum perspective: the glass is in a superposition of 'half empty' and 'half full' states.

So far, our simple question has motivated thoughts about the 'absence of correctness' and 'different types of computing' (i.e. digital and quantum: by filling and emptying the glass, we could also introduce analogue computing). Whilst they're interesting, perhaps the most intriguing aspect is how a constrained world view can affect mathematical analysis and associated conclusions.

As originally posed, the question focuses on the glass's contents, or lack of them. However, there's no reason why a response needs to adopt the same restrictions. A really imaginative reply could be 'the glass is twice as big as it needs to be'.

I think this question of world views is especially relevant when thinking about probabilities. By that statement, I'm not referring to the different theoretical bases of frequentist and Bayesian statistics. Instead, I'm referring to the set of outcomes to which we'll ascribe probabilities (using our preferred approach).

Consider the case of tossing a coin. If we view the possible outcomes as being the set {Heads, Tails} then we can easily determine whether the coin is biased or not. However, if we have not considered the possibility of the coin landing on its side then we will not be able to calculate an associated probability, because it will never occur to us to do so.

Note, there is a difference between not conceiving of the possibility of an outcome (and hence not including it in our world view) and not observing an outcome. In our coin tossing example, if we had done three tosses, all of which ended up as Heads, we could still calculate bounds on the probability of a toss coming up Tails.

Taking a more complex example, suppose we are conducting a test that has a small failure rate. To avoid getting an incorrect result, we repeat the test several times. If all of these repetitions give us the same result, we should end up with greater confidence in that result.

Now, suppose we do many, many more repetitions, all of which produce the same result. If our world view only concerns the test result then this situation will just give us greater and greater confidence in the result. However, if we expand our world view to include the possibility that there is an error, or malfunction, in the test equipment then at some point we'll stop believing the test result and start thinking there's something wrong with the test (because we're not observing the small number of failures we expect to see).

Dating from 2016, it's not the most recent paper, but *Too Good To Be True: When Overwhelming Evidence Fails To Convince* [1] develops this theme. The paper makes some interesting observations about how random bit flips could undermine tests of primality, which is a key aspect of modern cryptography. It also considers legal situations, where, for example, too many witnesses picking the same individual from a line-up may cast doubt on the way the line-up has been organised. In essence, there is a rigorous mathematical sense in which something may be 'too good to be true'.

Ultimately, this discourse has usefully reminded me that I need to be careful about adopting a world view that is too narrow or unimaginative. Unfortunately, it hasn't helped me navigate social situations, especially those where there are glasses, whether they be filled with water or something else!

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References

1 Gunn, L.J., et al. (2016) Too good to be true: when overwhelming evidence fails to convince, *Proc. Math. Phys. Eng. Sci.*, vol. 472, no. 2187, p. 20150748.