IMA Presidential Address – A Mathematical Eye

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Abstract from the Presidential Address in 2022

The mathematical sciences have never been more visible. Modellers and statisticians have informed policy during the pandemic and big data impinges on everybody's daily lives. For a mathematician there is so much more: mathematics is all around us.

In this talk I will explore the world with a mathematical eye. This leads to questions about how a mathematician sees ordinary objects, mathematics in society, and the role and responsibilities of the mathematical community in helping to unlock everyone's inner mathematician.

The only prerequisite for this talk is an enquiring mind.

My Mathematical Eye

Do not always believe what you read. The abstract and title for the IMA Presidential Address was given almost a year in advance of the first event in 2022, and even then circumstances had changed. I have since toured the talk at the IMA's branches before writing this version of the Address. I had thought of weaving together the many important applications of mathematical sciences and their impact on society (big data, pandemic modelling, energy pricing, etc.), how those ideas became accepted into policy and practice (including the demonisation of algorithms) and the imaginative thinking behind the creation of these solutions. This would then lead to a mathematical eye that distinguishes the way mathematicians tackle problems from the approaches of other disciplines.

It was a good idea. But it is not what I do as a mathematician, so all I could have done was to report and attempt to assess what works and what does not in the transfer of ideas between research and policy. Many others could do this with greater knowledge and depth of understanding than me. My mathematical eye – the mathematical me/I – is different, and so I will think about the personal aspects of mathematics and mathematical practice, rather than how this is translated into actions in society.

This is already controversial. To the extent that we are taught to think about what mathematics is, it is presented as *objective*, *unbiased* and so *impersonal*. A (correctly stated) theorem is a theorem. Although the last statement is true, it is also somewhat vacuous. Surely the real issue is whether it is interesting, surprising or impactful, either in terms of applicability or in the development of the subject. In the more applied areas of mathematics, is it enough to be 'fairly correct' or 'contain a grain of truth' to be useful and to provide insights? Answers to these questions, and I emphasise that to me these are the important questions, depend on judgement, experience, knowledge and gut feeling. They may also be time dependent.

So much for objective, unbiased and impersonal mathematics. Or maybe it would be more accurate to say objective, unbiased and impersonal mathematicians and their interpretations of mathematics. This is one of the reasons I think it is so important that we try to include an element of ethical thinking in mathematics degrees.

In my experience, the mathematical eye – the curiosity, the 'taste' for particular mathematical problems and the inevitable biases these create – spills over into ordinary life. There are few things I interact with that do not have a mathematical aspect somewhere and I will look at just a few examples in this article. Through this I also want to look at how my own bias works – and the importance of recognising this. None of the examples are meant to be deep, but I would encourage you to think about your own practice as a mathematician and how it influences the way you see the world around you. What examples would you have used to illustrate your own interests?

Knitting

Knitting may not seem like a topic for mathematical investigation. But knitting is pretty extraordinary. Pull a strand that has not been tied off and it unravels. It is not a knot. Yet it holds together and is a barrier against the cold. That in itself could be the topic of a mathematical discussion, but instead I want to look at another aspect of knitting and weaving: the pattern.

Knitting involves a small number of actions: knitting (k), purling (p), returning, casting off .... This means that it can be codified into a set of simple instructions. Moreover, as there are relatively few possible actions, the instructions can be written down in shorthand, for example a moss stitch and a seed stitch [1] can be written as:

> Row 1: *k1, p1; rep from * Rows 2: *p1, k1; rep from * or Row 1: (K1, P1) rep to end Row 2: (P1, K1) rep to end Repeat these 2 rows for length desired.

From this point of view, a knitting pattern is essentially equivalent to a computer program.

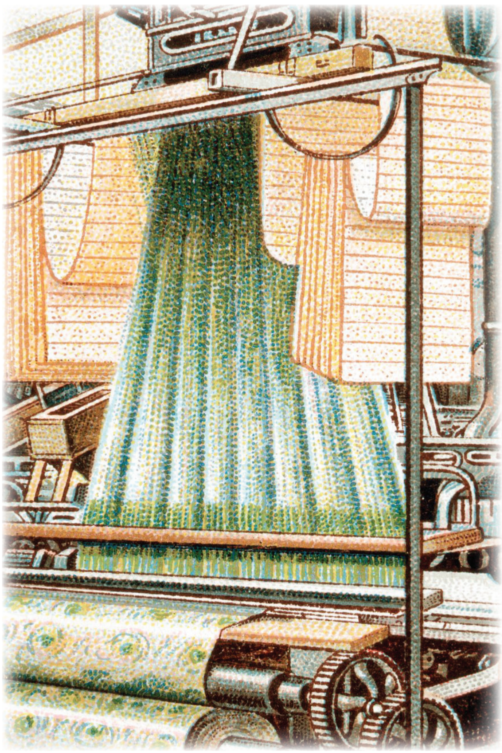

Figure 1: The Jacquard loom

Indeed, the Jacquard loom shown in Figure 1, developed in 1801 and one of the innovations of the Industrial Revolution, used punch cards to encode the movement of the weaving machinery needed to create different types of cloth. This innovation was known to Babbage and Lovelace as they developed their mechanical computer, known as the Analytical Engine, in the mid-19th century. As Ada Lovelace put it [2]:

> We may say most aptly that the Analytical Engine weaves algebraic patterns just as the Jacquard-loom weaves flowers and leaves.

Lovelace even had the idea that coding could go beyond science and that using scientific principles of harmony:

> The engine might compose elaborate and scientific pieces of music of any degree of complexity or extent.

This 'programmable' aspect also means that a piece of knitting can be decoded to provide the set of instructions used to create it. This had military applications during the Second World War. Phyllis Latour Doyle (a secret agent, b. 1921) used combinations of knits and purls to send information learned by talking to German soldiers in France. The Belgian Resistance used knitting to monitor and send information on train movements; old ladies in rooms overseeing the train station used knits and purls to indicate the number and times of trains passing through the station. The US Office of Censorship was worried that knitting patterns might contain hidden messages and banned people from posting them abroad [3]!

Paper folding

Paper folding can become obsessive, and there are many features of origami and pop-up books that can be looked at mathematically. Even the simplest folding can be interesting. Take a sheet of paper and fold it in half, right over left. Opening it out again you will find that the fold is a valley. As with knitting, we can start coding [4], for example, denote a valley by a V.

Now fold right over left again and note that the lower sheet has not been moved at all, but the upper sheet has been inverted. Its upper surface is now downwards and its right-hand side is now on the left. Repeat the right over left fold and think about what has happened: the lower (originally left-hand half) sheet has been folded in exactly the same way as the original sheet, so it will have a valley V when unfolded, but the upper sheet has a valley in reverse – a peak denoted by Λ. The order of any folds is also reversed, although at this stage there is only one. So the newly opened out sheet will have a sequence of folds $VV\Lambda$, where the middle V is the original fold.

Can we generalise the process? Of course. If we repeat, then the lower sheet simply inherits the earlier pattern of folds of the original sheet. We have the original central V and then the right-hand sheet has the earlier pattern reversed and inverted, so the next sequence is

$$VV\Lambda\ V\ V\Lambda\Lambda. \tag{1}$$

This is demonstrated in Figure 2. Of course, as mathematicians we prefer 0s and 1s to Vs and Λs, so we might prefer to label the sequence of folds as

$$0 \to 0(0)1 \to 001(0)011 \to 0010011(0)0011011 \to \dots.$$

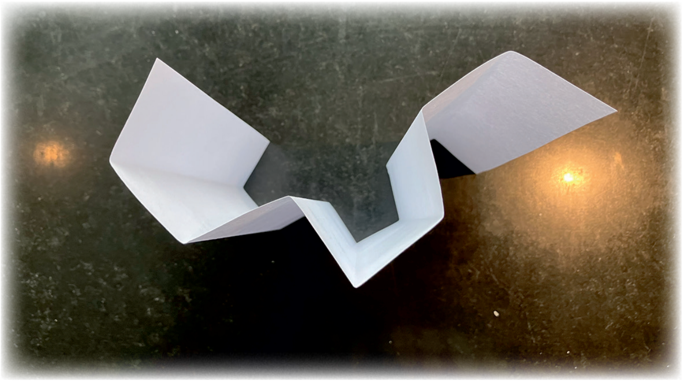

Figure 2: The unfolding of a folded piece of paper. From left to right, the sequence of valleys and peaks is shown in (1).

These series can be described precisely mathematically, and with a little more work, the same principles can be used for folds that mix left over right with right over left [4].

The point of both these examples is that mathematicians naturally code and generalise and that they find it hard to avoid doing this even in their ordinary lives. But these ideas underpin many descriptions of mathematical phenomenon: very similar ideas (*substitution sequences*) can be used to describe the bifurcations leading to chaos via period-doubling in simple systems. A lot of my own work concerns coding dynamics and understanding the dynamical information contained in those codings.

Patterns

This tendency to look for patterns in everyday objects from a mathematical point of view seems to be one of the ways I define my mathematical I/eye. Figure 3 is another example. The Kuba people of the Democratic Republic of the Congo create these wonderful textiles, many of which are given as wedding presents. One of the principles for such gifts is that they should be unique. The geometric designs and their possible permutations make me immediately think of combinatorics.

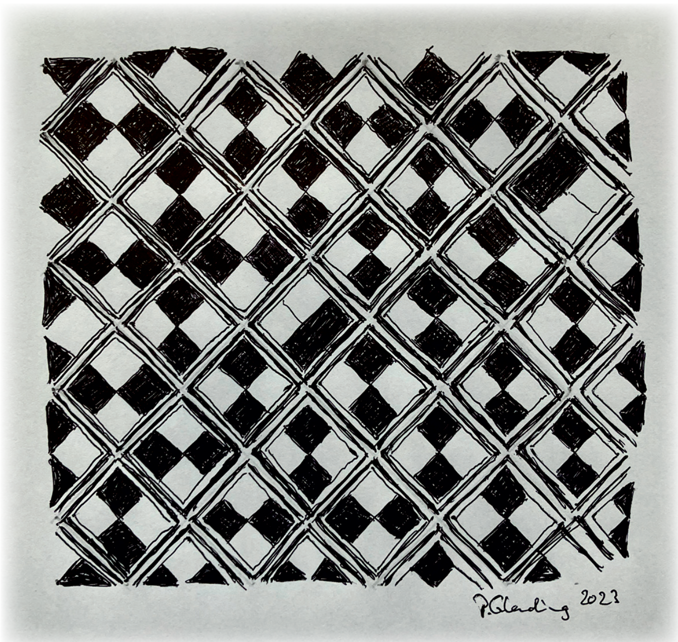

Figure 3: Sketch by the author inspired by a Kuba textile

The cloth on which the sketch is based shows a basic design that allows the weaver to work quickly, with a combinatorial complexity that makes it possible to create an almost unlimited number of personalised cloth pieces. The weavers often use dislocations in interesting ways too, although this is not present in the example in Figure 3.

The cloth on which the sketch of Figure 3 is based has 40 squares (the sketch has 28 full squares), each of which is divided into four smaller squares. Each of the four smaller squares can be shaded or not shaded, so each square has $2^4 = 16$ possible configurations. Ignoring equivalence under rotations, this means that there are 16^{40} different designs that can be created from the same basic template. That's a very big number. To give it some context,

$$16^{40} = (2^4)^{40} = 2^{160} = (2^{10})^{16},$$

so since $2^{10} = 1024 > 1000 = 10^3$, then $16^{40} > 10^{48}$. For comparison, the number of grains of sand on the world's beaches has been estimated at 10^{18} and this does not get close, nor does the number of stars in the universe, though there are around 10^{80} atoms in the universe.

In the sketch, only designs in which two of the four smaller squares are coloured appear, providing a balance between light and dark. This, and other stylistic choices, limit the number of combinations. It is left to the reader to determine the extent of this limitation.

Spiders

The photograph in Figure 4 is another example of how I can get swept away by a series of thoughts from a seemingly ordinary observation. This spider's web after some rain has a beautiful array of droplets studding the web. But why? Why droplets and not a coating of water? I do not know the answer so the remainder of this section is pure speculation – but it shows how hard I find it not to think about the mathematics underlying the everyday.

Photograph by the author

Figure 4: A spider's web after rain

First note that the drops are not at the minima of the web strings, so gravity is unlikely to be playing an important role here. Although, as IMA Council member Chris Budd OBE CMath FIMA pointed out, gravity is definitely present in the photograph, playing a role in the brachistochrone-shaped curve of the filaments – his eye picking out different features to mine. So without gravity, this suggests that surface tension is the driving force and makes me think of the way that water from a tap changes from a continuous stream to a drip as the flow is reduced. Only here there is no flow, so is it that the amount of water that can adhere to a strand is too small to provide a continuous coating? But then what does 'adhere' mean in this context? The spider's web is sticky, but the positioning of the drops seems quite regular. Does that mean there is a break-up of a continuous film into droplets, perhaps with an instability of the film with a particular wave number? Or is it that the stickiness of the web is not uniform and that sticky lumps create barriers that hold the droplets in place?

A quick literature review shows that there is evidence for the non-uniform stickiness hypothesis, possibly because the spider needs to move around the web without getting trapped itself – what would this mean about the way a spider sees its own web? Former IMA President Robert MacKay FRS CMath FIMA tells me he read about this as a schoolboy. Is there a more mundane explanation? Droplets form on clothes lines. Is it, perhaps, dirt or past positions of clothes pegs that determine their position? Or is surface tension on its own actually enough? Perhaps a stream of water breaks up into droplets when the flow becomes weak enough? Mathematicians clearly enjoy engaging in such speculation – even an anonymous reviewer could not resist adding another suggestion!

The point about this little monologue is not whether I am right or wrong about the droplets, but that I cannot see something like the spider's web without getting into an internal argument about the mathematics behind what I am seeing, even when it touches on areas of mathematics such as fluid dynamics that is not one of my strengths! Those familiar with Industrial Study Groups might also recognise this sort of thinking as the first steps towards identifying the mathematical features relevant to a particular problem.

Art

It would feel remiss of me to talk about a mathematical eye without talking about the visual arts. There are so many connections between art and mathematics that it is hard to know what to choose: perspective, abstraction and the rigours of geometry, the creation of imaginary spaces or one of the many other intersections and synergies.

In the end, I have chosen a case in which the mathematical eye, namely the sensibility of someone working to interpret numbers and statistics in the late 1890s, prefigured the bold, colourful abstract art of the 1930s and beyond. I am not advancing any causal relationship here. After all, there are only so many different colours and simple shapes to choose from. What I think Figure 5 does show, however, is how those who work with images and those who work with numbers can come up with similar solutions to problems of representing abstract quantities.

The left-hand image is a painting from 1958: *Beginning* by Kenneth Noland. Noland was one of the colour field artists, which included Mark Rothko. This technique involves the use of large blocks of colour. The second is a cropped version of a panel by W.E.B. Du Bois and his students from the 1900 Paris Exposition, where Du Bois won a Gold Medal for his exhibition of a range of achievements of the Black American population.

My point is that both use concentric circles and strong colour as the basis for their composition and that both break up this simple structure, though in different ways. Noland breaks the

Figure 5: Beginning *by Kenneth Noland [5] and a panel from the exhibit at the 1900 Paris Exposition by W.E.B. Du Bois [6].*

perfection of the circles by making them rougher, with an asymmetric background with obvious brush strokes drawing attention to the materials used in the painting. Du Bois breaks the circles by inserting cuts towards the centre, which can be used to provide the space for more quantitative information. This is part of a series and the example in Figure 5 shows the valuation of taxable property held by the Black American community in the State of Georgia in the different years indicated, drawing attention to the growth of ownership over the period 1875 to 1899.

Du Bois was the first Black American to gain a PhD from Harvard (1895), a sociologist and a civil rights activist [7]. The Du Bois panels are early examples of what we now call infographics. This and the other panels created for the Paris Exposition provide wonderful examples of how images can be used to convey information. I particularly like the sequence of growing money bags to indicate income and the spirals rather than broken axes to show data of different magnitude on the same diagram. Although there is some controversy about whether Du Bois was using contemporary criteria of success uncritically, the panels certainly and very effectively convey the social changes in the 35 years following the abolition of slavery in 1865.

Conclusion

What I have tried to show in these little case studies is that being a mathematician is intensely personal: we choose the type of problem we work on, we choose the approaches we become expert in and we choose the people we work with. This has consequences. It means we have prejudices about what we like and consider important, and we have biases in terms of expectations and methodologies. This is inevitable, but it is equally important that it is recognised (cf. unconscious bias training). It makes it easy to accept the importance of a proposed programme of work or the standard narrative of an area of mathematics without questioning it. Although we would go mad if we spent the whole time questioning orthodoxy, a healthy scepticism from time to time – reflection on why and how we are doing something – seems to me to be a good thing.

This means that there are different types of mathematician, not just in their areas of study but also in the way they approach that study. There are those with a big picture and those fascinated by some curious corner, and there are those thrilled by the possibilities of the latest ideas and those meandering through the alleyways of their subject. There is no right or wrong here, just different personal choices. I hope that this article makes you more aware of your own mathematical eye.

Mathematics is personal, but it also matters. It matters as part of our cultural life and it also matters because of the ways it can help society address problems. So on a global level we need to recognise the complex ecosystem of mathematical approaches and attitudes that produce progress and maintain its diversity.

And at a personal level, *embrace your inner mathematician*.

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