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ABSTRACT BOOK

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Contents

Photoacoustic and Ultrasonic Tomography for Breast Imaging, Felix Lucka

New high-resolution, three-dimensional imaging techniques are being developed that probe the breast without delivering harmful radiation. In particular, photoacoustic tomography (PAT) and ultrasound tomography (UST) promise to give access to high-quality images of tissue parameters with important diagnostic value such as optical absorption, blood oxygen saturation, sound speed, acoustic density and acoustic attenuation. However, the involved inverse problems are very challenging from an experimental, mathematical and computational perspective. In this talk, we want to give an overview of these challenges and illustrate them using data from a clinical prototype scanner for combined PAT and UST. We show results from different experimental phantoms, healthy volunteers and breast cancer patients from two ongoing clinical feasibility studies.

Sequential Experimental Design for X-ray CT, Tianyuan Wang

March 14, 2024

In X-ray Computed Tomography (CT), obtaining projections from various angles is crucial for 3D reconstruction. To adapt CT for real-time quality con- trol, it's essential to reduce the scan angles while preserving reconstruction quality. Sparse-angle tomography, which achieves 3D reconstructions with fewer data, necessitates selecting the most informative angles—a challenge equivalent to solving a sequential optimal experimental design (OED) problem. However, OED issues are marked by complexity, including high-dimensional, non-convex optimization that makes adaptive solutions during scanning difficult. To navigate these complexities, we approach the sequential OED problem through a Bayesian framework, modeling it as a partially observable Markov decision process and employing deep reinforcement learning for solutions. The approach learns efficient non-greedy policies to solve a given class of OED problems through extensive offline training rather than solving a given OED problem directly via numerical optimization. Consequently, our policy efficiently identifies the most informative angles for real-time operations, significantly enhancing the practicality of sparse-angle CT in quality control scenarios. This stream- lined approach ensures that CT can be efficiently integrated into quality control processes, with the potential to significantly impact the field.

Stable determination of a complex anisotropic conductivity of a medium from local measurements, Jessica Crosse

We study the inverse problem in Electrical Impedance Tomography (EIT) of determining the anisotropic conductivity σ of a medium $\Omega \subset R^n$, with $n \geq 3$. Specifically, we assume that σ is a complex-symmetric matrix-valued function on Ω satisfying some ellipticity condition that allows the resulting linear system (due to the fact that σ is complex-valued) to be strongly elliptic. Specifically, σ is assumed to be of type $\sigma(\cdot) = A(a(\cdot))$ in Ω, where one-parameter family

$$t \rightarrow A(t), \quad t \in [\lambda^{-1}, \lambda],$$

is assumed to be a-priori known. The inverse problem we consider is the stable determination at the boundary $\partial\Omega$ of a (and A(a)) from the local Dirichlet-to-Neumann map. Our work is based on the construction of singular solutions to the underlying second order elliptic partial differential equation (the inverse conductivity equation) with complex coefficients $\sigma \in W^{1,p}(\Omega)$, $p > n$. Such solutions have an isolated singularity of any order.

An inverse problem from engineering geology: a regularization approach for landslide thickness estimation, Liwei Hu

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ABSTRACT

Accurately estimating landslides' depth is essential for hazard prediction. However, most of the classical methods rely on overly simplistic assumptions [1].

In this work [a], we will present the landslide thickness estimation problem as an inverse problem $Aw = b$ obtained from discretization of the thickness equation

$$\frac{\partial(h_f v_x)}{\partial x} + \frac{\partial(h_f v_y)}{\partial y} = -\frac{\partial \zeta}{\partial t}, \tag{1}$$

where the forward operator A contains information on the surface velocity (v_x, v_y), the right-hand side b corresponds to the surface elevation change $\frac{\partial \zeta}{\partial t}$, and w is the thickness h_f. By employing a regularization approach, the inverse problem is reformulated as an optimization problem.

Using real-world data implies that this minimization problem is of such large dimensionality that downsampling is required. We will investigate different strategies to reduce the dimensionality of the problem. In particular, a gradient descent based approach will be compared with the bootstrapping approach applied in [2]. Particular attention will also be given to analysis of the method for choosing the regularization parameter in an automatic way as proposed in [3].

Finally, we will show results obtained both on synthetic data generated by landslide simulation softwares and on data measured from real-world landslides.

[a]intended for oral presentation, otherwise poster presentation

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Estimating Ozone Profiles from Satellite Radiometer Data, Colin Fox

Abstract:
Radiometers on satellites at heights of 500-800 km measure the thermal microwave radiation of various gases in the stratosphere, such as ozone.
Two examples are the MLS (microwave limb sounder) on NASA's Aura Mission and the MIPAS (Michelson interferometer for passive atmospheric sounding) on the ESA's Envisat.
Measurements are a path integral along the-line-of-sight of the radiometer, subject to absorption, modelled as the the weakly non-linear
radiative transfer equation (RTE). We ignore absorption and treat the inverse problem as a linear-Gaussian inference problem, allowing use of the
"marginal then conditional" (MTC) sampler that calculates the posterior mean faster than regularised inversion, and allows full uncertainty quantification.
In a simulated example using default priors, the posterior mean closely matches the regularised solution while posterior uncertainty correctly locates the high altitudes
where both solutions deviate from the ground-truth profile. Improved prior modeling corrects the bias in the posterior mean while a block-Gibbs extension to MTC
treats the full nonlinear problem, for minimal increase in computational cost.

Presenter: Colin Fox. Joint work with Lennart Golks.

Learning Preconditioners for Inverse Problems, Patrick Fahy

We explore the application of preconditioning in optimisation algorithms, specifically those appearing in Inverse Problems in imaging. Such problems often contain an ill-posed forward operator and are large-scale. Therefore, computationally efficient algorithms which converge quickly are desirable. To remedy these issues, learning-to-optimise leverages training data to accelerate solving certain optimisation problems. Traditional optimisation methods often use scalar hyperparameters, especially limiting their convergence speed when applied to ill-posed problems.
In contrast, we propose a novel approach that replaces these scalar quantities with matrices learned using data. We consider multiple parametrisations of the preconditioner, including full matrices, diagonal matrices, and convolutions. The convergence properties of these methods have been analysed, and a comparison is made against both classical and learned optimisation algorithms. Generalisation performance of these methods is also considered, both for in-distribution and out-of-distribution data.

Identification of time-wise thermal conductivity and diffusion on free-boundary inverse coefficient problem, Taysir Emhemed Ail Dyhoum

By

M.S. Hussein , Taysir E. Dyhoum, S.O. Hussein and M. Qassim

This work concerns finding solutions to free-boundary inverse coefficient problems (ICPs). Mathematically, we handle a one-dimensional non-homogeneous heat equation, subject to initial and boundary conditions plus non-localized integral observations of zeroth and first-order heat momentums as additional data. The direct solver is represented in temperature distribution and non-localized integrals measurements and obtained using the Crank-Nicolson (CN) finite difference scheme. On the other hand, the inverse problem aims to find simultaneously temperature distribution, the time-dependent free-boundary function, which reports the location of the moving interface, and time-wise thermal conductivity or diffusion functions. We use lsqnonlin non-linear least-square solver from MATLAB optimization toolbox since the inverse problems here are recasted as non-linear optimization problems. Using examples and discussions, we explore the optimal value for the regulation parameters to ensure the accuracy, convergence, and stability of the obtained reconstructions. The existence and uniqueness constraints of these inverse problems are manipulated and verified considering the existing definitions and theorems.

Dynamic X-ray Computed Tomography using Deep Generative Networks, Xinyuan Wang

Abstract of IMA conference

X-ray Computed Tomography (CT) is an essential imaging modality to reveal the interior of static objects in many fields like science, healthcare and industrial inspection. However, the sequential nature of its data acquisition limits its capacity to visualize dynamic processes with high temporal resolution and conventional image reconstruction techniques applied to dynamic CT result in considerable motion artifacts. While more sophisticated dynamic image reconstruction methods can improve upon the spatial and temporal resolution to some extend, the full potential of dynamic CT is not yet realized.

We propose to use the expressive power of deep generative neural networks in an unsupervised learning framework for dynamic CT image reconstruction. In particular, we explore temporal extensions of the Deep Image Prior and Neural Radiance Fields, present a software framework designed to integrate them with real-world dynamic CT applications efficiently and show preliminary results from simulated data scenarios.

Hybrid Gaussian Beam Pseudo-spectral Method for Efficient Wave Propagation in PAT, Elliott Macneil

Photoacoustic tomography (PAT) is a hybrid imaging technique based on the photoacoustic effect. The PAT forward problem can be modelled as an initial value problem for the free space wave equation. The PAT inverse problem aims to recover an initial pressure from pressure time series recorded at sensors placed outside the region of interest. Despite the advances made in the recent years (parallel interrogation with up to 64 beams), the data acquisition time in state-of-the-art PAT scanners is still a bottle-neck resulting in sparse, limited angle data. The solution of inverse problems with incomplete data necessitate iterative methods involving repeated calls to the forward solver, which is the most compute intensive part of the process. Inspired by the Multiscale Gaussian Beam method proposed by Qian and Ying, we devise an efficient hybrid wave solver, leveraging Gaussian Beams for efficient and highly parallel propagation of high frequency components of the solution, and a pseudo-spectral method for accurate solution of the low frequency components. We discuss the accuracy and performance of our method on an example of solution of the forward problem in PAT.

Parameter Identification for a Two-Compartment Contrast Flow Field Model, Sophie Externbrink

Sophie Externbrink * Daniel Ruprecht * Sebastian Götschel *

Tumor perfusion and vascular properties are important determinants of a cancer's response to therapy. Being able to determine those parameters from patient-specific data collected at the bedside would allow for better, more individual tumor treatment.

Models describing the transport of contrast agent based on advection-diffusion equations are commonly used, but lack the ability to derive physically accurate solutions for the transportation of tracer through an organ. Therefore, Sourbron[1] proposed a two-compartment model, where the flow of contrast agent is modeled by separating the arterial and venous flows into a system of transport equations, coupled by a transfer coefficient function which describes the exchange of the contrast agent from arteries to veins through capillaries.

In this talk we discuss the parameter identification problem, i.e., how to estimate flow velocities and the conversion coefficient function, given the concentration of contrast agent over time, which will be obtained via 3D dynamic contrast-enhanced ultrasound measurements. We derive adjoint equations for efficient gradient computation, discuss the discretization of state and adjoint equation and the use of Leray projection within the optimization algorithm to ensure a divergence free velocity field, and present numerical examples.

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Deterministic and Stochastic Optimisation Framework using the Core Imaging Library and Synergistic Image Reconstruction Framework for CT and PET Reconstruction, Margaret Anne Georgina Duff

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In recent years, a multitude of complex tomography challenges have emerged, requiring collaborative efforts across mathematics, algorithm design, and numerical software development. To support this collaboration, we have built the Core Imaging Library (CIL) [1, 2], providing a comprehensive toolkit for solving inverse problems in mathematical and computational imaging. This talk explores some of CIL's successful applications within tomography and general imaging tasks, emphasizing its modular optimization framework and introducing a new stochastic optimization approach.

Firstly, we illustrate our work on hyperspectral neutron tomography [3], where CIL is used to resolve materials spatially and spectrally based on Bragg edges in energy-resolved neutron data.

Secondly, we demonstrate a directional total variation reconstruction method in CIL, which won a prize at the Helsinki Tomography Challenge 2022 for limited-angle X-ray CT reconstruction [4].

Finally, we present recent advancements in CIL, introducing a framework for stochastic algorithms which have the potential to increase speed and efficiency with the increasingly large datasets encountered with modern imaging methods. The CIL optimisation framework can now integrate stochastic gradient estimators into base algorithms like gradient descent and iterative soft thresholding, so we can switch between stochastic and non-stochastic algorithms; including stochastic gradient descent (SGD), stochastic average gradient (SAG), and stochastic variance reduced gradient (SVRG). The plug-and-play nature of the software library enables easy comparison between different stochastic methods. We showcase the functionality of the framework with a comparative study against a deterministic algorithm on a PET dataset, with the use of the open-source Synergistic Image Reconstruction Framework (SIRF) [5].

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The Inverse Problem in Diffuse Optical Tomography Reconstruction: a Modular Deep Learning-based Approach, Paola Causin

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Medical imaging is nowadays a pillar in diagnostics and therapeutic follow–up. Current research tries to integrate established - but ionizing - tomographic techniques with technologies offering reduced radiation exposure. Diffuse Optical Tomography (DOT) uses non–ionizing light in the Near-Infrared (NIR) window to reconstruct optical coefficients in living beings, providing functional indications about the composition of the investigated organ/tissue. Due to predominant light scattering at NIR wavelengths, DOT reconstruction is, however, a severely ill–conditioned inverse problem.

Conventional reconstruction approaches based on variational methods show severe weaknesses when dealing also with mildly complex cases and/or are computationally very intensive. In this work we explore deep learning techniques for DOT inversion. Namely, we propose a fully data–driven approach based on a modularity concept: first data and originating signal are separately processed via autoencoders, then the corresponding low–dimensional latent spaces are connected via a bridging network which acts at the same time as regularizer. Numerical results are shown, accompanied by a theoretical analysis which helps to shed light on the pros and cons of this strategy.

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On Optimal Regularisation Parameters via Bilevel Learning, Sebastian James Scott

Variational regularisation is commonly used to solve linear inverse problems and involves augmenting a data fidelity by a regulariser, weighted by a regularisation parameter. Often the regularisation parameter is assumed to be strictly positive which implicitly assumes the regulariser is a good choice for the given application - but what does it mean for a regulariser to be "good"? One characterisation is offered via bilevel learning, a powerful framework to determine optimal parameters which involves solving a nested optimisation problem. Indeed, by optimising over the regularisation parameter we can determine conditions which guarantee that zero is not an optimal parameter. While existing conditions primarily focus on the denoising application, in this talk a new condition involving Bregman distances will be introduced that offers a better characterisation than existing theory and is applicable to a wide class of inverse problems and regularisers.

Infimal Convolution of Curvelet Sparsity and Total Variation for Photoacoustic Reconstruction, Marta Becke

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Analytic regularisation functionals such as ℓ_1 norm of Curvelet coefficients or total variation are designed to excel in regularisation of inverse problems for certain image classes, e.g. Curvelets for images with edges along smooth curves and oscillatory textures, total variation for piece-wise constant images. These two regularisation paradigms perform relatively complementary tasks therefore it is of interest to investigate their combination. In this work we consider the framework of infimal convolution and study the effect of the composite functional. We propose an efficient ADMM solver for infimal convolution and integrate it with limited angle PAT reconstruction utilising Fourier domain closed form solutions for proximal problems involving total variation, Curvelets and forward/adjoint limited angle PAT operators. We compare our approach with recently proposed backward-backward splitting for complementary reconstruction with these two regularisation functionals which we reformulated to use the same closed form proximal problem solvers.

Statistical inverse learning of randomly sampled sparse dynamic tomography, Tommi Heikklä

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Abstract

We tackle the problem of sparse and dynamic tomography by randomizing the projection angles for each time step and regularizing in both spatial- and temporal domains using cylindrical shearlets. Our choice of regularization is motivated by the ability to optimally approximate functions in the class of cartoon-like videos, and properties of the (quasi-)Banach decomposition spaces for $p > 0$.

Using statistical inverse learning methods we obtain convergence rates for $p > 1$ in different noise conditions which are supported by numerical tests using both simulated and real dynamic tomography measurements.

Inverse learning in Hilbert scales, Abhishake Rastogi

We study ill-posed inverse problems with noisy data in the framework of statistical learning. The corresponding operator equation is assumed to fit a given Hilbert scale, generated by some unbounded self-adjoint operator. Approximate reconstructions from random noisy data are obtained with general regularization schemes in such a way that these belong to the domain of the generator. The analysis has thus to distinguish two cases, the regular one, when the true solution also belongs to the domain of the generator, and the 'oversmoothing' one, when this is not the case. Rates of convergence for the regularized solutions will be expressed in terms of certain distance functions. For solutions with smoothness given in terms of source conditions with respect to the scale generating operator, then the error bounds can then be made explicit in terms of the sample size.

This is a joint work with Prof. Peter Mathé.

Improving the scalability and tractability of optimization algorithms, Coralia Cartis

Abstract:
We discuss random and deterministic subspace methods for nonconvex optimization problems. We are interested in the optimisation of functions with low effective dimensionality, that vary only along certain important directions or components. We show that the effective subspace of variation can be efficiently learned in advance of the optimization process; we contrast this with random embedding techniques that focus directly on optimization rather than learning.

Fictitious null spaces in inverse problems, Ole Løseth Elvetun

Ole Løseth Elvetun*, Kim Knudsen and Bjørn Fredrik Nielsen

For non-injective operators, Tikhonov regularization and truncated SVD typically yield solutions which are close to the minimum norm solution of linear inverse problems. We have therefore previously developed a weighting procedure which produce more "unbiased" solutions for such problems.

In this talk we discuss the use of this weighting method applied to injective operators: The image under a compact operator F of the singular vectors/functions associated with very small singular values will be almost zero. Consequently, one may regard these singular vectors/functions to constitute a basis for a fictitious null space for F. In fact, if TSVD is employed, then F is approximated by an operator F_k which has a proper null space. Furthermore, the purpose of Tikhonov regularization is to reduce the error amplification caused by the presence of "almost null space" components.

It turns out that the weighting procedure also can improve the solution of injective inverse problems, compared with traditional approaches. We exemplify this numerically, using sparsity regularization, for an inverse heat conduction problem, a Cauchy type of problem for Laplace's equation and with phantom data for a linearized EIT problem.

Microlocal Analysis of ISAR Imaging, Tiernan Brosnan

Tiernan Brosnan* and Clifford Nolan
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*Lead Presenter

We consider Inverse Synthetic Aperture Radar (ISAR) imaging for a rotating scatterer. In ISAR, a stationary transceiver is used to image a moving object. The idealised inverse problem to be solved is to reconstruct the scene from measurements of the scattered field. In the rest frame of the object, the transceiver is moving. Thus, the measurements are equivalently obtained in a certain time interval, as the transceiver moves along a curve/surface in the scatterer's rest frame. We examine the case of a rotating scatterer on a horizontal target plane being imaged by a point-like transceiver, as well as the case of an unconstrained rotating scatterer being imaged by a linear transceiver array. For each scenario, the wavefront relation of the scattering operator, which is an FIO, is used to examine the nature of fictitious artifacts that arise in the reconstructed image. This analysis is used to suggest several experimental setups whereby the artifacts can be guaranteed to be outside a region selected for imaging. We also investigate the conditions under which the backprojection operator composed with the scattering operator is a pseudodifferential operator. When these conditions are satisfied, no artifacts appear in the reconstructed image.

Non-Uniqueness and reconstructability for the atmospheric tomography problem, Ronny Ramlau

Earth bound astronomical telescopes rely on Adaptive Optics systems in order to achieve a high imaging quality. In order to observe objects like extended galaxies, a correction for a larger field of view has to be obtained. This can be achieved by the use of the measurements of incoming wavefronts of multiple guide stars that allow a tomography based correction of the incoming light of the scientific object of interest.

The atmospheric tomography operator describes the impact of turbulent atmospheric layers on light passing through the atmosphere. Given wavefronts from different guide stars, measured by an (astronomical) telescope, the inverse problem consists in the reconstruction of the turbulence above the telescope. We show that the collected data is not sufficient to reconstruct the atmosphere uniquely. Additionally, we show that classical regularization methods as Tikhonov regularization or Landweber iteration will always fail to reconstruct a physically meaningful turbulence distribution. On a brighter side, we nevertheless achieve a good correction of the scientific images by using the reconstructed turbulence.

Computational improvements of Plug-and-Play methods with proximable denoisers, Andrea Sebastiani

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Very recently, Gradient Step denoisers have been proposed to overcome the main limitations of Plug-and-Play algorithms interpretation 1]. More in detail, it is possible to derive the functionals whose proximal operator corresponds to these denoisers. This result allows to define a non-convex objective function, that is minimized by the arising Plug-and-Play method involving a denoiser of this specific class. Despite the state-of-the-art results obtained by these methods, their execution requires an extensive amount of memory depending both on the size of the denoiser and the size of the images. The considerable computational demand motivates several acceleration strategies which aim to reduce the cost of each block update. The previous convergence results can be extended to this improved version of the method, characterizing the limit points. Numerical experiments demonstrate the advantages of such acceleration, reducing the resources necessary to compute the reconstruction of various images on different tasks.

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Single-stage approach for estimating optical parameters in spectral quantitative photoacoustic tomography, Miika Suhonen

Miika Suhonen, Aki Pulkkinen, Tanja Tarvainen

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Quantitative photoacoustic tomography (QPAT) is a biomedical imaging technique utilizing the photoacoustic effect. In QPAT, an external illumination of light causes pressure waves to propagate in the tissue, and these pressure waves can be measured outside the target with ultrasound sensors. The aim of QPAT is to estimate the optical parameters of the target, most importantly concentrations of light absorbing molecules (chromophores). Conventionally, the image reconstruction of QPAT can be seen to consist of two sequentially solved inverse problems. In the first inverse problem, the initial pressure distribution is estimated from the photoacoustic time-series. In the second inverse problem, the optical parameters are estimated from the estimated initial pressure. In this work, we propose a methodology for estimating spectral optical parameters in a single-stage i.e., estimating chromophore concentrations directly from photoacoustic time-series. The forward model is constructed by combining the models of light and ultrasound propagation and by representing the optical absorption and scattering with their spectral models. The methodology is evaluated using numerical simulations in different full-view and limited-view situations. Concentrations of four chromophores, two scattering related parameters, and the Grüneisen parameter were estimated directly from photoacoustic time-series data.

Hybrid knowledge and data-driven approaches for Diffuse Optical Tomography reconstruction, Alessandra Serianni

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Diffuse Optical Tomography (DOT) is a non-invasive medical imaging technique which employs Near- Infrared (NIR) light to recover the spatial distribution of optical coefficients in biological tissues. Due to the limited availability of boundary measurements and the intense light scattering, DOT reconstruction is a severely ill-posed problem [1]. Recently, the success of deep learning methods has shifted the focus of tomographic imaging from purely knowledge-driven to data-driven approaches.
In this contribution, we propose a hybrid approach that combines model-based and deep learning tech- niques. Our idea is to leverage Graph Neural Networks (GNNs), that -once trained- we use as a fast forward model that solves partial differential equations [2], into an iterative optimization-based method for solving the inverse problem. Due to the severe ill-conditioning of the reconstruction problem, we also learn a prior over the space of solutions using an autoencoder-type neural network which maps the latent code to the estimated physical parameter, that is passed to the GNN to obtain the prediction. The latent code is finally optimized to minimize the difference between the recorded and predicted data.

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Inverse magnetisation problem in paleomagnetic context: Field extrapolation and asymptotic estimates, Dmitry Ponomarev

The process of extraction of relict magnetic information from geosamples and meteorites is a challenging but important task in paleomagnetic research. Due to the weak intensity of the field produced by a magnetised rock, the measurements have to be performed in direct vicinity of the sample and using highly sensitive magnetometric devices such as SQUID and QDM. The basic quantity of interest is the net magnetisation (magnetisation moment vector). Reconstruction of this quantity hinges on effective processing of the experimental data, with the main challenges being the limited measurement area and the noise contamination. Motivated by a concrete experimental setting in the Paleomagnetism lab at EAPS department of MIT (USA), we will focus on constructive issues. Namely, using asymptotic analysis, one can obtain explicit formulas estimating the net magnetisation vector. However, since the measurement area is usually not sufficiently large, we face an intermediate problem of the field extrapolation. We propose and analyse some extrapolation strategies (allowing presence of noise) and illustrate them numerically.

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Variational Bayes image restoration with compressive autoencoders, Maud Biquard

Regularization of inverse problems is of paramount importance in computational imaging. The ability of neural networks to learn efficient image representations has been recently exploited to design powerful data-driven regularizers. While state-of-the-art plug-and-play methods rely on an implicit regularization provided by neural denoisers, alternative Bayesian approaches consider Maximum A Posteriori (MAP) estimation in the latent space of a generative model, thus with an explicit regularization.
However, state-of-the-art deep generative models require a huge amount of training data compared to denoisers. Besides, their complexity hampers the optimization involved in latent MAP derivation.
In this work, we first propose to use compressive autoencoders instead. These networks, which can be seen as variational autoencoders with a flexible latent prior, are smaller and easier to train than state-of-the-art generative models. As a second contribution, we introduce the Variational Bayes Latent Estimation (VBLE) algorithm, which performs latent estimation within the framework of variational inference. Thanks to a simple yet efficient parameterization of the variational posterior, VBLE allows for fast and easy (approximate) posterior sampling.
Experimental results on image datasets BSD and FFHQ demonstrate that VBLE reaches similar performance than state-of-the-art plug-and-play methods, while being able to quantify uncertainties faster than other existing posterior sampling techniques.

Network Inpainting via Optimal Transport, Enrico Facca

Reconstructing natural networks such as blood vessels digitally is essential to maintain the accuracy of simulations. However, these structures are typically only reachable through non-invasive imaging methods. This may introduce errors, such as lost data or the presence of artifacts, that affect the reliability of the data.

The main idea of this presentation is to address these issues thanks to recent advances in the branched transport theory, which studies these ramified structures as the result of an optimization process. This is done by including a physics-based regularization term in a variational inverse problem. We call the resulting approach Network Inpainting via Optimal Transport (NIOT).

We present a series of numerical experiments that present the capabilities and limitations of the NIOT approach.

Dynamic Learning Rate Adaptation for Stochastic Variational Inference, Maximillian Dinkel

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Like many optimization algorithms, Stochastic Variational Inference (SVI) is sensitive to the choice of the learning rate. If the learning rate is too small, the optimization process may be slow, and the algorithm might get stuck in local minima. On the other hand, if the learning rate is too large, the algorithm may oscillate or diverge, failing to converge to a solution. Adaptive learning rate methods such as Adam, Adagrad, or RMSprop automatically adjust the learning rate based on the history of gradients. Nevertheless, if the base learning rate is too large, the variational parameters might still oscillate around the optimal solution. With learning rate schedules, the learning rate can be reduced gradually to mitigate this problem. However, the amount at which the learning rate should be decreased in each iteration is not known a priori, which can significantly impact the performance of the optimization.

In this presentation, we propose a method to adapt the learning rate based on the history of the variational parameters. We use an empirical measure to quantify the amount of oscillations against the progress of the variational parameters to adapt the learning rate. The approach requires little memory and is computationally efficient. We demonstrate in various numerical examples that our method reduces the sensitivity of the optimization performance to the learning rate and can also be used in combination with other adaptive learning rate methods.

Invertible Model Error Transport for Bayesian Inversion, Maren Casfor

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Bayesian inversion is a powerful tool for solving inverse problems, but limited by the complexity of the forward model. For computationally expensive forward problems the forward model usually has to be simplified. The inaccuracy of the forward model, the so called model error [4] is often ignored in Baysian setting what propagates to an error in the posterior distribution [2],[4]. Including the model error in the Bayesian setting allowes us to use a simplified forward model $f \approx F$ for faster computation, but even so get a posterior distribution close to the exact one.
In the presented approach from [3] the model error is included as a stochastic variable ε distributed by the push forward $M_{\#}\pi_x$ under the model error function $M(x) = F(x) - f(x)$. Samples of the model error are thus given by samples drawn from π_x, which are mapped under M.

The model error approach will be combined with the concept of transport maps in order to sample from the posterior distribution π_x. Herby we replace the common sampling algorithm Markov Chain Monte Carlo by a transport map T which approximates the posterior distribution by the push forward

$$T_{\#}\pi_{\text{ref}} = \pi_{\text{ref}} \circ T^{-1} |\det \nabla_x T^{-1}|$$

for an arbitrary reference density π_{ref}. The transport map T is given by an Invertibal Neural Network (INN) [1] and trained by minimizing the Kullback-Leibler divergence of the push forward $T_{\#}\pi_{\text{ref}}$ and the posterior π_x.

One application where we can observe this type of modelling error is an inverse problem in scatterometry, since the simulation by a Finite Element Method (FEM) is very expensive. Instead of using a very fine mesh for the FEM, a coarse mesh can be used and the so caused error is then corrected by including the the model error.

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Graphite Material Property Model Calibration for Advanced Gas-Cooled Reactors, Daniel Timothy Kent

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A physically informed material property model is used to predict graphite material properties, stress, and dimensional change to support operation of the UK's fleet of Advanced Gas-cooled Reactors throughout their lifetime. To produce up-to-date analyses that support lifetime safety cases, the model is routinely validated and calibrated against measurements taken from the core and material test reactors.

'Forward' calculations using the model make use of both a computationally intensive finite element model where stresses and distortions are required, and a rapid, MATLAB-based formulation where properties at individual points are required. Calibration of this model consists of the solution of a complex inverse problem, in which both the finite element model and MATLAB formulation are solved 'backwards' to determine the response of the graphite to irradiation and radiolytic oxidation.

A probabilistic methodology is employed in order to quantify both the aleatoric and epistemic uncertainties within the model parameters. This allows uncertainties to be propagated through finite element models, which informs predictions of when safety case metric limits will be reached.

An iterative solver for nonlinear Fourier-type wavefront sensing, Victoria Hutterer

Victoria Hutterer, Andreas Neubauer, Julia Shatokhina, Simon Hubmer, Bernadett Stadler and Ronny Ramlau

Pyramid wavefront sensors (PWFSs) are commonly included in the design of advanced adaptive optics (AO) instruments for the future generation of extremely large telescopes (ELTs). Furthermore, they are now also considered for free-space optical communications (FSOC) and ophthalmic imaging. Recently, a generalization of the PWFS has been introduced called Fourier-type wavefront sensors (WFSs). They perform phase measurements by optical Fourier filtering with different optical elements in the focal plane (e.g., 4-sided or 3-sided glass pyramids). AO control with Fourier-type WFSs is a nonlinear inverse problem. However, in existing systems Fourier-type wavefront sensing is conventionally based on linearization of the underlying model. Therefore, in nonlinear regimes, the image quality can be critically degraded due to approximation errors.
Here, we present the iterative solver Nonlinear Pyramid Extension (NOPE) for Fourier-type wavefront sensing. It is a generalized wavefront reconstruction method for all Fourier-type sensors. The algorithm has been designed to be effective in nonlinear regimes.
The robustness of nonlinear Fourier-type wavefront sensing with the NOPE is demonstrated for ELT-scale and ophthalmic AO instruments. Moreover, we show how digital Fourier-type wavefront sensing can be used for phase unwrapping. Different realizations of Fourier-type WFSs are investigated and their performance compared to linear approaches.

Utilising Monte Carlo method for light transport in optical tomography, Jonna Kangasniemi

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Abstract

In optical tomography, optical properties of an imaged target are estimated from boundary measurements of near-infrared light. The potential applications include, for example, breast cancer imaging, functional brain studies and small animal studies. Numerical solution of the inverse problem in optical tomography requires modelling of light transport. In general, the diffusion approximation to the radiative transport equation has been used. However, when the size of the target is less than a few scattering lengths, light transport has to be modelled as a radiative transfer. In biological tissues this means that the thickness of the tissue is less than a few centimetres and/or it has low scattering regions.

In this work, we utilised the Monte Carlo method for light transport to numerically approximate the solution of the radiative transfer equation. In the minimisation algorithm to solve the inverse problem, the derivatives for scattering are evaluated utilising a perturbation approximation for Monte Carlo. The number of photon packets in Monte Carlo simulation are selected with an adaptive approach. The proposed method is evaluated with numerical simulations. This is joint work with Meghdoot Mozumder, Aki Pulkkinen and Tanja Tarvainen.

Optimal State and Parameter Estimation Algorithms, Olga Mula

Abstract: This talk presents an overview of recent works aiming at solving inverse problems (state and parameter estimation) by combining optimally measurement observations and parametrized PDE models. After defining a notion of optimal performance in terms of the smallest possible reconstruction error that any reconstruction algorithm can achieve, I will present practical numerical algorithms based on nonlinear reduced models for which we can prove that they can deliver a performance close to optimal. The proposed concepts may be viewed as exploring alternatives to Bayesian inversion in favor of more deterministic notions of accuracy quantification.

Data-driven approaches to the Kuopio Tomography Challenge 2023, Alexander Denker

Author: Alexander Denker, Zeljko Kereta, Imraj Singh, Tom Freudenberg, Tobias Kluth, Peter Maass, Simon Arridge

Electrical impedance tomography (EIT) plays a crucial role in non-invasive imaging, with both medical and industrial applications.
In this talk, we present three data-driven reconstruction methods for EIT imaging: a postprocessing network, a fully learned network and a conditional diffusion model.
These approaches were originally submitted to the Kuopio tomography challenge 2023 (KTC2023).

The first method describes a postprocessing approach, a conventional technique in data-driven approaches to inverse problems.
Here, the goal is to remove noise and artefacts inherent in another (classical) reconstruction method.
In contrast, the fully learned network directly processes measurements, learning the complete inversion process from the data itself.
Lastly, the conditional diffusion model is trained to approximate the conditional density of conductivity distribution based on boundary measurements. All three methods share a similar neural network architecture and were trained using a synthetically generated data set.

The postprocessing model was able to win the KTC2023 with the fully learned approach showing a similar performance.
This presentation will expound on the rationale behind the development of these models, insights and practical considerations.

Source Design Optimization for Depth Image Reconstruction in X-ray Imaging, Hamid Fathi

Abstract

In this research, we investigate the Optimal Experimental Design (OED) problem for laminog- raphy in X-ray imaging. This is particularly relevant for industrial quality control where non- destructive testing is essential. Traditional tomography often requires full object rotation, which is not feasible for large, flat objects. Laminography addresses this by reconstructing images of specific slices (which we call depth images) from limited angle projections, reducing data requirements and speeding up acquisition.

We pose the experimental design problem within the Bayes risk framework and empirical Bayes risk minimization. Then, we employ bilevel optimization and implement it by gradient-based methods for solving high-dimensional problems. This approach can mitigate imaging artifacts and speed up the acquisition process in cases where moving the source and detector is time-consuming (e.g., in robotic arm imaging systems). Additionally, we incorporate various regularization terms in the lower- level optimization to suppress artifacts that arise with the reconstructed depth image. Numerical experiments on simulated data validate our approach, demonstrating its effectiveness in reducing artifacts and optimizing source positioning compared to random designs.

Our contributions extend to analyzing image reconstruction processes, proposing a Bayes risk minimization method for experimental design, and providing comprehensive evaluations through simulated data. The study promises enhancements in imaging efficiency and quality, crucial for industrial applications relying on laminography for defect detection and quality assessment.

Presentation Preference: oral presentation

Keywords: X-ray imaging, Laminography, experimental design, bilevel optimization

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Infinite-dimensional compressed sensing and applications to inverse problems in medical imaging, Anupam Gumber

University of Genova, Italy

Compressed sensing has had a considerable impact on inverse problems and sam- pling theory over the last several years. In this talk, we delve into the application of ideas stemming from applied harmonic analysis and approximation theory to inverse problems in medical imaging. Specifically, we explore sampling theory and sparsity- promoting schemes grounded in the theory of infinite-dimensional compressed sens- ing (CS). We provide new results concerning a general theory of non-uniform recov- ery guarantees in infinite-dimensional CS for abstract inverse problems, even those potentially ill-posed, involving an arbitrary forward operator. This is achieved by adopting a CS framework within the realm of Hilbert spaces. From an infinite- dimensional perspective, we examine signals defined in the continuous domain, al- lowing the measurement operator to deviate from an orthonormal transformation, while the unknown signal exhibits sparsity properties relative to a dictionary. Sub- sequently, we delve into the design of the sampling scheme to effectively exploit this additional structural information. As a notable application, we obtain rigorous re- covery estimates for the sparse Radon transform, a crucial component in modeling computed tomography in medical imaging. This is a joint work with G. S. Alberti and M. Santacesaria.

Generative imaging for fast image reconstruction and uncertainty quantification in radio interferometry. Matthijs Mars

The advent of the next generation of interferometric telescopes, exemplified by the Square Kilometre Array (SKA), presents novel challenges in managing the vast quantities of data they will acquire. This presentation explores innovative machine learning methods that balance computational efficiency with state-of-the-art image reconstruction quality. Current image reconstruction techniques in radio interferometry primarily rely on variations of the CLEAN algorithm, which are computationally efficient but tend to fall short in reconstructing extended sources. In contrast, the latest approaches employ proximal optimisation techniques, which offer superior reconstruction quality but are computationally expensive and rely on manual priors. This talk delves into our research on leveraging machine learning for image reconstruction and discusses the specific challenges that arise within the context of radio interferometry. Specifically, we discuss how to deal with a measurement operator that varies on an observation-by-observation basis. Our work showcases the application of generative imaging methods for image reconstruction, using conditional generative adversarial networks (cGAN). Despite past concerns regarding cGANs' limited diversity in the generated posterior samples, recent advancements in regularised cGANs have led to models that produce samples closely approximating the underlying posterior distribution. These novel generative models offer both computational efficiency and robustness to variations in the measurement operator, allowing for fast image reconstruction as well as uncertainty estimation through (approximate) posterior sampling.

Utilising Monte Carlo method for light transport in optical imaging, Tanja Tarvainen

We study the inverse problem in optical imaging when the forward operator is the Monte Carlo method for light transport. Monte Carlo a stochastic method that can be used to simulate the solution of the radiative transfer equation. In the approach, paths of photons are simulated when they undergo absorption and scattering events in a turbid medium. In the inverse problem of optical imaging, estimates for absorption and scattering are computed. Now, due to the stochastic nature of the forward operator, also the search direction of a minimisation algorithm for solving these estimates is stochastic. We discuss the inverse problems and their solutions in two optical imaging modalities: diffuse optical tomography and quantitative photoacoustic tomography.

Uncertainty quantification in statistical imaging sciences: 40 years of muddling through, Marcelo Pereyra

Abstract: Probability theory and statistical science are cornerstones of imaging sciences, underpinning many and varied approaches from Markov random fields to score-based denoising diffusion models and stochastic flow-matching techniques. In addition to powerful image estimation methods, statistical science provides a framework for uncertainty quantification and for using image data as quantitative evidence. These capabilities are important for the rigorous interpretation of experimental results and for robust interfacing of quantitative imaging pipelines with scientific and decision-making processes. This talk explores the following question: four decades after the publication of the first seminal papers on the topic, are the probabilities and statistical inferences delivered by existing probabilistic and statistical imaging methods meaningful under replication of an experiment? or are they still only meaningful as subjective measures of belief?

Subgradient Langevin Methods For Sampling From Non-Smooth Potentials, Martin Holler

MARTIN HOLLER
JOINT WORK WITH A. HABRING AND T. POCK

In regularization approaches for inverse problems, the Bayesian viewpoint continues to gain importance. This is true in particular in the context of generative machine learning approaches, but also for classical variational methods. A main advantage of the Bayesian viewpoint is that it provides a natural basis for uncertainty quantification, e.g., via the estimation of error bounds. The latter requires techniques to sample from the posterior distribution, which is often of the form $\pi(x) \propto e^{-U(x)}$ with $U(x) = F(x) + G(Kx)$ and K a linear operator.

In this talk, we introduce and analyze sampling techniques for probability distributions of this form, that are applicable in the particularly challenging setting where G is non-differentiable. Two different methods are proposed. Both employ a subgradient step with respect to $G \circ K$, but, depending on the regularity of F, they employ either an explicit or an implicit gradient step with respect to F. For both methods, non-asymptotic convergence proofs are provided, with improved convergence results for more regular F. Further, numerical experiments are conducted for simple 2D examples, illustrating the convergence rates, and for examples of Bayesian imaging, showing the practical feasibility of the proposed methods for high dimensional data.

A uniqueness result for the anisotropic Schrödinger type equation from local measurements, Niall Donlon

We discuss the inverse problem of uniquely identifying the coefficients σ and q of an anisotropic Schrodinger-type equation $div(\sigma \nabla u) + qu = 0$ from the knowledge of the local Neumann-to-Dirichlet (D-N) map. This model includes a large class of inverse problems. When $q=0$, σ represents the conductivity in a conductor Ω. Here the determination of σ from the Dirichlet-to-Neumann (or similarly the N-D) map is the celebrated Calderón's problem. When σ is the identity matrix the equation under consideration is the Schrödinger equation and when $q>0$, it is the reduced wave equation (or Helmholtz equation). In the latter case q encodes information about the wave speed of the media in $Omega$. When $q\leq 0$, the equation is associated with models of propagation of light through a body. Specifically, the coefficients σ and q model the diffusion tensor and absorption coefficients, respectively, in the inverse problem known as Diffuse Optical Tomography (DOT).

In this work, we assume that σ and q are both *a-priori* known to be piecewise constant on a given partition of $\Omega \subset \mathbb{R}^n$, $n\geq 3$. In this setting, we investigate the unique (possibly simultaneous) determination of σ and q from the local D-N map, given on a non-empty 'curved' portion Σ of the boundary $\partial\Omega$.

Semiparametric Theory for Bayesian Inverse Problems, Adel Magra

April 2024

We explore statistical inverse problems where the forward map is partially unknown up to a finite dimensional parameter. Our focus is on making inferences on this parameter from noisy observations of the transformed signal. Employing a Bayesian framework, we consider two examples: semi-blind deconvolution and thermal diffusivity recovery in a heat equation.

In semi-blind deconvolution, we deal with uncertainty in the convolution kernel's location. We establish a Bernstein-von Mises (BvM) theorem for the marginal posterior of the location that depends on the ill poshness of the problem, the smoothness of the signal that is convoluted and on the regularity of the prior put on that signal. In particular, we observe a wider BvM applicability as the problem's ill-posedness increases.

For the second example, we consider a heat equation with unknown thermal diffusivity and initial temperature map. We put a Gaussian process prior on the initial condition and derive a BvM theorem for the diffusivity's marginal posterior. This theorem's validity depends on the prior's regularity matching the initial condition's smoothness, as revealed through numerical investigations.

Our Bayesian framework offer robust uncertainty quantification for these problems, demonstrating how prior assumptions and problem characteristics influence statistical inferences. These findings provide insights into handling uncertainty in real-world scenarios where incomplete knowledge of system parameters is common.

Image Reconstruction via Deep Image Prior Subspaces, Zeljko Kereta

Abstract:

Deep learning has been widely used for solving imaging inverse problems but its deployability has been held back due to the shortage of high-quality paired training data.

Unsupervised learning methods, e.g., deep image prior (DIP), naturally fill this gap, but bring a host of new issues: the susceptibility to overfitting due to a lack of robust early stopping strategies and unstable convergence.

We present a novel approach to tackle these issues by restricting DIP optimisation to a sparse linear subspace of its parameters, employing a synergy of dimensionality reduction techniques and second order optimisation methods.

The low-dimensionality of the subspace reduces DIP's tendency to fit noise and allows the use of stable second order optimisation methods, e.g., natural gradient descent or L-BFGS.

Experiments across both image restoration and tomographic tasks of different geometry and ill-posedness show that second order optimisation within a low-dimensional subspace is favourable in terms of optimisation stability to reconstruction fidelity trade-off.

A bi-convex lifted Bregman training strategy for unfolded proximal neural networks, Xiaoyu Wang

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Abstract

For a few decades, proximal algorithms have been state-of-the-art approaches for solving ill-posed inverse problems, using variational formulations often involving non-differentiable sparsity regularisations. In the recent years, these methods have been combined with deep learning strategies to replace standard sparsity regularisations by learned denoisers, and yield better solution estimates. In particular, unfolded proximal neural networks (PNNs) have been proposed to design denoisers by unrolling proximal algorithms for a fixed number of iterations, enabling the regularisation' linearity parameters to be learned from training data. In this work, we follow this line of development and propose a lifted training formulation based on Bregman distances for unfolded PNNs. This formulation exploits the proximal structure of the network, in particular leading to a bi-convex penalised loss. Leveraging deterministic block-coordinate proximal-gradient methods, we design a bespoke computational strategy beyond traditional back-propagation methods for solving the learning problem efficiently. We further propose a deterministic batch formulation of the learning strategy. We assess the behaviour of the proposed training approaches for PNNs through numerical simulations, by designing a denoising unfolded network whose structure is based on dual proximal-gradient iterations.

Bayesian Active Learning in the Presence of Nuisance Parameters, Sabina Sloman

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[2]Department of Computer Science, Aalto University, Helsinki, Finland

Abstract

In many settings, such as scientific inference, optimization, and transfer learning, the learner has a well-defined objective, which can be treated as estimation of a *target parameter*, and no intrinsic interest in characterizing the entire data-generating process. Usually, the learner must also contend with additional sources of uncertainty or variables — with *nuisance parameters*. Bayesian active learning, or sequential optimal experimental design, can straightforwardly accommodate the presence of nuisance parameters, and so is a natural active learning framework for such problems. However, the introduction of nuisance parameters can lead to bias in the Bayesian learner's estimate of the target parameters, a phenomenon we refer to as *negative interference*. We characterize the threat of negative interference and how it fundamentally changes the nature of the Bayesian active learner's task. We show that the extent of negative interference can be extremely large, and that accurate estimation of the nuisance parameters is critical to reducing it. The Bayesian active learner is confronted with a dilemma: whether to spend a finite acquisition budget in pursuit of estimation of the target or of the nuisance parameters. Our setting encompasses Bayesian transfer learning as a special case, and our results shed light on the phenomenon of negative transfer between learning environments.

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Numerical and Sensitivity Analysis of Modeling the Newcastle Disease Dynamics, Nurudeen Oluwasola Lasisi

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Abstract:

Newcastle disease is highly contagious disease of birds caused by a para-myxo virus. In this paper, we presented Novel quarantine-adjusted incident and linear incident of Newcastle disease model equations. We considered the dynamics transmission and control of Newcastle disease. The existence and uniqueness of the solutions were obtained. The existence of disease free point was showed, and the model threshold parameter was examined using next generation operator method. The sensitivity analysis was carried out in order to indentify the most sensitive parameters on the disease transmission. This revealed that as parameters $\beta\beta$, $\omega\omega$, $aaaaaa$ ∧ increase while keeping other parameters constant, the effective reproduction number RR_{eeee} increases. This implies that, the parameters increase the endemicity of the infection of individuals. More so, when the parameters $\mu\mu, \varepsilon\varepsilon, \gamma\gamma, \delta\delta_1, aaaaaa\ \alpha\alpha$ increase, while keeping other parameters constant, the effective reproduction number RR_{eeee} decreases. This implies, the parameters decrease the endemicity of the infection as they have negative indices. Analytical results were numerically verified by the Differential Transformation Method (DTM) and quantitative views of the model equations were showcased. We established that as contact rate ($\beta\beta$) increases, the effective reproduction number RR_{eeee} increases, as effectiveness of drug usage increases, the RR_{eeee} decreases and as quarantine individual decreases, the RR_{eeee} decreases. The results of the simulations showed that infected individual increases when the susceptible person approaches zero, also vaccination individual increases when infected individual decreases and simultaneously increases the recovery individual.

Keywords: Disease free equilibrium; Effective reproduction number; Endemicity; Newcastle disease model; Numerical; Sensitivity Analysis.

Enhancing Physical Properties of Powder Systems through Particle Size Distribution Optimization and Non-Differentiable Stochastic Models, Gil Robalo Rei

4th IMA Conference on Inverse Problems from Theory to Application

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Abstract

In various engineering applications such as all-solid-state batteries (ASSB) or additive manufacturing, the physical domain comprises a particle-based microstructure. These spherical particles, with varying radii governed by their particle size distribution (PSD), strongly affect the global properties of the underlying physical system. Given the stochastic nature of arranging random-sized particles in predefined spaces, the forward model is inherently of stochastic nature without a functional relationship between particle positions and sizes. Therefore, to optimize system properties, we developed a stochastic variational approach where the derivatives of the objective function w.r.t. the variational parameters of the PSD are obtained using a score function estimator, elimi- nating the differentiability requirements on the forward model. This approach allows for PSD optimization independent of the underlying physical problem. To demonstrate the effectiveness of this method, we optimize various geometri- cal properties of which the mechanical behavior of the particles in a constrained space is enforced via a discrete element method. Lastly, for an ASSB system, we minimize the expected tortuosity factor, representing geometrical resistance to an ion flux through the microstructure, and quantify the uncertainty in performance gain. This example showcases the effectiveness of this approach, enabling efficient optimization of performance-critical metrics of multi-physics systems.

Persistent Sequential Monte Carlo, Minas Karamanis

Sequential Monte Carlo (SMC) methods have emerged as powerful tools for solving inverse problems in complex, multimodal, and non-linear settings. However, their efficiency is often hampered by the need for a large number of particles to obtain accurate posterior and marginal likelihood estimates, leading to high computational costs. We introduce Persistent Sequential Monte Carlo (PSMC), a novel extension that mitigates these limitations through an inclusive reweighting and resampling strategy. By leveraging information across all previous iterations, PSMC constructs a rich, diverse ensemble of persistent particles, overcoming particle impoverishment and mode collapse in complex and multimodal targets. PSMC achieves significantly lower variance marginal likelihood estimates, crucial for model comparison, while requiring fewer overall likelihood evaluations than standard SMC. The persistent particles facilitate better transition kernel adaptation, enabling more efficient exploration. Extensive experiments across challenging distributions demonstrate PSMC's consistent advantages over standard techniques, exhibiting lower squared bias in posterior moment estimation and substantially lower marginal likelihood errors, even at reduced computational cost. PSMC represents a significant methodological advancement, offering a more robust, efficient, and scalable sampling framework for tackling complex inverse problems.

Solving Dynamic Inverse Problems with Neural Fields, Pablo Arratia L´opez

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In Dynamic Inverse Problems we are interested in the reconstruction of a space-time process from time-dependent measurements. In several cases of interest, such as dynamic free-breathing cardiac MRI, the acquisition of measurements is slow with respect to the motion of the process (e.g., moving organs), hence, only highly under-sampled measurements can be acquired. A naive approach to solving this problem is to neglect the dynamic nature of the process and proceed with a frame-by-frame reconstruction, however, this may lead to oversmoothed results. To regularize this problem a motion model that relates the image sequence and the motion expressed in terms of a velocity field is introduced. Instead of the commonly used grid-based methods, we use space- time continuous representations via Neural Fields, this is, both images and motion are parametrized as neural networks. We investigate the advantages of mesh-free representations and how do they compare to traditional grid-based methods, the implicit regularization that neural fields impose, and different motion models.

Adaptive Higher Order Derivative Explicit Runge-Kutta-Nystrom Method For The Direct Integration of Second Order Ordinary Differential Equations, Moses Adebowale Akanbi

Akanbi, Moses Adebowale[a,*], Olaniyan, Adegoke Stephen[a], Kazeem, Moshood Tolulope[a], Wusu, Ashiribo Senapon[a], Okugbesan, Basheerat Oluwaseun[a], Mustapha, Adewale Rilwan[a]

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Abstract

In this paper, a two-stage explicit Runge-Kutta-Nystrom Method with higher order derivatives for the numerical integration of second order ordinary differential equations is presented. It describes how a two-stage multi-derivative explicit Runge-Kutta method is adapted to be a Runge-Kutta-Nystrom Method. The order condition of the method is obtained up to order four. The stability analysis of the method is investigated and using some standard test problems, the accuracy and efficiency of the method are established. The numerical results compare favourably with standard methods in the literature.

Keywords: Algorithms, Initial Value Problems, Second Order Differential Equations, Multi-derivative Runge-Kutta, Runge-Kutta-Nystrom Method, Higher-Order Derivatives.
2020 MSC: 65L05, 65L06, 65L20.

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Practical approaches to inverse problems in photoacoustic tomography, Jaakko Kultima

March 15, 2024

Abstract

Numerical solutions for inverse problems arising from photoacoustic tomography (PAT) are generally large in size, and computationally costly. Considering just the acoustic part of PAT, one attempts to reconstruct the initial pressure distribution inside the object of interest based on the time-series pressure data recorded on (some part of) the boundary. Initial pressure distribution being a real-valued function over the support of the target, one is left with a parameter reconstruction of N parameters, where N is the number of basis vectors used to present the initial pressure distribution, e.g., the number of pixels/voxels.

In reconstructions we rely on iterative solvers for the linear inverse problem. Direct approaches are often extremely time consuming due to expensive forward evaluations, and rapid convergence of the algorithm is crucial for the utility. To this end we propose to use the so-called sketch-to-precondition scheme. We will further look into efficient presentations for the initial pressure distribution as well as pre-processing data.

Bregman-Based Inversion of Residual Neural Networks, Alexandra Valavanis

Abstract: Many imaging problems such as blind image deconvolution require the solution of nonlinear inverse problems. We study a residual neural network architecture for the approximation of nonlinear forward problems, aiming to invert the forward problem. An interesting aspect of this network architecture is that it will allow us to formulate a convex variational regularisation method for the inversion of this residual network, where the data fidelity term is based on a tailored Bregman distance. We will discuss theoretical aspects of this regularisation method, propose algorithms to solve the underlying optimisation problem and present numerical results for selected imaging problems. This is joint work with Martin Benning from University College London.

Fibre-optic communication based on the inverse scattering transform for finite-genus solutions, Stepan Bogdanov

The inverse scattering transform (IST), also known as the nonlinear Fourier transform, is a technique to integrate certain nonlinear partial differential equations. It is based on the concept of scattering data having linear spectral properties to convert the nonlinear dynamics of an equation's solution into a linear evolutionary problem. This method is applied in fiber-optic communication, offering transmission without nonlinearity-induced distortions. Specifically, integrating the nonlinear Schrodinger equation with periodic boundary conditions is deeply related to finite-genus solutions, parametrizing with a discrete set of scattering data. The IST framework for these solutions has been developed for a specific case of exact periodicity (frequency commensurability among composite nonlinear modes). However, this theoretical limitation restricts the practical use of finite-genus solutions.

To address the problem, we developed a neural network-based approach to construct the IST for finite-genus solutions. These solutions are parametrized in terms of the Riemann-Hilbert problem with "phases" entering the jump matrices. A neural network demonstrated its ability to retrieve the "phases" from given solutions. We made a numerical simulation of the fiber-optic communication system with finite-genus solutions used as data carriers. We applied our neural network-based IST at the receiver of the communication system and demonstrated reliable and flexible data transmission.

Edge preserving priors for inverse problems, Hanne Kekkonen

The Bayesian approach to inverse problems allows us to encode our *a priori* knowledge of the unknown function of interest as a probability distribution. Gaussian process priors are often used in Bayesian inverse problems due to their fast computational properties. However, the smoothness of the resulting estimates is not well suited for modelling functions with sharp changes, such as images. Smooth functions with few local irregularities have a sparse expansion in the wavelet basis, making wavelet-based Besov priors a good candidate for modelling spatially inhomogeneous functions. The sparsity-promoting and edge-preservation properties of Besov priors can be further enhanced by introducing a new random variable that takes values in the space of 'trees,' ensuring that the realisations have jumps only on a small set. We will also discuss how to estimate the optimal value for the hyperparameter controlling the sparsity of the solution from the data.

Sequential optimal experimental design for single-pixel image reconstruction using reinforcement learning, Marcos Obando

A prominent example of Compressed Sensing is given by single-pixel imaging, where an image is reconstructed from a sequence of measurements, each consisting of the sum of intensities obtained by multiplying the image with a different binary sampling pattern. Optimal experimental design provides a framework to choose the most informative set of sampling patterns for a given class of images a-priori, i.e., before the measurement. In sequential optimal experimental design (sOED), this choice is made a-posteriori, i.e., we adapt the sampling pattern during the measurement, based on the data acquired so far. Conventional sOED approaches lead to challenging optimization problems, prohibiting real-time implementations. In contrast, we explore sOED using reinforcement learning (RL): RL attempts to learn a policy, namely a mapping of the representation of the information gathered so far to a probability distribution over the next sampling pattern. We use the actor-critic RL formalism, which finds a design policy proposing sampling patterns by maximising an image quality based reward function. In a proof-of-concept study we investigate different representations of this problem, their effect on the chosen policy and discuss their applicability to single-pixel imaging in real-world applications.

Tomography of the fast-ion velocity distribution in tokamak plasmas with a physics-based prior, Per Christian Hansen

DTU Compute, Technical University of Denmark

A tokamak reactor uses a magnetic field to confine plasma, with a temperature of several million degrees, in the shape of a torus. The plasma is heated by injection of neutral particles. Energy is then produced through fusion in the form of heat that produces electricity by way of turbines.

The goal of this inverse problem is to monitor the dynamics of the fast plasma ions via indirect measurements. Specifically, we compute the velocity distribution of the ions in a small volume of plasma from measurements of the Doppler shift of photons emitted from the plasma when it is heated.

This inverse problem is severely underdetermined, and there is a demand for incorporating prior information about the solution to stabilize the computation of a regularized solution. A successful way to do this is to express the solution in terms of a physics-informed basis consisting of so-called slowing-down functions that reflect the statistical behavior of fast ions in the plasma.

We show that this approach is equivalent to using a general-form Tikhonov regularization term that dampens undesired high-frequency components. We demonstrate its usefulness with numerical simulations of plasma in Wendelstein 7X (Max Planck Institute) and TCV (EPFL).

Joint work with

- Mirko Salewski, DTU Physics, Technical University of Denmark
- Bo Simmendefeldt Schmidt, Dept. of Physics and Astronomy, University of California, Irvine

A primal-dual data-driven method for computational optical imaging with a photonic lantern, Audrey Repetti

Optical fibres aim to image in-vivo biological processes. In this context, high spatial resolution and stability to fibre movements are key to enable decision-making processes (e.g., for microendoscopy). Recently, a single-pixel imaging technique based on a multicore fibre photonic lantern has been designed, named computational optical imaging using a lantern (COIL). A proximal algorithm based on a sparsity prior, dubbed SARA-COIL, has been further proposed to solve the associated inverse problem, to enable image reconstructions for high resolution COIL microendoscopy.
In this work, we develop a data-driven approach for COIL. We replace the sparsity prior in the proximal algorithm by a learned denoiser, leading to a plug-and-play (PnP) algorithm. The resulting PnP method, based on a proximal primal-dual algorithm, enables to solve the Morozov formulation of the inverse problem. We use recent results in learning theory to train a network with desirable Lipschitz properties, and we show that the resulting primal-dual PnP algorithm converges to a solution to a monotone inclusion problem.
Our simulations highlight that the proposed data-driven approach improves the reconstruction quality over variational SARA-COIL method on both simulated and real data.